16

SOUND AND HEARING

16.1. **IDENTIFY** and **SET UP:** Eq. (15.1) gives the wavelength in terms of the frequency. Use Eq. (16.5) to relate the pressure and displacement amplitudes.

EXECUTE: **(a)** $\lambda = v/f = (344\text{ m/s})/1000\text{ Hz} = 0.344\text{ m}$

(b) $p_{\text{max}} = BkA$ and Bk is constant gives $p_{\text{max}1}/A_1 = p_{\text{max}2}/A_2$

$$A_2 = A_1\left(\frac{p_{\text{max}2}}{p_{\text{max}1}}\right) = 1.2\times10^{-8}\text{ m}\left(\frac{30\text{ Pa}}{3.0\times10^{-2}\text{ Pa}}\right) = 1.2\times10^{-5}\text{ m}$$

(c) $p_{\text{max}} = BkA = 2\pi BA/\lambda$

$p_{\text{max}}\lambda = 2\pi BA = \text{constant}$ so $p_{\text{max}1}\lambda_1 = p_{\text{max}2}\lambda_2$ and

$$\lambda_2 = \lambda_1\left(\frac{p_{\text{max}1}}{p_{\text{max}2}}\right) = (0.344\text{ m})\left(\frac{3.0\times10^{-2}\text{ Pa}}{1.5\times10^{-3}\text{ Pa}}\right) = 6.9\text{ m}$$

$f = v/\lambda = (344\text{ m/s})/6.9\text{ m} = 50\text{ Hz}$

EVALUATE: The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

16.2. **IDENTIFY:** Apply $p_{\text{max}} = BkA$ and solve for A.

SET UP: $k = \dfrac{2\pi}{\lambda}$ and $v = f\lambda,$ so $k = \dfrac{2\pi f}{v}$ and $p = \dfrac{2\pi fBA}{v}.$

EXECUTE: $A = \dfrac{p_{\text{max}}v}{2\pi Bf} = \dfrac{(3.0\times10^{-2}\text{ Pa})(1480\text{ m/s})}{2\pi(2.2\times10^{9}\text{ Pa})(1000\text{ Hz})} = 3.21\times10^{-12}\text{ m}.$

EVALUATE: Both v and B are larger, but B is larger by a much greater factor, so v/B is a lot smaller and therefore A is a lot smaller.

16.3. **IDENTIFY:** Use Eq. (16.5) to relate the pressure and displacement amplitudes.

SET UP: As stated in Example 16.1 the adiabatic bulk modulus for air is $B = 1.42\times10^{5}\text{ Pa}.$ Use Eq. (15.1) to calculate λ from f, and then $k = 2\pi/\lambda.$

EXECUTE: **(a)** $f = 150\text{ Hz}$

Need to calculate k: $\lambda = v/f$ and $k = 2\pi/\lambda$ so $k = 2\pi f/v = (2\pi\text{ rad})(150\text{ Hz})/344\text{ m/s} = 2.74\text{ rad/m}.$ Then $p_{\text{max}} = BkA = (1.42\times10^{5}\text{ Pa})(2.74\text{ rad/m})(0.0200\times10^{-3}\text{ m}) = 7.78\text{ Pa}.$ This is below the pain threshold of 30 Pa.

(b) f is larger by a factor of 10 so $k = 2\pi f/v$ is larger by a factor of 10, and $p_{\text{max}} = BkA$ is larger by a factor of 10. $p_{\text{max}} = 77.8\text{ Pa},$ above the pain threshold.

(c) There is again an increase in f, k, and p_{max} of a factor of 10, so $p_{\text{max}} = 778\text{ Pa},$ far above the pain threshold.

EVALