31

ALTERNATING CURRENT

31.1. **IDENTIFY:** The maximum current is the current amplitude, and it must not ever exceed 1.50 A.

SET UP: $I_{\text{rms}} = I/\sqrt{2}$. I is the current amplitude, the maximum value of the current.

EXECUTE: $I = 1.50\text{A}$ gives $I_{\text{rms}} = \dfrac{1.50\text{ A}}{\sqrt{2}} = 1.06\text{ A}$.

EVALUATE: The current amplitude is larger than the root-mean-square current.

31.2. **IDENTIFY** and **SET UP:** Apply Eqs. (31.3) and (31.4).

EXECUTE: **(a)** $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(2.10\text{ A}) = 2.97\text{ A}$.

(b) $I_{\text{rav}} = \dfrac{2}{\pi}I = \dfrac{2}{\pi}(2.97\text{ A}) = 1.89\text{ A}$.

EVALUATE: **(c)** The root-mean-square current is always greater than the rectified average, because squaring the current before averaging, and then taking the square root to get the root-mean-square value will always give a larger value than just averaging.

31.3. **IDENTIFY** and **SET UP:** Apply Eq. (31.5).

EXECUTE: **(a)** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0\text{ V}}{\sqrt{2}} = 31.8\text{ V}$.

(b) Since the voltage is sinusoidal, the average is zero.

EVALUATE: The voltage amplitude is larger than V_{rms}.

31.4. **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the capacitance if we know the current amplitude.

SET UP: $X_C = \dfrac{V}{I}$ and $X_C = \dfrac{1}{2\pi fC}$.

EXECUTE: **(a)** $\phi = -90°$. The source voltage lags the current by $90°$.

(b) $X_C = \dfrac{V}{I} = \dfrac{60.0\text{ V}}{5.30\text{ A}} = 11.3\ \Omega$. Solving $X_C = \dfrac{1}{2\pi fC}$ for C gives

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(80.0\text{ Hz})(11.3\ \Omega)} = 1.76\times10^{-4}\text{ F}.$$

EVALUATE: This is a $176\text{-}\mu\text{F}$ capacitor, which is not unreasonable.

31.5. **IDENTIFY:** We want the phase angle for the source voltage relative to the current, and we want the inductance if we know the current amplitude.

SET UP: $X_L = \dfrac{V}{I}$ and $X_L = 2\pi fL$.

EXECUTE: **(a)** $\phi = +90°$. The source voltage leads the current by $90°$.

(b) $X_L = \dfrac{V}{I} = \dfrac{45.0\text{ V}}{3.90\text{ A}} = 11.54\ \Omega$. Solving $X_L = 2\pi fL$ for f gives $f = \dfrac{X_L}{2\pi L} = \dfrac{11.54\ \Omega}{2\pi(9.50\times10^{-3}\text{ H})} = 193\text{ Hz}$.

EVALUATE: The angular frequency is about 1200 rad/s.

31.6. **IDENTIFY:** The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

SET UP: The reactances are $X_C = 1/\omega C$ and $X_L = \omega L.$

EXECUTE: **(a)** Equating the reactances gives $\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}$

(b) Using the numerical values we get $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5.00 \text{ mH})(3.50\ \mu\text{F})}} = 7560 \text{ rad/s}$

$$X_C = X_L = \omega L = (7560 \text{ rad/s})(5.00 \text{ mH}) = 37.8\ \Omega$$

EVALUATE: At other angular frequencies, the two reactances could be very different.

31.7. **IDENTIFY** and **SET UP:** Apply Eqs. (31.18) and (31.19).

EXECUTE: $V = IX_C$ so $X_C = \dfrac{V}{I} = \dfrac{170 \text{ V}}{0.850 \text{ A}} = 200\ \Omega$

$X_C = \dfrac{1}{\omega C}$ gives $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(60.0 \text{ Hz})(200\ \Omega)} = 1.33\times10^{-5} \text{ F} = 13.3\ \mu\text{F}$

EVALUATE: The reactance relates the voltage amplitude to the current amplitude and is similar to Ohm's law.

31.8. **IDENTIFY:** The reactance of an inductor is $X_L = \omega L = 2\pi f L.$ The reactance of a capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}.$

SET UP: The frequency *f* is in Hz.

EXECUTE: **(a)** At 60.0 Hz, $X_L = 2\pi(60.0 \text{ Hz})(0.450 \text{ H}) = 170\ \Omega.$ X_L is proportional to *f* so at 600 Hz, $X_L = 1700\ \Omega.$

(b) At 60.0 Hz, $X_C = \dfrac{1}{2\pi(60.0 \text{ Hz})(2.50\times10^{-6} \text{ F})} = 1.06\times10^{3}\ \Omega.$ X_C is proportional to $1/f$, so at 600 Hz, $X_C = 106\ \Omega.$

(c) $X_L = X_C$ says $2\pi f L = \dfrac{1}{2\pi f C}$ and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.450 \text{ H})(2.50\times10^{-6} \text{ F})}} = 150 \text{ Hz}.$

EVALUATE: X_L increases when *f* increases. X_C decreases when *f* increases.

31.9. **IDENTIFY** and **SET UP:** Use Eqs. (31.12) and (31.18).

EXECUTE: **(a)** $X_L = \omega L = 2\pi f L = 2\pi(80.0 \text{ Hz})(3.00 \text{ H}) = 1510\ \Omega$

(b) $X_L = 2\pi f L$ gives $L = \dfrac{X_L}{2\pi f} = \dfrac{120\ \Omega}{2\pi(80.0 \text{ Hz})} = 0.239 \text{ H}$

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(80.0 \text{ Hz})(4.00\times10^{-6} \text{ F})} = 497\ \Omega$

(d) $X_C = \dfrac{1}{2\pi f C}$ gives $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(80.0 \text{ Hz})(120\ \Omega)} = 1.66\times10^{-5} \text{ F}$

EVALUATE: X_L increases when *L* increases; X_C decreases when *C* increases.

31.10. **IDENTIFY:** $V_L = I\omega L$

SET UP: ω is the angular frequency, in rad/s. $f = \dfrac{\omega}{2\pi}$ is the frequency in Hz.

EXECUTE: $V_L = I\omega L$ so $f = \dfrac{V_L}{2\omega IL} = \dfrac{(12.0 \text{ V})}{2\pi(2.60\times10^{-3} \text{ A})(4.50\times10^{-4} \text{ H})} = 1.63\times10^{6} \text{ Hz}.$

EVALUATE: When *f* is increased, *I* decreases.

31.11. **IDENTIFY:** In an *L-R* ac circuit, we want to find out how the voltage across a resistor varies with time if we know how the voltage varies across the inductor.

SET UP: $v_L = -I\omega L\sin\omega t$ and $v_R = V_R\cos(\omega t)$.

EXECUTE: **(a)** $v_L = -I\omega L\sin\omega t$. $\omega = 480$ rad/s. $I\omega L = 12.0$ V.

$I = \dfrac{12.0\text{ V}}{\omega L} = \dfrac{12.0\text{ V}}{(480\text{ rad/s})(0.180\text{ H})} = 0.1389$ A. $V_R = IR = (0.1389\text{ A})(90.0\ \Omega) = 12.5$ V.

$v_R = V_R\cos(\omega t) = (12.5\text{ V})\cos[(480\text{ rad/s})t]$.

(b) $v_R = (12.5\text{ V})\cos[(480\text{ rad/s})(2.00\times10^{-3}\text{ s})] = 7.17$ V.

EVALUATE: The instantaneous voltage (7.17 V) is less than the voltage amplitude (12.5 V).

31.12. **IDENTIFY:** Compare v_C that is given in the problem to the general form $v_C = \dfrac{I}{\omega C}\sin\omega t$ and determine ω.

SET UP: $X_C = \dfrac{1}{\omega C}$. $v_R = iR$ and $i = I\cos\omega t$.

EXECUTE: **(a)** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(120\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 1736\ \Omega$

(b) $I = \dfrac{V_C}{X_C} = \dfrac{7.60\text{ V}}{1736\ \Omega} = 4.378\times10^{-3}$ A and $i = I\cos\omega t = (4.378\times10^{-3}\text{ A})\cos[(120\text{ rad/s})t]$. Then

$v_R = iR = (4.38\times10^{-3}\text{ A})(250\ \Omega)\cos((120\text{ rad/s})t) = (1.10\text{ V})\cos((120\text{ rad/s})t)$.

EVALUATE: The voltage across the resistor has a different phase than the voltage across the capacitor.

31.13. **IDENTIFY** and **SET UP:** The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in Eq. (31.12) to calculate the inductive reactance. Eq. (31.10) gives the voltage across the inductor.

EXECUTE: **(a)** $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$

$v_R = iR$, so $i = \dfrac{v_R}{R} = \left(\dfrac{3.80\text{ V}}{150\ \Omega}\right)\cos[(720\text{ rad/s})t] = (0.0253\text{ A})\cos[(720\text{ rad/s})t]$

(b) $X_L = \omega L$

$\omega = 720$ rad/s, $L = 0.250$ H, so $X_L = \omega L = (720\text{ rad/s})(0.250\text{ H}) = 180\ \Omega$

(c) If $i = I\cos\omega t$ then $v_L = V_L\cos(\omega t + 90°)$ (from Eq. 31.10).

$V_L = I\omega L = IX_L = (0.02533\text{ A})(180\ \Omega) = 4.56$ V

$v_L = (4.56\text{ V})\cos[(720\text{ rad/s})t + 90°]$

But $\cos(a + 90°) = -\sin a$ (Appendix B), so $v_L = -(4.56\text{ V})\sin[(720\text{ rad/s})t]$.

EVALUATE: The current is the same in the resistor and inductor and the voltages are $90°$ out of phase, with the voltage across the inductor leading.

31.14. **IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan\phi = \dfrac{X_L}{R}$. $X_L = \omega L$. $I = \dfrac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$.

For an inductor, the voltage leads the current.

EXECUTE: **(a)** $X_L = \omega L = (250\text{ rad/s})(0.400\text{ H}) = 100\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (100\ \Omega)^2} = 224\ \Omega$.

(b) $I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{224\ \Omega} = 0.134$ A

(c) $V_R = IR = (0.134\text{ A})(200\ \Omega) = 26.8$ V. $V_L = IX_L = (0.134\text{ A})(100\ \Omega) = 13.4$ V.

(d) $\tan\phi = \dfrac{X_L}{R} = \dfrac{100\ \Omega}{200\ \Omega}$ and $\phi = +26.6°$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.14.

EVALUATE: Note that $V_R + V_L$ is greater than V. The loop rule is satisfied at each instance of time but the voltages across R and L reach their maxima at different times.

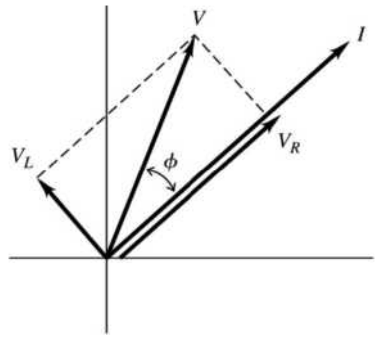


Figure 31.14

31.15. **IDENTIFY:** Apply the equations in Section 31.3.

SET UP: $\omega = 250$ rad/s, $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F and $V = 30.0$ V.

EXECUTE: **(a)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})(0.400\text{ H}) - 1/((250\text{ rad/s})(6.00\times 10^{-6}\text{ F})))^2} = 601\ \Omega$

(b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499$ A.

(c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6°$, and the voltage lags the current.

(d) $V_R = IR = (0.0499\text{ A})(200\ \Omega) = 9.98$ V; $V_L = I\omega L = (0.0499\text{ A})(250\text{ rad/s})(0.400\text{ H}) = 4.99$ V;

$V_C = \dfrac{I}{\omega C} = \dfrac{(0.0499\text{ A})}{(250\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 33.3$ V.

EVALUATE: **(e)** At any instant, $v = v_R + v_C + v_L$. But v_C and v_L are 180° out of phase, so v_C can be larger than v at a value of t, if $v_L + v_R$ is negative at that t.

31.16. **IDENTIFY:** For an *L-R-C* series ac circuit, we want to find the voltages and voltage amplitudes across all the circuit elements.

SET UP: $X_C = \dfrac{1}{\omega C}$, $X_L = \omega L$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $I = \dfrac{V}{Z}$ and $\tan\phi = \dfrac{X_L - X_C}{R}$. The instantaneous voltages are $v_R = V_R\cos(\omega t) = IR\cos(\omega t)$, $v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t)$, $v_C = V_C\sin(\omega t) = IX_C\sin(\omega t)$ and $v = V\cos(\omega t + \phi)$.

EXECUTE: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 666.7\ \Omega$.

$X_L = \omega L = (250\text{ rad/s})(0.900\text{ H}) = 225\ \Omega$.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (225\ \Omega - 666.7\ \Omega)^2} = 484.9\ \Omega$.

$I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{484.9\ \Omega} = 0.06187\text{ A} = 61.87$ mA.

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{225\ \Omega - 666.7\ \Omega}{200\ \Omega} = -2.2085$ and $\phi = -1.146$ rad.

(a) $v_R = V_R\cos(\omega t) = IR\cos(\omega t) = (0.06187\text{ A})(200\ \Omega)\cos[(250\text{ rad/s})(20.0\times 10^{-3}\text{ s})] = 3.51$ V.

$v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t) = -(0.06187\text{ A})(225\ \Omega)\sin[(250\text{ rad/s})(20.0\times 10^{-3}\text{ s})] = 13.35$ V.

$v_C = V_C \sin(\omega t) = IX_C \sin(\omega t) = (0.06187\text{ A})(666.7\ \Omega)\sin[(250\text{ rad/s})(20.0\times10^{-3}\text{ s})] = -39.55\text{ V}.$

$v = V\cos(\omega t + \phi) = (30.0\text{ V})\cos[(250\text{ rad/s})(20.0\times10^{-3}\text{ s}) - 1.146\text{ rad}] = -22.70\text{ V}.$

$v_R + v_L + v_C = 3.51\text{ V} + 13.35\text{ V} + (-39.55\text{ V}) = -22.7\text{ V}.$ $v_R + v_L + v_C$ is equal to v.

(b) $V_R = IR = 12.4\text{ V}.$ $V_C = 41.2\text{ V}.$ $V_L = 13.9\text{ V}.$

$V_R + V_C + V_L = 12.4\text{ V} + 41.2\text{ V} + 13.9\text{ V} = 67.5\text{ V}.$ $V_R + V_C + V_L$ is not equal to V.

EVALUATE: The instantaneous voltages do add up to v because they all occur at the same time, so they must add to v by Kirchhoff's loop rule. The amplitudes do not add to V because the maxima do not occur at the same time due to phase differences between the inductor, capacitor and resistor.

31.17. **IDENTIFY** and **SET UP:** Use the equation that preceeds Eq. (31.20): $V^2 = V_R^2 + (V_L - V_C)^2$

EXECUTE: $V = \sqrt{(30.0\text{ V})^2 + (50.0\text{ V} - 90.0\text{ V})^2} = 50.0\text{ V}$

EVALUATE: The equation follows directly from the phasor diagrams of Fig. 31.13 (b or c) in the textbook. Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

31.18. **IDENTIFY:** For an *L-R* ac circuit, we want to use the resistance, voltage amplitude of the source and power in the resistor to find the impedance, the voltage amplitude across the inductor and the power factor.

SET UP: $P_{av} = \frac{1}{2}I^2R,\ Z = \frac{V}{I},\ V_R = IR,$ and $\tan\phi = \frac{X_L}{R}.$

EXECUTE: **(a)** $P_{av} = \frac{1}{2}I^2R.$ $I = \sqrt{\frac{2P_{av}}{R}} = \sqrt{\frac{2(216\text{ W})}{300\ \Omega}} = 1.20\text{ A}.$ $Z = \frac{V}{I} = \frac{500\text{ V}}{1.20\text{ A}} = 417\ \Omega.$

(b) $V_R = IR = (1.20\text{ A})(300\ \Omega) = 360\text{ V}.$ $V_L = \sqrt{V^2 - V_R^2} = \sqrt{(500\text{ V})^2 - (360\text{ V})^2} = 347\text{ V}.$

(c) $\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{347\text{ V}}{360\text{ V}}$ gives $\phi = 43.95°.$ The power factor is $\cos\phi = 0.720.$

EVALUATE: The voltage amplitude across the resistor cannot exceed the voltage amplitude (500 V) of the ac source.

31.19. **IDENTIFY:** For a pure resistance, $P_{av} = V_{rms}I_{rms} = I_{rms}^2R.$

SET UP: 20.0 W is the average power P_{av}.

EXECUTE: **(a)** The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

(b) $I_{rms} = \frac{P_{av}}{V_{rms}} = \frac{20.0\text{ W}}{120\text{ V}} = 0.167\text{ A}$

(c) $R = \frac{P_{av}}{I_{rms}^2} = \frac{20.0\text{ W}}{(0.167\text{ A})^2} = 720\ \Omega$

EVALUATE: We can also calculate the average power as $P_{av} = \frac{V_{R,rms}^2}{R} = \frac{V_{rms}^2}{R} = \frac{(120\text{ V})^2}{720\ \Omega} = 20.0\text{ W}.$

31.20. **IDENTIFY:** The average power supplied by the source is $P_{av} = V_{rms}I_{rms}\cos\phi.$ The power consumed in the resistance is $P_{av} = I_{rms}^2R.$

SET UP: $\omega = 2\pi f = 2\pi(1.25\times10^3\text{ Hz}) = 7.854\times10^3\text{ rad/s}.$ $X_L = \omega L = 157\ \Omega.$ $X_C = \frac{1}{\omega C} = 909\ \Omega.$

EXECUTE: **(a)** First, let us find the phase angle between the voltage and the current:

$\tan\phi = \frac{X_L - X_C}{R} = \frac{157\ \Omega - 909\ \Omega}{350\ \Omega}$ and $\phi = -65.04°.$ The impedance of the circuit is

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350\ \Omega)^2 + (-752\ \Omega)^2} = 830\ \Omega.$ The average power provided by the generator is then $P_{av} = V_{rms}I_{rms}\cos(\phi) = \frac{V_{rms}^2}{Z}\cos(\phi) = \frac{(120\text{ V})^2}{830\ \Omega}\cos(-65.04°) = 7.32\text{ W}.$

(b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\dfrac{120\text{ V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\text{ W}.$

EVALUATE: Conservation of energy requires that the answers to parts (a) and (b) are equal.

31.21. **IDENTIFY:** Relate the power factor to R and Z for an L-R-C series ac circuit. Then use this result to find the voltage amplitude across a resistor.

SET UP and **EXECUTE:** **(a)** From Figure 31.13(a) or (b), $\cos\phi = \dfrac{IR}{IZ} = \dfrac{R}{Z}.$

(b) Using the result from (a) gives $Z = \dfrac{R}{\cos\phi}.$ $I = \dfrac{V}{Z} = \dfrac{V\cos\phi}{R}.$

$V_R = IR = V\cos\phi = (90.0\text{ V})\cos(-31.5°) = 76.7\text{ V}.$

EVALUATE: The voltage amplitude for the resistor is less than the voltage amplitude of the ac source.

31.22. **IDENTIFY:** We want to relate the average power delivered by the source in an L-R-C circuit to the rms current and resistance.

SET UP: From Exercise 31.21 we know that the power factor is $\cos\phi = \dfrac{R}{Z}.$ We also know that $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi.$

EXECUTE: **(a)** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi.$ $\cos\phi = \dfrac{R}{Z}$ so $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \dfrac{R}{Z}.$ But $\dfrac{V_{\text{rms}}}{Z} = I_{\text{rms}}$ so $P_{\text{av}} = I_{\text{rms}}^2 R.$

(b) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R}$ and $V_{\text{rms}} = V/\sqrt{2},$ so $P_{\text{av}} = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{\left(\dfrac{36.0\text{ V}}{\sqrt{2}}\right)^2}{96.0\ \Omega} = 6.75\text{ W}.$

EVALUATE: The instantaneous power can be greater than 6.75 W at times, but it can also be less than that at other times, giving an average of 6.75 W.

31.23. **IDENTIFY** and **SET UP:** Use the equations of Section 31.3 to calculate ϕ, Z and V_{rms}. The average power delivered by the source is given by Eq. (31.31) and the average power dissipated in the resistor is $I_{\text{rms}}^2 R.$

EXECUTE: **(a)** $X_L = \omega L = 2\pi f L = 2\pi(400\text{ Hz})(0.120\text{ H}) = 301.6\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(400\text{ Hz})(7.3\times10^{-6}\text{ F})} = 54.51\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{301.6\ \Omega - 54.41\ \Omega}{240\ \Omega},$ so $\phi = +45.8°.$ The power factor is $\cos\phi = +0.697.$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240\ \Omega)^2 + (301.6\ \Omega - 54.51\ \Omega)^2} = 344\ \Omega$

(c) $V_{\text{rms}} = I_{\text{rms}} Z = (0.450\text{ A})(344\ \Omega) = 155\text{ V}$

(d) $P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = (0.450\text{ A})(155\text{ V})(0.697) = 48.6\text{ W}$

(e) $P_{\text{av}} = I_{\text{rms}}^2 R = (0.450\text{ A})^2 (240\ \Omega) = 48.6\text{ W}$

EVALUATE: The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

(f) All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

(g) The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

31.24. **IDENTIFY** and **SET UP:** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi.$ $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}.$ $\cos\phi = \dfrac{R}{Z}.$

EXECUTE: $I_{\text{rms}} = \dfrac{80.0\text{ V}}{105\ \Omega} = 0.762\text{ A}.$ $\cos\phi = \dfrac{75.0\ \Omega}{105\ \Omega} = 0.714.$ $P_{\text{av}} = (80.0\text{ V})(0.762\text{ A})(0.714) = 43.5\text{ W}.$

EVALUATE: Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{\text{av}} = I_{\text{rms}}^2 R = (0.762\text{ A})^2 (75.0\ \Omega) = 43.5\text{ W}.$

31.25. **IDENTIFY:** The angular frequency and the capacitance can be used to calculate the reactance X_C of the capacitor. The angular frequency and the inductance can be used to calculate the reactance X_L of the inductor. Calculate the phase angle ϕ and then the power factor is $\cos\phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{av} = V_{rms}I_{rms}\cos\phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

SET UP: The source has rms voltage $V_{rms} = \frac{V}{\sqrt{2}} = \frac{45\ \text{V}}{\sqrt{2}} = 31.8\ \text{V}$.

EXECUTE: **(a)** $X_L = \omega L = (360\ \text{rad/s})(15\times10^{-3}\ \text{H}) = 5.4\ \Omega$.

$X_C = \frac{1}{\omega C} = \frac{1}{(360\ \text{rad/s})(3.5\times10^{-6}\ \text{F})} = 794\ \Omega$. $\tan\phi = \frac{X_L - X_C}{R} = \frac{5.4\ \Omega - 794\ \Omega}{250\ \Omega}$ and $\phi = -72.4°$.

The power factor is $\cos\phi = 0.302$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\ \Omega)^2 + (5.4\ \Omega - 794\ \Omega)^2} = 827\ \Omega$. $I_{rms} = \frac{V_{rms}}{Z} = \frac{31.8\ \text{V}}{827\ \Omega} = 0.0385\ \text{A}$.

$P_{av} = V_{rms}I_{rms}\cos\phi = (31.8\ \text{V})(0.0385\ \text{A})(0.302) = 0.370\ \text{W}$.

(c) The average power delivered to the resistor is $P_{av} = I_{rms}^2 R = (0.0385\ \text{A})^2(250\ \Omega) = 0.370\ \text{W}$. The average power delivered to the capacitor and to the inductor is zero.

EVALUATE: On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

31.26. **IDENTIFY:** At resonance in an *L-R-C* ac circuit, we know the reactance of the capacitor and the voltage amplitude across it. From this information, we want to find the voltage amplitude of the source.

SET UP: At resonance, $Z = R$. $V_C = IX_C$.

EXECUTE: $I = \frac{V}{X} = \frac{600\ \text{V}}{200\ \Omega} = 3.00\ \text{A}$. $Z = R = 300\ \Omega$. $V = IZ = (3.00\ \text{A})(300\ \Omega) = 900\ \text{V}$.

EVALUATE: At resonance, $Z = R$, but X_C is not zero.

31.27. **IDENTIFY** and **SET UP:** The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R$. For part (b), calculate *Z* and use $I = V/Z$.

EXECUTE: **(a)** $f_0 = \frac{1}{2\pi\sqrt{LC}} = 113\ \text{Hz}$. $I = V/R = 15.0\ \text{mA}$.

(b) $X_C = 1/\omega C = 500\ \Omega$. $X_L = \omega L = 160\ \Omega$.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega$. $I = V/Z = 7.61\ \text{mA}$. $X_C > X_L$ so the source voltage lags the current.

EVALUATE: $\omega_0 = 2\pi f_0 = 710\ \text{rad/s}$. $\omega = 400\ \text{rad/s}$ and is less than ω_0. When $\omega < \omega_0$, $X_C > X_L$. Note that *I* in part (b) is less than *I* in part (a).

31.28. **IDENTIFY:** The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

SET UP: The impedance is $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$, the current amplitude is $I = V/Z$ and the instantaneous values of the potential and current are $v = V\cos(\omega t + \phi)$, where $\tan\phi = (X_L - X_C)/R$, and $i = I\cos\omega t$.

EXECUTE: **(a)** *Z* is a minimum when $\omega L = \frac{1}{\omega C}$, which gives

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(8.00\ \text{mH})(12.5\ \mu\text{F})}} = 3162\ \text{rad/s}$, which rounds to 3160 rad/s. $Z = R = 175\ \Omega$.

(b) $I = V/Z = (25.0\ \text{V})/(175\ \Omega) = 0.143\ \text{A}$

(c) $i = I\cos\omega t = I/2$, so $\cos\omega t = \frac{1}{2}$, which gives $\omega t = 60° = \pi/3\ \text{rad}$. $v = V\cos(\omega t + \phi)$, where $\tan\phi = (X_L - X_C)/R = 0/R = 0$. So, $v = (25.0\ \text{V})\cos\omega t = (25.0\ \text{V})(1/2) = 12.5\ \text{V}$.

$v_R = Ri = (175\ \Omega)(1/2)(0.143\text{ A}) = 12.5\text{ V}.$

$v_C = V_C\cos(\omega t - 90°) = IX_C\cos(\omega t - 90°) = \dfrac{0.143\text{ A}}{(3162\text{ rad/s})(12.5\ \mu\text{F})}\cos(60° - 90°) = +3.13\text{ V}.$

$v_L = V_L\cos(\omega t + 90°) = IX_L\cos(\omega t + 90°) = (0.143\text{ A})(3162\text{ rad/s})(8.00\text{ mH})\cos(60° + 90°).$

$v_L = -3.13\text{ V}.$

(d) $v_R + v_L + v_C = 12.5\text{ V} + (-3.13\text{ V}) + 3.13\text{ V} = 12.5\text{ V} = v_{\text{source}}$

EVALUATE: The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are $180°$ out of phase, so they add to zero, leaving all the potential difference across the resistor.

31.29. **IDENTIFY** and **SET UP:** At the resonance frequency, $Z = R$. Use that $V = IZ$, $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$. P_{av} is given by Eq. (31.31).

(a) EXECUTE: $V = IZ = IR = (0.500\text{ A})(300\ \Omega) = 150\text{ V}$

(b) $V_R = IR = 150\text{ V}$

$X_L = \omega L = L(1/\sqrt{LC}) = \sqrt{L/C} = 2582\ \Omega;\ V_L = IX_L = 1290\text{ V}$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega;\ V_C = IX_C = 1290\text{ V}$

(c) $P_{\text{av}} = \frac{1}{2}VI\cos\phi = \frac{1}{2}I^2R$, since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{\text{av}} = \frac{1}{2}(0.500\text{ A})^2(300\ \Omega) = 37.5\text{ W}$

EVALUATE: At resonance $V_L = V_C$. Note that $V_L + V_C > V$. However, at any instant $v_L + v_C = 0$.

31.30. **IDENTIFY:** The current is maximum at the resonance frequency, so choose C such that $\omega = 50.0\text{ rad/s}$ is the resonance frequency. At the resonance frequency $Z = R$.

SET UP: $V_L = I\omega L$

EXECUTE: **(a)** The amplitude of the current is given by $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$. Thus, the current will have a maximum amplitude when $\omega L = \dfrac{1}{\omega C}$. Therefore, $C = \dfrac{1}{\omega^2 L} = \dfrac{1}{(50.0\text{ rad/s})^2(9.00\text{ H})} = 44.4\ \mu\text{F}.$

(b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \dfrac{V}{R} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}$. Thus, the amplitude of the voltage across the inductor is

$V_L = I(\omega L) = (0.300\text{ A})(50.0\text{ rad/s})(9.00\text{ H}) = 135\text{ V}.$

EVALUATE: Note that V_L is greater than the source voltage amplitude.

31.31. **IDENTIFY** and **SET UP:** At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150\ \Omega$, $L = 0.750\text{ H}$, $C = 0.0180\ \mu\text{F}$, $V = 150\text{ V}$

EXECUTE: **(a)** At the resonance frequency $X_L = X_C$ and from $\tan\phi = \dfrac{X_L - X_C}{R}$ we have that $\phi = 0°$ and the power factor is $\cos\phi = 1.00$.

(b) $P_{\text{av}} = \frac{1}{2}VI\cos\phi$ (Eq. 31.31)

At the resonance frequency $Z = R$, so $I = \dfrac{V}{Z} = \dfrac{V}{R}$.

$$P_{\text{av}} = \tfrac{1}{2}V\left(\frac{V}{R}\right)\cos\phi = \tfrac{1}{2}\frac{V^2}{R} = \tfrac{1}{2}\frac{(150\text{ V})^2}{150\ \Omega} = 75.0\text{ W}$$

(c) EVALUATE: When C and f are changed but the circuit is kept on resonance, nothing changes in $P_{av} = V^2/(2R)$, so the average power is unchanged: $P_{av} = 75.0$ W. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

31.32. IDENTIFY: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V_C = IX_C$. $V = IZ$.

SET UP: At resonance, $Z = R$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.350\text{ H})(0.0120\times 10^{-6}\text{ F})}} = 1.54\times 10^4$ rad/s

(b) $V = IZ = \left(\dfrac{V_C}{X_C}\right)Z = \left(\dfrac{V_C}{X_C}\right)R$. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(1.54\times 10^4\text{ rad/s})(0.0120\times 10^{-6}\text{ F})} = 5.41\times 10^3\ \Omega$.

$V = \left(\dfrac{550\text{ V}}{5.41\times 10^3\ \Omega}\right)(400\ \Omega) = 40.7$ V.

EVALUATE: The voltage amplitude for the capacitor is more than a factor of 10 times greater than the voltage amplitude of the source.

31.33. IDENTIFY and SET UP: The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $X_L = \omega L$. $X_C = \dfrac{1}{\omega C}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At the resonance frequency $X_L = X_C$ and $Z = R$.

EXECUTE: **(a)** $Z = R = 115\ \Omega$

(b) $\omega_0 = \dfrac{1}{\sqrt{(4.50\times 10^{-3}\text{ H})(1.26\times 10^{-6}\text{ F})}} = 1.33\times 10^4$ rad/s. $\omega = 2\omega_0 = 2.66\times 10^4$ rad/s.

$X_L = \omega L = (2.66\times 10^4\text{ rad/s})(4.50\times 10^{-3}\text{ H}) = 120\ \Omega$. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(2.66\times 10^4\text{ rad/s})(1.25\times 10^{-6}\text{ F})} = 30\ \Omega$

$Z = \sqrt{(115\ \Omega)^2 + (120\ \Omega - 30\ \Omega)^2} = 146\ \Omega$

(c) $\omega = \omega_0/2 = 6.65\times 10^3$ rad/s. $X_L = 30\ \Omega$. $X_C = \dfrac{1}{\omega C} = 120\ \Omega$.

$Z = \sqrt{(115\ \Omega)^2 + (30\ \Omega - 120\ \Omega)^2} = 146\ \Omega$, the same value as in part (b).

EVALUATE: For $\omega = 2\omega_0$, $X_L > X_C$. For $\omega = \omega_0/2$, $X_L < X_C$. But $(X_L - X_C)^2$ has the same value at these two frequencies, so Z is the same.

31.34. IDENTIFY: At resonance $Z = R$ and $X_L = X_C$.

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.280\text{ H})(4.00\times 10^{-6}\text{ F})}} = 945$ rad/s.

(b) $I = 1.70$ A at resonance, so $R = Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{1.70\text{ A}} = 70.6\ \Omega$

(c) At resonance, $V_R = 120$ V, $V_L = V_C = I\omega L = (1.70\text{ A})(945\text{ rad/s})(0.280\text{ H}) = 450$ V.

EVALUATE: At resonance, $V_R = V$ and $V_L - V_C = 0$.

31.35. IDENTIFY and SET UP: Eq. (31.35) relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{rms}^2 = V_{rms}^2/R$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

EXECUTE: **(a)** $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ so $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{120\text{ V}}{12.0\text{ V}} = 10$

(b) $I_2 = \frac{V_2}{R} = \frac{12.0\text{ V}}{5.00\ \Omega} = 2.40\text{ A}$

(c) $P_{\text{av}} = I_2^2 R = (2.40\text{ A})^2(5.00\ \Omega) = 28.8\text{ W}$

(d) The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$$P_{\text{av}} = \frac{V_1^2}{R} \text{ so } R = \frac{V_1^2}{P_{\text{av}}} = \frac{(120\text{ V})^2}{28.8\text{ W}} = 500\ \Omega$$

And $\left(\frac{N_1}{N_2}\right)^2 (5.00\ \Omega) = (10)^2(5.00\ \Omega) = 500\ \Omega,$ as was to be shown.

EVALUATE: The resistance is "transformed." A load of resistance R connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

31.36. IDENTIFY: $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}$ and $P_{\text{av},1} = P_{\text{av},2}$. $\frac{N_1}{N_2} = \frac{V_1}{V_2}$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

SET UP: $V_1 = 120\text{ V}$. $V_2 = 13{,}000\text{ V}$.

EXECUTE: **(a)** $\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{13{,}000\text{ V}}{120\text{ V}} = 108$

(b) $P_{\text{av}} = V_2 I_2 = (13{,}000\text{ V})(8.50 \times 10^{-3}\text{ A}) = 110\text{ W}$

(c) $I_1 = \frac{P_{\text{av}}}{V_1} = \frac{110\text{ W}}{120\text{ V}} = 0.917\text{ A}$

EVALUATE: Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

31.37. IDENTIFY: Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary. A transformer transforms voltages according to $\frac{V_2}{V_1} = \frac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance R is $R_{\text{eff}} = \frac{R}{(N_2/N_1)^2}$. Resistance R is related to P_{av} and V_{rms} by $P_{\text{av}} = \frac{V_{\text{rms}}^2}{R}$. Conservation of energy requires $P_{\text{av},1} = P_{\text{av},2}$ so $V_1 I_1 = V_2 I_2$.

SET UP: Let $V_1 = 240\text{ V}$ and $V_2 = 120\text{ V}$, so $P_{2,\text{av}} = 1600\text{ W}$. These voltages are rms.

EXECUTE: **(a)** $V_1 = 240\text{ V}$ and we want $V_2 = 120\text{ V}$, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

(b) $P_{\text{av}} = V_1 I_1$, so $I_1 = \frac{P_{\text{av}}}{V_1} = \frac{1600\text{ W}}{240\text{ V}} = 6.67\text{ A}.$

(c) The resistance R of the blower is $R = \frac{V_1^2}{P_{\text{av}}} = \frac{(120\text{ V})^2}{1600\text{ W}} = 9.00\ \Omega.$ The effective resistance of the blower is $R_{\text{eff}} = \frac{9.00\ \Omega}{(1/2)^2} = 36.0\ \Omega.$

EVALUATE: $I_2 V_2 = (13.3\text{ A})(120\text{ V}) = 1600\text{ W}.$ Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

31.38. IDENTIFY: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.

SET UP: The woofer has a R and L in series and the tweeter has a R and C in series.

EXECUTE: **(a)** $Z_{\text{tweeter}} = \sqrt{R^2 + (1/\omega C)^2}$

(b) $Z_{\text{woofer}} = \sqrt{R^2 + (\omega L)^2}$

(c) If $Z_{\text{tweeter}} = Z_{\text{woofer}}$, then the current splits evenly through each branch.

(d) At the crossover point, where currents are equal, $R^2 + (1/\omega C^2) = R^2 + (\omega L)^2$. $\omega = \dfrac{1}{\sqrt{LC}}$ and

$f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$.

EVALUATE: The crossover frequency corresponds to the resonance frequency of a *R-C-L* circuit, since the crossover frequency is where $X_L = X_C$.

31.39. **IDENTIFY** and **SET UP:** Use Eq. (31.24) to relate *L* and *R* to ϕ. The voltage across the coil leads the current in it by $52.3°$, so $\phi = +52.3°$.

EXECUTE: $\tan\phi = \dfrac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus

$\tan\phi = \dfrac{X_L}{R}$ and $X_L = R\tan\phi = (48.0\ \Omega)\tan 52.3° = 62.1\ \Omega$.

$X_L = \omega L = 2\pi f L$ so $L = \dfrac{X_L}{2\pi f} = \dfrac{62.1\ \Omega}{2\pi(80.0\text{ Hz})} = 0.124\text{ H}$.

EVALUATE: $\phi > 45°$ when $(X_L - X_C) > R$, which is the case here.

31.40. **IDENTIFY:** $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$. $V_{\text{rms}} = I_{\text{rms}}R$. $V_{C,\text{rms}} = I_{\text{rms}}X_C$. $V_{L,\text{rms}} = I_{\text{rms}}X_L$.

SET UP: $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{30.0\text{ V}}{\sqrt{2}} = 21.2\text{ V}$.

EXECUTE: **(a)** $\omega = 200$ rad/s, so $X_L = \omega L = (200\text{ rad/s})(0.400\text{ H}) = 80.0\ \Omega$ and

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(200\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 833\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (80.0\ \Omega - 833\ \Omega)^2} = 779\ \Omega$.

$I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\text{ V}}{779\ \Omega} = 0.0272\text{ A}$. V_1 reads $V_{R,\text{rms}} = I_{\text{rms}}R = (0.0272\text{ A})(200\ \Omega) = 5.44\text{ V}$. V_2 reads $V_{L,\text{rms}} = I_{\text{rms}}X_L = (0.0272\text{ A})(80.0\ \Omega) = 2.18\text{ V}$. V_3 reads $V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.0272\text{ A})(833\ \Omega) = 22.7\text{ V}$.

V_4 reads $\left|\dfrac{V_L - V_C}{\sqrt{2}}\right| = |V_{L,\text{rms}} - V_{C,\text{rms}}| = |2.18\text{ V} - 22.7\text{ V}| = 20.5\text{ V}$. V_5 reads $V_{\text{rms}} = 21.2\text{ V}$.

(b) $\omega = 1000$ rad/s so $X_L = \omega L = (5)(80.0\ \Omega) = 400\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{833\ \Omega}{5} = 167\ \Omega$.

$Z = \sqrt{(200\ \Omega)^2 + (400\ \Omega - 167\ \Omega)^2} = 307\ \Omega$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\text{ V}}{307\ \Omega} = 0.0691\text{ A}$. V_1 reads $V_{R,\text{rms}} = 13.8\text{ V}$.

V_2 reads $V_{L,\text{rms}} = 27.6\text{ V}$. V_3 reads $V_{C,\text{rms}} = 11.5\text{ V}$.

V_4 reads $|V_{L,\text{rms}} - V_{C,\text{rms}}| = |27.6\text{ V} - 11.5\text{ V}| = 16.1\text{ V}$. V_5 reads $V_{\text{rms}} = 21.2\text{ V}$.

EVALUATE: The resonance frequency for this circuit is $\omega_0 = \dfrac{1}{\sqrt{LC}} = 645$ rad/s. 200 rad/s is less than the resonance frequency and $X_C > X_L$. 1000 rad/s is greater than the resonance frequency and $X_L > X_C$.

31.41. **IDENTIFY:** We can use geometry to calculate the capacitance and inductance, and then use these results to calculate the resonance angular frequency.

SET UP: The capacitance of an air-filled parallel plate capacitor is $C = \dfrac{\varepsilon_0 A}{d}$. The inductance of a long solenoid is $L = \dfrac{\mu_0 A N^2}{l}$. The inductor has $N = (125\text{ coils/cm})(9.00\text{ cm}) = 1125$ coils. The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$. $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$.

EXECUTE: $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.50\times10^{-2}\ \text{m})^2}{8.00\times10^{-3}\ \text{m}} = 2.24\times10^{-12}\ \text{F}.$

$L = \dfrac{\mu_0 AN^2}{l} = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.250\times10^{-2}\ \text{m})^2(1125)^2}{9.00\times10^{-2}\ \text{m}} = 3.47\times10^{-4}\ \text{H}.$

$\omega_0 = \dfrac{1}{\sqrt{(3.47\times10^{-4}\ \text{H})(2.24\times10^{-12}\ \text{F})}} = 3.59\times10^7\ \text{rad/s}.$

EVALUATE: The result is a rather high angular frequency.

31.42. **IDENTIFY:** Use geometry to calculate the self-inductance of the toroidal solenoid. Then find its reactance and use this to find the impedance, and finally the current amplitude, of the circuit.

SET UP: $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, $X_L = 2\pi f L$, $Z = \sqrt{R^2 + X_L^2}$, and $I = V/Z$.

EXECUTE: $L = \dfrac{\mu_0 N^2 A}{2\pi r} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(2900)^2(0.450\times10^{-4}\ \text{m}^2)}{9.00\times10^{-2}\ \text{m}} = 8.41\times10^{-4}\ \text{H}.$

$X_L = 2\pi f L = (2\pi)(365\ \text{Hz})(8.41\times10^{-4}\ \text{H}) = 1.929\ \Omega.$ $Z = \sqrt{R^2 + X_L^2} = 3.40\ \Omega.$ $I = \dfrac{V}{Z} = \dfrac{24.0\ \text{V}}{3.40\ \Omega} = 7.06\ \text{A}.$

EVALUATE: The inductance is physically reasonable.

31.43. **IDENTIFY:** An L-R-C ac circuit operates at resonance. We know L, C, and V and want to find R.

SET UP: At resonance, $Z = R$ and $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$. $X_C = \dfrac{1}{\omega C}$, $I = V/Z$.

EXECUTE: $\omega = \dfrac{1}{\sqrt{LC}} = 633.0\ \text{rad/s}$ $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(633\ \text{rad/s})(4.80\times10^{-6}\ \text{F})} = 329.1\ \Omega.$

$I = \dfrac{V_C}{X_C} = \dfrac{80.0\ \text{V}}{329.1\ \Omega} = 0.2431\ \text{A}.$ At resonance $Z = R$, so $I = \dfrac{V}{R}$. $R = \dfrac{V}{I} = \dfrac{56.0\ \text{V}}{0.2431\ \text{A}} = 230\ \Omega.$

EVALUATE: At resonance, the impedance is a minimum.

31.44. **IDENTIFY:** $X_L = \omega L$. $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$

SET UP: $f = 120\ \text{Hz}$; $\omega = 2\pi f$.

EXECUTE: **(a)** $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega} = \dfrac{250\ \Omega}{2\pi(120\ \text{Hz})} = 0.332\ \text{H}$

(b) $Z = \sqrt{R^2 + X_L^2} = \sqrt{(400\ \Omega)^2 + (250\ \Omega)^2} = 472\ \Omega.$ $\cos\phi = \dfrac{R}{Z}$ and $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$. $P_{\text{av}} = \dfrac{V_{\text{rms}}^2}{Z}\dfrac{R}{Z}$, so

$V_{\text{rms}} = Z\sqrt{\dfrac{P_{\text{av}}}{R}} = (472\ \Omega)\sqrt{\dfrac{800\ \text{W}}{400\ \Omega}} = 668\ \text{V}.$

EVALUATE: $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{668\ \text{V}}{472\ \Omega} = 1.415\ \text{A}.$ We can calculate P_{av} as

$I_{\text{rms}}^2 R = (1.415\ \text{A})^2(400\ \Omega) = 800\ \text{W},$ which checks.

31.45. **(a) IDENTIFY** and **SET UP:** Source voltage lags current so it must be that $X_C > X_L$ and we must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional L needed to raise X_L to equal X_C.

(b) EXECUTE: Power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z\cos\phi = (60.0\ \Omega)(0.720) = 43.2\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi$

$\cos\phi = 0.720$ gives $\phi = -43.95°$ (ϕ is negative since the voltage lags the current)

Then $X_L - X_C = R\tan\phi = (43.2\ \Omega)\tan(-43.95°) = -41.64\ \Omega.$

Therefore need to add $41.64\ \Omega$ of X_L.

$X_L = \omega L = 2\pi fL$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{41.64\ \Omega}{2\pi(50.0\ \text{Hz})} = 0.133\ \text{H}$, amount of inductance to add.

EVALUATE: From the information given we can't calculate the original value of L in the circuit, just how much to add. When this L is added the current in the circuit will increase.

31.46. **IDENTIFY:** We know $R, X_L, X_C,$ and V_L for a series L-R-C ac circuit. We want to find V_R, V_C, V and the power delivered by the source.

SET UP: $I = \dfrac{V_L}{X_L},\ V = IX,\ P_{av} = I_{rms}^2 R.$

EXECUTE: **(a)** $I = \dfrac{V_L}{X_L} = \dfrac{450\ \text{V}}{900\ \Omega} = 0.500\ \text{A}.$ $V_R = IR = (0.500\ \text{A})(300\ \Omega) = 150\ \text{V}.$

(b) $V_C = IX_C = (0.500\ \text{A})(500\ \Omega) = 250\ \text{V}.$

(c) $V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(150\ \text{V})^2 + (450\ \text{V} - 250\ \text{V})^2} = 250\ \text{V}.$

(d) $P_{av} = I_{rms}^2 R = \dfrac{1}{2}I^2 R = \dfrac{1}{2}\dfrac{V_R^2}{R} = \dfrac{1}{2}\dfrac{(150\ \text{V})^2}{300\ \Omega} = 37.5\ \text{W}.$

EVALUATE: The voltage amplitude of the source is not the sum of the voltage amplitudes of the other circuit elements since the voltages have their maxima at different times and are hence out of phase.

31.47. **IDENTIFY:** We know the impedances and the average power consumed. From these we want to find the power factor and the rms voltage of the source.

SET UP: $P = I_{rms}^2 R.$ $\cos\phi = \dfrac{R}{Z}.$ $Z = \sqrt{R^2 + (X_L - X_C)^2}.$ $V_{rms} = I_{rms}Z.$

EXECUTE: **(a)** $I_{rms} = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{60.0\ \text{W}}{300\ \Omega}} = 0.447\ \text{A}.$ $Z = \sqrt{(300\ \Omega)^2 + (500\ \Omega - 300\ \Omega)^2} = 361\ \Omega.$

$\cos\phi = \dfrac{R}{Z} = \dfrac{300\ \Omega}{361\ \Omega} = 0.831.$

(b) $V_{rms} = I_{rms}Z = (0.447\ \text{A})(361\ \Omega) = 161\ \text{V}.$

EVALUATE: The voltage amplitude of the source is $V_{rms}\sqrt{2} = 228\ \text{V}.$

31.48. **IDENTIFY:** Use $V_{rms} = I_{rms}Z$ to calculate Z and then find R. $P_{av} = I_{rms}^2 R$

SET UP: $X_C = 50.0\ \Omega$

EXECUTE: $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{240\ \text{V}}{3.00\ \text{A}} = 80.0\ \Omega = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (50.0\ \Omega)^2}.$ Thus,

$R = \sqrt{(80.0\ \Omega)^2 - (50.0\ \Omega)^2} = 62.4\ \Omega.$ The average power supplied to this circuit is equal to the power dissipated by the resistor, which is $P = I_{rms}^2 R = (3.00\ \text{A})^2(62.4\ \Omega) = 562\ \text{W}.$

EVALUATE: $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{-50.0\ \Omega}{62.4\ \Omega}$ and $\phi = -38.7°.$

$P_{av} = V_{rms}I_{rms}\cos\phi = (240\ \text{V})(3.00\ \text{A})\cos(-38.7°) = 562\ \text{W},$ which checks.

31.49. **IDENTIFY:** The voltage and current amplitudes are the maximum values of these quantities, not necessarily the instantaneous values.

SET UP: The voltage amplitudes are $V_R = RI, V_L = X_L I,$ and $V_C = X_C I,$ where $I = V/Z$ and

$Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}.$

EXECUTE: **(a)** $\omega = 2\pi f = 2\pi(1250\ \text{Hz}) = 7854\ \text{rad/s}.$ Carrying extra figures in the calculator gives

$X_L = \omega L = (7854\ \text{rad/s})(3.50\ \text{mH}) = 27.5\ \Omega;$ $XC = 1/\omega C = 1/[(7854\ \text{rad/s})(10.0\ \mu\text{F})] = 12.7\ \Omega;$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (27.5\ \Omega - 12.7\ \Omega)^2} = 57.5\ \Omega;$

$I = V/Z = (60.0\ \text{V})/(52.1\ \Omega) = 1.15\ \text{A};$ $V_R = RI = (50.0\ \Omega)(1.15\ \text{A}) = 57.5\ \text{V};$

$V_L = X_L I = (27.5\ \Omega)(1.15\ \text{A}) = 31.6\ \text{V};\ V_C = X_C I = (12.7\ \Omega)(1.15\ \text{A}) = 14.6\ \text{V}.$

The voltage amplitudes can add to more than 60.0 V because the voltage maxima do not all occur at the same instant of time. At any instant, the instantaneous voltages across the resistor, inductor and capacitor all add to equal the instantaneous source voltage.

(b) All of them will change because they all depend on ω. $X_L = \omega L$ will double to $55.0\ \Omega$, and $X_C = 1/\omega C$ will decrease by half to 6.35 Ω. Therefore $Z = \sqrt{(50.0\ \Omega)^2 + (55.0\ \Omega - 6.35\ \Omega)^2} = 69.8\ \Omega;$

$I = V/Z = (60.0\text{V})/(69.8\ \Omega) = 0.860\ \text{A};\ V_R = IR = (0.860\ \text{A})(50.0\ \Omega) = 43.0\ \text{V};$

$V_L = IX_L = (0.860\ \text{A})(55.0\ \Omega) = 47.3\ \text{V};\ V_C = IX_C = (0.860\ \text{A})(6.35\ \Omega) = 5.46\ \text{V}.$

EVALUATE: The new amplitudes in part (b) are not simple multiples of the values in part (a) because the impedance and reactances are not all the same simple multiple of the angular frequency.

31.50. **IDENTIFY** and **SET UP:** $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$.

EXECUTE: **(a)** $\dfrac{1}{\omega_1 C} = \omega_1 L$ and $LC = \dfrac{1}{\omega_1^2}$. At angular frequency ω_2,

$$\frac{X_L}{X_C} = \frac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \frac{1}{\omega_1^2} = 4.\quad X_L > X_C.$$

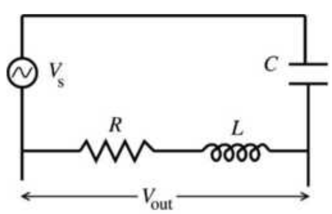
(b) At angular frequency ω_3, $\dfrac{X_L}{X_C} = \omega_3^2 LC = \left(\dfrac{\omega_1}{3}\right)^2 \left(\dfrac{1}{\omega_1^2}\right) = \dfrac{1}{9}.\quad X_C > X_L.$

EVALUATE: When ω increases, X_L increases and X_C decreases. When ω decreases, X_L decreases and X_C increases.

(c) The resonance angular frequency ω_0 is the value of ω for which $X_C = X_L$, so $\omega_0 = \omega_1$.

31.51. **IDENTIFY** and **SET UP:** Express Z and I in terms of ω, L, C and R. The voltages across the resistor and the inductor are 90° out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.51.



$$X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Figure 31.51

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_s \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

<u>ω small</u>

As ω gets small, $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 \to \dfrac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \to R^2$.

Therefore $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

ω large

As ω gets large, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2, R^2 + \omega^2 L^2 \to \omega^2 L^2$

Therefore, $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

EVALUATE: $V_{\text{out}}/V_s \to 0$ as ω becomes small, so there is V_{out} only when the frequency ω of V_s is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

31.52. **IDENTIFY:** $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}$.

If ω is small: $\frac{V_{\text{out}}}{V_s} \approx \frac{1}{\omega C\sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1$.

EVALUATE: When ω is large, X_C is small and X_L is large so Z is large and the current is small. Both factors in $V_C = IX_C$ are small. When ω is small, X_C is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

31.53. **IDENTIFY:** $I = V/Z$ and $P_{\text{av}} = \frac{1}{2}I^2 R$.

SET UP: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

EXECUTE: **(a)** $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

(b) $P_{\text{av}} = \frac{1}{2}I^2 R = \frac{1}{2}\left(\frac{V}{Z}\right)^2 R = \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}$.

(c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C}$, so $\omega_0 = \frac{1}{\sqrt{LC}}$.

(d) $P_{\text{av}} = \frac{(100\text{ V})^2(200\ \Omega)/2}{(200\ \Omega)^2 + (\omega(2.00\text{ H}) - 1/[\omega(0.500\times10^{-6}\text{ F})])^2} = \frac{25\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000\text{ s}^{-2})^2}\text{ W}$.

The graph of P_{av} versus ω is sketched in Figure 31.53.

EVALUATE: Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

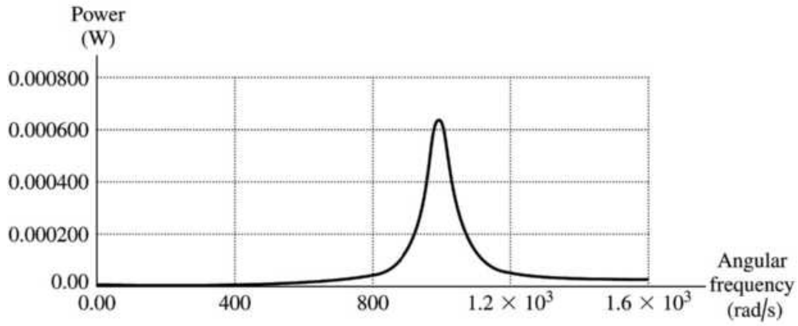


Figure 31.53

31.54. **IDENTIFY:** $V_L = I\omega L$ and $V_C = \dfrac{I}{\omega C}$.

SET UP: Problem 31.53 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

EXECUTE: **(a)** $V_L = I\omega L = \dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(b) $V_C = \dfrac{I}{\omega C} = \dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(c) The graphs are given in Figure 31.54.

EVALUATE: **(d)** When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000 \text{ rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

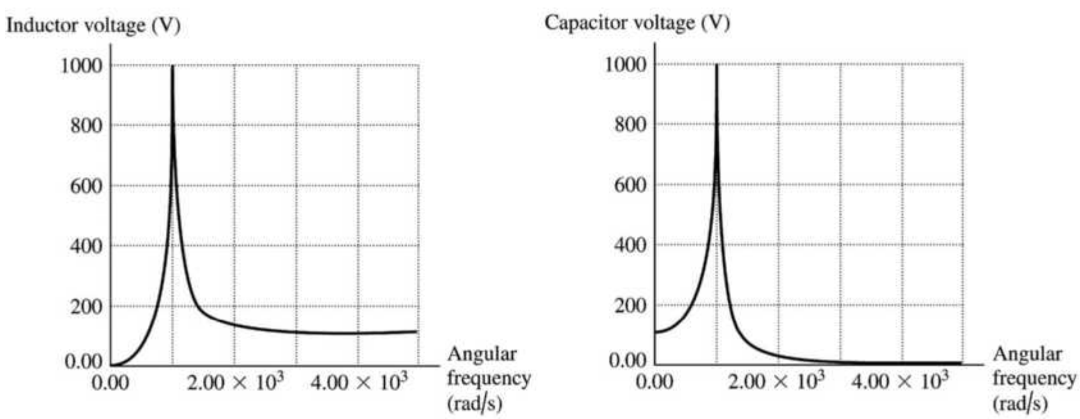


Figure 31.54

31.55. **IDENTIFY:** We know R, X_C and ϕ so Eq. (31.24) tells us X_L. Use $P_{av} = I_{rms}^2 R$ to calculate I_{rms}. Then calculate Z and use Eq. (31.26) to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{av} = 140$ W

EXECUTE: **(a)** $\tan\phi = \dfrac{X_L - X_C}{R}$

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega$

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.22). $I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{140\ \text{W}}{180\ \Omega}} = 0.882\ \text{A}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega$

$V_{rms} = I_{rms}Z = (0.882\ \text{A})(306\ \Omega) = 270\ \text{V}.$

EVALUATE: We could also use Eq. (31.31): $P_{av} = V_{rms}I_{rms}\cos\phi$

$V_{rms} = \dfrac{P_{av}}{I_{rms}\cos\phi} = \dfrac{140\ \text{W}}{(0.882\ \text{A})\cos(-54.0°)} = 270\ \text{V},$ which agrees. The source voltage lags the current when $X_C > X_L$, and this agrees with what we found.

31.56. **IDENTIFY:** At any instant of time the same rules apply to the parallel ac circuit as to the parallel dc circuit: the voltages are the same and the currents add.

SET UP: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by $90°$ and for a capacitor the voltage lags the current by $90°$.

EXECUTE: **(a)** The parallel *L-R-C* circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v.$ Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$. The phase diagram is sketched in Figure 31.56.

(c) From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2.$

(d) From part (c): $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$. But $I = \dfrac{V}{Z}$, so $\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}.$

EVALUATE: For large ω, $Z \to \dfrac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small ω, $Z \to \omega L$. The current in the inductive branch is much larger than the current in the other branches.

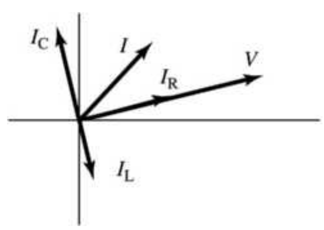


Figure 31.56

31.57. **IDENTIFY:** Apply the expression for *I* from Problem 31.56 when $\omega_0 = 1/\sqrt{LC}$.

SET UP: From Problem 31.56, $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}.$

EXECUTE: **(a)** At resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}} \Rightarrow \omega_0 C = \dfrac{1}{\omega_0 L} \Rightarrow I_C = V\omega_0 C = \dfrac{V}{\omega_0 L} = I_L$ so $I = I_R$ and *I* is a minimum.

(b) $P_{av} = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V^2}{R}$ at resonance where $R < Z$ so power is a maximum.

(c) At $\omega = \omega_0$, I and V are in phase, so the phase angle is zero, which is the same as a series resonance.

EVALUATE: **(d)** The parallel circuit is sketched in Figure 31.57. At resonance, $|i_C| = |i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between *a* and *b* is always zero.

(e) If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C + i_L = 0$ and the statement in part (d) isn't valid.

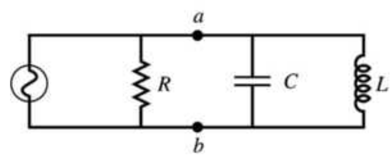


Figure 31.57

31.58. **IDENTIFY:** Refer to the results and the phasor diagram in Problem 31.56. The source voltage is applied across each parallel branch.

SET UP: $V = \sqrt{2}V_{rms} = 311\text{ V}$

EXECUTE: **(a)** $I_R = \frac{V}{R} = \frac{311\text{ V}}{400\ \Omega} = 0.778\text{ A}.$

(b) $I_C = V\omega C = (311\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.672\text{ A}.$

(c) $\phi = \arctan\left(\frac{I_C}{I_R}\right) = \arctan\left(\frac{0.672\text{ A}}{0.778\text{ A}}\right) = 40.8°.$

(d) $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.778\text{ A})^2 + (0.672\text{ A})^2} = 1.03\text{ A}.$

(e) Leads since $\phi > 0$.

EVALUATE: The phasor diagram shows that the current in the capacitor always leads the source voltage.

31.59. **IDENTIFY** and **SET UP:** Refer to the results and the phasor diagram in Problem 31.56. The source voltage is applied across each parallel branch.

EXECUTE: **(a)** $I_R = \frac{V}{R};\ I_C = V\omega C;\ I_L = \frac{V}{\omega L}.$

(b) The graph of each current versus ω is given in Figure 31.59a.

(c) $\omega \to 0: I_C \to 0; I_L \to \infty.\ \ \omega \to \infty: I_C \to \infty; I_L \to 0.$

At low frequencies, the current is not changing much so the inductor's back-emf doesn't "resist." This allows the current to pass fairly freely. However, the current in the capacitor goes to zero because it tends to "fill up" over the slow period, making it less effective at passing charge. At high frequency, the induced emf in the inductor resists the violent changes and passes little current. The capacitor never gets a chance to fill up so passes charge freely.

(d) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.0\text{ H})(0.50\times10^{-6}\text{ F})}} = 1000\text{ rad/sec}$ and $f = 159\text{ Hz}$. The phasor diagram is sketched in Figure 31.59b.

(e) $I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2}.$

$$I = \sqrt{\left(\frac{100\text{ V}}{200\ \Omega}\right)^2 + \left((100\text{ V})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) - \frac{100\text{ V}}{(1000\text{ s}^{-1})(2.0\text{ H})}\right)^2} = 0.50\text{ A}$$

(f) At resonance $I_L = I_C = V\omega C = (100\text{ V})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) = 0.0500\text{ A}$ and $I_R = \dfrac{V}{R} = \dfrac{100\text{ V}}{200\ \Omega} = 0.50\text{ A}.$

EVALUATE: At resonance $i_C = i_L = 0$ at all times and the current through the source equals the current through the resistor.

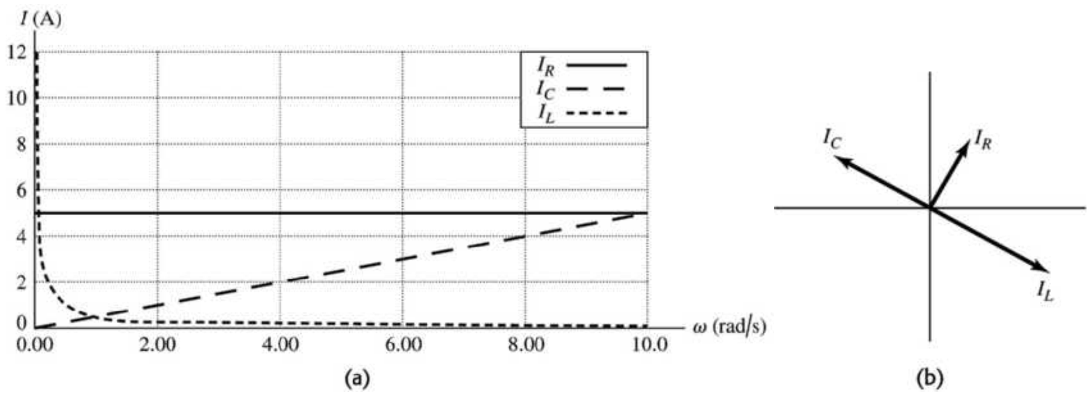


Figure 31.59

31.60. **IDENTIFY:** The circuit is in resonance, and we know *R*, *L*, *C* and *V*. We want the resonance angular frequency, the current amplitude through the source and resistor and the maximum energy stored in the inductor and capacitor.

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and at resonance, $Z = R.$ $I = \dfrac{V}{Z}.$ $V_R = V_C = V_L = V.$ $V_R = I_R R,$ $V_C = I_C X_C,$ $V_L = I_L X_L.$ The maximum energy stored in the inductor is $U_L = \frac{1}{2}LI_L^2.$ The maximum energy stored in the capacitor is $U_C = \frac{1}{2}CV_C^2.$

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{(0.300\text{ H})(0.100\times10^{-6}\text{ F})}} = 5.77\times10^3\text{ rad/s}.$

(b) $I = \dfrac{V}{Z} = \dfrac{V}{R} = \dfrac{240\text{ V}}{100\ \Omega} = 2.40\text{ A}.$

(c) $I_R = \dfrac{V}{R} = 2.40\text{ A}.$

(d) $X_L = \omega L = (5.77\times10^3\text{ rad/s})(0.300\text{ H}) = 1.73\times10^3\ \Omega.$

$I_L = \dfrac{V}{X_L} = \dfrac{240\text{ V}}{1.73\times10^3\ \Omega} = 0.139\text{ A}.$

(e) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(5.77\times10^3\text{ rad/s})(0.100\times10^{-6}\text{ F})} = 1.73\times10^3\ \Omega.$ $I_C = I_L = 0.139\text{ A}.$

(f) $U_L = \frac{1}{2}LI_L^2 = \frac{1}{2}(0.300\text{ H})(0.139\text{ A})^2 = 2.90\times10^{-3}\text{ J} = 2.90\text{ mJ}.$

$U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}(0.100\times10^{-6}\text{ F})(240\text{ V})^2 = 2.90\times10^{-3}\text{ J} = 2.90\text{ mJ}.$

EVALUATE: The maximum energy stored in the inductor and capacitor is the same, but not at the same time.

31.61. **IDENTIFY:** The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and the resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}.$

SET UP: ω_0 is independent of *R*.

EXECUTE: **(a)** ω_0 (or f_0) depends only on *L* and *C* so change these quantities.

(b) To double ω_0, decrease *L* and *C* by multiplying each of them by $\frac{1}{2}$.

EVALUATE: Increasing L and C decreases the resonance frequency; decreasing L and C increases the resonance frequency.

31.62. IDENTIFY: The average power depends on the phase angle ϕ.

SET UP: The average power is $P_{av} = V_{rms}I_{rms}\cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** $P_{av} = V_{rms}I_{rms}\cos\phi = \frac{1}{2}(V_{rms}I_{rms})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60°$. $\tan\phi = (X_L - X_C)/R$, which gives $\tan 60° = (\omega L - 1/\omega C)/R$. Using $R = 75.0\ \Omega$, $L = 5.00$ mH and $C = 2.50\ \mu$F and solving for ω we get $\omega = 28760$ rad/s $= 28{,}800$ rad/s.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28{,}760\text{ rad/s})(5.00\text{ mH}) = 144\ \Omega$ and $X_C = 1/\omega C = 1/[(28{,}760\text{ rad/s})(2.50\ \mu\text{F})] = 13.9\ \Omega$, giving $Z = \sqrt{(75\ \Omega)^2 + (144\ \Omega - 13.9\ \Omega)^2} = 150\ \Omega$; $I = V/Z = (15.0\text{ V})/(150\ \Omega) = 0.100$ A and $P_{av} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(15.0\text{ V})(0.100\text{ A})(1/2) = 0.375$ W.

EVALUATE: All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

31.63. IDENTIFY and SET UP: Eq. (31.19) allows us to calculate I and then Eq. (31.22) gives Z. Solve Eq. (31.21) for X_L.

EXECUTE: **(a)** $V_C = IX_C$ so $I = \frac{V_C}{X_C} = \frac{360\text{ V}}{480\ \Omega} = 0.750$ A

(b) $V = IZ$ so $Z = \frac{V}{I} = \frac{120\text{ V}}{0.750\text{ A}} = 160\ \Omega$

(c) $Z^2 = R^2 + (X_L - X_C)^2$

$X_L - X_C = \pm\sqrt{Z^2 - R^2}$, so

$X_L = X_C \pm \sqrt{Z^2 - R^2} = 480\ \Omega \pm \sqrt{(160\ \Omega)^2 - (80.0\ \Omega)^2} = 480\ \Omega \pm 139\ \Omega$

$X_L = 619\ \Omega$ or $341\ \Omega$

(d) EVALUATE: $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. At resonance, $X_C = X_L$. As the frequency is lowered below the resonance frequency X_C increases and X_L decreases. Therefore, for $\omega < \omega_0$, $X_L < X_C$. So for $X_L = 341\ \Omega$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L = 619\ \Omega$. But at these two values of X_L, the magnitude of $X_L - X_C$ is the same so Z and I are the same. In one case $(X_L = 691\ \Omega)$ the source voltage leads the current and in the other $(X_L = 341\ \Omega)$ the source voltage lags the current.

31.64. IDENTIFY and SET UP: Calculate Z and $I = V/Z$.

EXECUTE: **(a)** For $\omega = 800$ rad/s:

$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{(500\ \Omega)^2 + ((800\text{ rad/s})(2.0\text{ H}) - 1/((800\text{ rad/s})(5.0\times10^{-7}\text{ F})))^2}$. $Z = 1030\ \Omega$.

$I = \frac{V}{Z} = \frac{100\text{ V}}{1030\ \Omega} = 0.0971$ A. $V_R = IR = (0.0971\text{ A})(500\ \Omega) = 48.6$ V,

$V_C = \frac{1}{\omega C} = \frac{0.0971\text{ A}}{(800\text{ rad/s})(5.0\times10^{-7}\text{ F})} = 243$ V and $V_L = I\omega L = (0.0971\text{ A})(800\text{ rad/s})(2.00\text{ H}) = 155$ V.

$\phi = \arctan\left(\frac{\omega L - 1/(\omega C)}{R}\right) = -60.9°$. The graph of each voltage versus time is given in Figure 31.64a.

(b) Repeating exactly the same calculations as above for $\omega = 1000$ rad/s:

$Z = R = 500\ \Omega$; $\phi = 0$; $I = 0.200$ A; $V_R = V = 100$ V; $V_C = V_L = 400$ V. The graph of each voltage versus time is given in Figure 31.64b.

(c) Repeating exactly the same calculations as part (a) for $\omega = 1250$ rad/s:
$Z = 1030\ \Omega$; $\phi = +60.9°$; $I = 0.0971$ A; $V_R = 48.6$ V; $V_C = 155$ V; $V_L = 243$ V. The graph of each voltage versus time is given in Figure 31.64c.

EVALUATE: The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.00\ \text{H})(0.500\ \mu\text{F})}} = 1000$ rad/s. For $\omega < \omega_0$ the phase angle is negative and for $\omega > \omega_0$ the phase angle is positive.

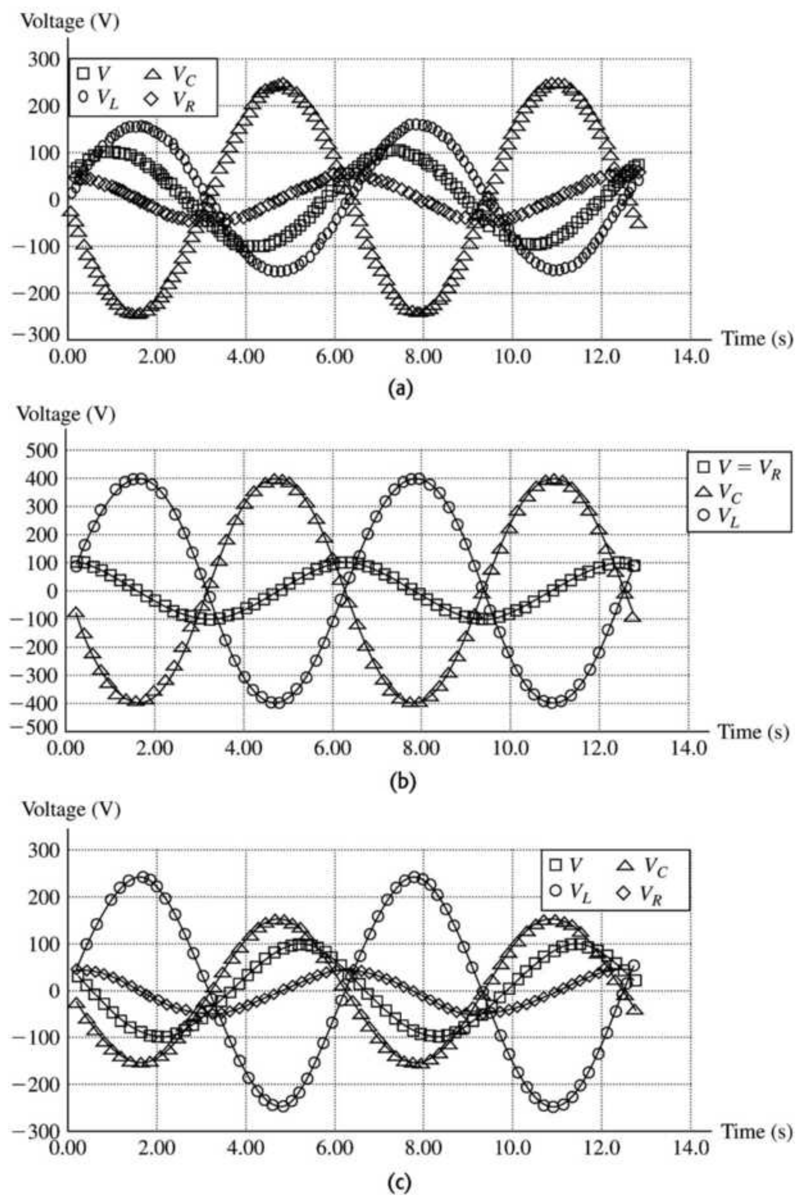


Figure 31.64

31.65. IDENTIFY and **SET UP:** Consider the cycle of the repeating current that lies between $t_1 = \tau/2$ and $t_2 = 3\tau/2$. In this interval $i = \dfrac{2I_0}{\tau}(t - \tau)$. $I_{\text{av}} = \dfrac{1}{t_2 - t_1}\int_{t_1}^{t_2} i\,dt$ and $I_{\text{rms}}^2 = \dfrac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2 dt.$

EXECUTE: $I_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i\,dt = \frac{1}{\tau}\int_{\tau/2}^{3\tau/2} \frac{2I_0}{\tau}(t-\tau)\,dt = \frac{2I_0}{\tau^2}\left[\frac{1}{2}t^2 - \tau t\right]_{\tau/2}^{3\tau/2}$

$$I_{av} = \left(\frac{2I_0}{\tau^2}\right)\left(\frac{9\tau^2}{8} - \frac{3\tau^2}{2} - \frac{\tau^2}{8} + \frac{\tau^2}{2}\right) = (2I_0)\tfrac{1}{8}(9 - 12 - 1 + 4) = \frac{I_0}{4}(13 - 13) = 0.$$

$$I_{rms}^2 = (I^2)_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2 dt = \frac{1}{\tau}\int_{\tau/2}^{3\tau/2} \frac{4I_0^2}{\tau^2}(t-\tau)^2\,dt$$

$$I_{rms}^2 = \frac{4I_0^2}{\tau^3}\int_{\tau/2}^{3\tau/2}(t-\tau)^2 dt = \frac{4I_0^2}{\tau^3}\left[\tfrac{1}{3}(t-\tau)^3\right]_{\tau/2}^{3\tau/2} = \frac{4I_0^2}{3\tau^3}\left[\left(\frac{\tau}{2}\right)^3 - \left(-\frac{\tau}{2}\right)^3\right]$$

$$I_{rms}^2 = \frac{I_0^2}{6}[1+1] = \tfrac{1}{3}I_0^2$$

$$I_{rms} = \sqrt{I_{rms}^2} = \frac{I_0}{\sqrt{3}}.$$

EVALUATE: In each cycle the current has as much negative value as positive value and its average is zero. i^2 is always positive and its average is not zero. The relation between I_{rms} and the current amplitude for this current is different from that for a sinusoidal current (Eq. 31.4).

31.66. **IDENTIFY:** Apply $V_{rms} = I_{rms}Z.$

SET UP: $\omega_0 = \frac{1}{\sqrt{LC}}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}.$

EXECUTE: **(a)** $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.80\text{ H})(9.00\times 10^{-7}\text{ F})}} = 786\text{ rad/s}.$

(b) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}.$

$$Z = \sqrt{(300\ \Omega)^2 + ((786\text{ rad/s})(1.80\text{ H}) - 1/((786\text{ rad/s})(9.00\times 10^{-7}\text{ F})))^2} = 300\ \Omega.$$

$$I_{rms\text{-}0} = \frac{V_{rms}}{Z} = \frac{60\text{ V}}{300\ \Omega} = 0.200\text{ A}.$$

(c) We want $I = \frac{1}{2}I_{rms\text{-}0} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$ $R^2 + (\omega L - 1/\omega C)^2 = \frac{4V_{rms}^2}{I_{rms\text{-}0}^2}.$

$$\omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} + R^2 - \frac{4V_{rms}^2}{I_{rms\text{-}0}^2} = 0 \text{ and } (\omega^2)^2 L^2 + \omega^2\left(R^2 - \frac{2L}{C} - \frac{4V_{rms}^2}{I_{rms\text{-}0}^2}\right) + \frac{1}{C^2} = 0.$$

Substituting in the values for this problem, the equation becomes $(\omega^2)^2(3.24) + \omega^2(-4.27\times 10^6) + 1.23\times 10^{12} = 0.$ Solving this quadratic equation in ω^2 we find $\omega^2 = 8.90\times 10^5\text{ rad}^2/\text{s}^2$ or $4.28\times 10^5\text{ rad}^2/\text{s}^2$ and $\omega = 943\text{ rad/s}$ or $654\text{ rad/s}.$

(d) (i) $R = 300\ \Omega,\ I_{rms\text{-}0} = 0.200\text{ A},\ |\omega_1 - \omega_2| = 289\text{ rad/s}.$ (ii) $R = 30\ \Omega,\ I_{rms\text{-}0} = 2\text{A},\ |\omega_1 - \omega_2| = 28\text{ rad/s}.$
(iii) $R = 3\ \Omega,\ I_{rms\text{-}0} = 20\text{ A},\ |\omega_1 - \omega_2| = 2.88\text{ rad/s}.$

EVALUATE: The width gets smaller as R gets smaller; I_{rms-0} gets larger as R gets smaller.

31.67. **IDENTIFY:** The resonance frequency, the reactances, and the impedance all depend on the values of the circuit elements.

SET UP: The resonance frequency is $\omega_0 = 1/\sqrt{LC},$ the reactances are $X_L = \omega L$ and $X_C = 1/\omega C,$ and the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}.$

EXECUTE: **(a)** $\omega_0 = 1/\sqrt{LC}$ becomes $\frac{1}{\sqrt{2L}\sqrt{2C}} \to 1/2,$ so ω_0 decreases by $\frac{1}{2}.$

(b) Since $X_L = \omega L,$ if L is doubled, X_L increases by a factor of 2.

(c) Since $X_C = 1/\omega C$, doubling C decreases X_C by a factor of $\frac{1}{2}$.

(d) $Z = \sqrt{R^2 + (X_L - X_C)^2} \rightarrow Z = \sqrt{(2R)^2 + (2X_L - \frac{1}{2}X_C)^2}$, so Z does not change by a simple factor of 2 or $\frac{1}{2}$.

EVALUATE: The impedance does not change by a simple factor, even though the other quantities do.

31.68. IDENTIFY: At resonance, $Z = R$. $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_C = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time-dependent quantity correspond to the maximum values of those quantities.

EXECUTE: **(a)** $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. At resonance $\omega L = \dfrac{1}{\omega C}$ and $I_{max} = \dfrac{V}{R}$.

(b) $V_C = IX_C = \dfrac{V}{R\omega_0 C} = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}$.

(c) $V_L = IX_L = \dfrac{V}{R}\omega_0 L = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}$.

(d) $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{1}{2}C\dfrac{V^2}{R^2}\dfrac{L}{C} = \dfrac{1}{2}L\dfrac{V^2}{R^2}$.

(e) $U_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2}L\dfrac{V^2}{R^2}$.

EVALUATE: At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

31.69. IDENTIFY: $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_C = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time-dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = \dfrac{\omega_0}{2}$.

(a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \left(\dfrac{\omega_0 L}{2} - 2/\omega_0 C\right)^2}} = \dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(b) $V_C = IX_C = \dfrac{2}{\omega_0 C}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{2V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(c) $V_L = IX_L = \dfrac{\omega_0 L}{2}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{V/2}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(d) $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{2LV^2}{R^2 + \dfrac{9}{4}\dfrac{L}{C}}$.

(e) $U_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2}\dfrac{LV^2}{R^2 + \dfrac{9}{4}\dfrac{L}{C}}$.

EVALUATE: For $\omega < \omega_0, V_C > V_L$ and the maximum energy stored in the capacitor is greater than the maximum energy stored in the inductor.

31.70. **IDENTIFY:** $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_C = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = 2\omega_0$.

(a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (2\omega_0 L - 1/2\omega_0 C)^2}} = \dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(b) $V_C = IX_C = \dfrac{1}{2\omega_0 C}\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{V/2}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(c) $V_L = IX_L = 2\omega_0 L\dfrac{V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}} = \sqrt{\dfrac{L}{C}}\dfrac{2V}{\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(d) $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{LV^2}{8\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

(e) $U_L = \dfrac{1}{2}LI^2 = \dfrac{LV^2}{2\sqrt{R^2 + \dfrac{9}{4}\dfrac{L}{C}}}$.

EVALUATE: For $\omega > \omega_0, V_L > V_C$ and the maximum energy stored in the inductor is greater than the maximum energy stored in the capacitor.

31.71. **IDENTIFY:** A transformer transforms voltages according to $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance R is $R_{\text{eff}} = \dfrac{R}{(N_2/N_1)^2}$.

SET UP: $N_2 = 275$ and $V_1 = 25.0\text{ V}$.

EXECUTE: **(a)** $V_2 = V_1(N_2/N_1) = (25.0\text{ V})(834/275) = 75.8\text{ V}$

(b) $R_{\text{eff}} = \dfrac{R}{(N_2/N_1)^2} = \dfrac{125\,\Omega}{(834/275)^2} = 13.6\,\Omega$

EVALUATE: The voltage across the secondary is greater than the voltage across the primary since $N_2 > N_1$. The effective load resistance of the secondary is less than the resistance R connected across the secondary.

31.72. **IDENTIFY:** $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$ and $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$. Calculate Z. $R = Z\cos\phi$.

SET UP: $f = 50.0\text{ Hz}$ and $\omega = 2\pi f$. The power factor is $\cos\phi$.

EXECUTE: **(a)** $P_{\text{av}} = \dfrac{V_{\text{rms}}^2}{Z}\cos\phi$. $Z = \dfrac{V_{\text{rms}}^2\cos\phi}{P_{\text{av}}} = \dfrac{(120\text{ V})^2(0.560)}{(220\text{ W})} = 36.7\,\Omega$.

$R = Z\cos\phi = (36.7\,\Omega)(0.560) = 20.6\,\Omega$.

(b) $Z = \sqrt{R^2 + X_L^2} \cdot X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\,\Omega)^2 - (20.6\,\Omega)^2} = 30.4\,\Omega$. But $\phi = 0$ is at resonance, so the inductive and capacitive reactances equal each other. Therefore we need to add $X_C = 30.4\,\Omega$. $X_C = \dfrac{1}{\omega C}$ therefore gives $C = \dfrac{1}{\omega X_C} = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(50.0\text{ Hz})(30.4\,\Omega)} = 1.05\times10^{-4}\text{ F}$.

(c) At resonance, $P_{av} = \dfrac{V^2}{R} = \dfrac{(120\text{ V})^2}{20.6\ \Omega} = 699\text{ W}.$

EVALUATE: $P_{av} = I_{rms}^2 R$ and I_{rms} is maximum at resonance, so the power drawn from the line is maximum at resonance.

31.73. **IDENTIFY:** $p_R = i^2 R.$ $p_L = iL\dfrac{di}{dt}.$ $p_C = \dfrac{q}{C}i.$

SET UP: $i = I\cos\omega t$

EXECUTE: **(a)** $p_R = i^2R = I^2\cos^2(\omega t)R = V_R I\cos^2(\omega t) = \dfrac{1}{2}V_R I(1+\cos(2\omega t)).$

$$P_{av}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_R I}{2T}\int_0^T (1+\cos(2\omega t))dt = \frac{V_R I}{2T}[t]_0^T = \tfrac{1}{2}V_R I.$$

(b) $p_L = Li\dfrac{di}{dt} = -\omega L I^2\cos(\omega t)\sin(\omega t) = -\tfrac{1}{2}V_L I\sin(2\omega t).$ But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(L) = 0.$

(c) $p_C = \dfrac{q}{C}i = v_C i = V_C I\sin(\omega t)\cos(\omega t) = \tfrac{1}{2}V_C I\sin(2\omega t).$ But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(C) = 0.$

(d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \tfrac{1}{2}V_L I\sin(2\omega t) + \tfrac{1}{2}V_C I\sin(2\omega t)$ and

$p = I\cos(\omega t)(V_R\cos(\omega t) - V_L\sin(\omega t) + V_C\sin(\omega t)).$ But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V}$, so

$p = VI\cos(\omega t)(\cos\phi\cos(\omega t) - \sin\phi\sin(\omega t))$, at any instant of time.

EVALUATE: At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

31.74. **IDENTIFY:** $V_L = IX_L.$ $\dfrac{dV_L}{d\omega} = 0$ at the ω where V_L is a maximum. $V_C = IX_C.$ $\dfrac{dV_C}{d\omega} = 0$ at the ω where V_C is a maximum.

SET UP: Problem 31.53 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

EXECUTE: **(a)** $V_R = \text{maximum}$ when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}.$

(b) $V_L = \text{maximum}$ when $\dfrac{dV_L}{d\omega} = 0.$ Therefore: $\dfrac{dV_L}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right).$

$$0 = \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = -\frac{1}{\omega^2 C^2}.\quad \frac{1}{\omega^2} = LC - \frac{R^2C^2}{2}\ \text{and}\ \omega = \frac{1}{\sqrt{LC - R^2C^2/2}}.$$

(c) $V_C = \text{maximum}$ when $\dfrac{dV_C}{d\omega} = 0.$ Therefore: $\dfrac{dV_C}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right).$

$$0 = -\frac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2\ \text{and}\ \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

$$R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2.$$

EVALUATE: V_L is maximum at a frequency greater than the resonance frequency and V_C is a maximum at a frequency less than the resonance frequency. These frequencies depend on R, as well as on L and on C.

31.75. IDENTIFY: Follow the steps specified in the problem.

SET UP: In part (a) use Eq. (31.23) to calculate Z and then $I = V/Z$ ϕ is given by Eq. (31.24). In part (b) let $Z = R + iX$.

EXECUTE: **(a)** From the current phasors we know that $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$$Z = \sqrt{(400\ \Omega)^2 + \left((1000\text{ rad/s})(0.50\text{ H}) - \frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}\right)^2} = 500\ \Omega.$$

$$I = \frac{V}{Z} = \frac{200\text{ V}}{500\ \Omega} = 0.400\text{ A}.$$

(b) $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right)$. $\phi = \arctan\left(\dfrac{(1000\text{ rad/s})(0.500\text{ H}) - 1/(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}{400\ \Omega}\right) = +36.9°$

(c) $Z_{\text{cpx}} = R + i\left(\omega L - \dfrac{1}{\omega C}\right)$. $Z_{\text{cpx}} = 400\ \Omega - i\left((1000\text{ rad/s})(0.50\text{ H}) - \dfrac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}\right) =$
$400\ \Omega - 300\ \Omega i$.

$$Z = \sqrt{(400\ \Omega)^2 + (-300\ \Omega)^2} = 500\ \Omega.$$

(d) $I_{\text{cpx}} = \dfrac{V}{Z_{\text{cpx}}} = \dfrac{200\text{ V}}{(400 - 300i)\ \Omega} = \left(\dfrac{8+6i}{25}\right)\text{A} = (0.320\text{ A}) + (0.240\text{ A})i$. $I = \sqrt{\left(\dfrac{8+6i}{25}\right)\left(\dfrac{8-6i}{25}\right)}\text{ A} = 0.400\text{ A}$.

(e) $\tan\phi = \dfrac{\text{Im}(I_{\text{cpx}})}{\text{Re}(I_{\text{cpx}})} = \dfrac{6/25}{8/25} = 0.75 \Rightarrow \phi = +36.9°$.

(f) $V_{R\text{cpx}} = I_{\text{cpx}} R = \left(\dfrac{8+6i}{25}\right)(400\ \Omega) = (128 + 96i)\text{V}$.

$$V_{L\text{cpx}} = iI_{\text{cpx}}\omega L = i\left(\frac{8+6i}{25}\right)(1000\text{ rad/s})(0.500\text{ H}) = (-120 + 160i)\text{V}.$$

$$V_{C\text{cpx}} = i\frac{I_{\text{cpx}}}{\omega C} = i\left(\frac{8+6i}{25}\right)\frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})} = (+192 - 256i)\text{V}.$$

(g) $V_{\text{cpx}} = V_{R\text{cpx}} + V_{L\text{cpx}} + V_{C\text{cpx}} = (128 + 96i)\text{ V} + (-120 + 160i)\text{V} + (192 - 256i)\text{ V} = 200\text{ V}$.

EVALUATE: Both approaches yield the same value for I and for ϕ.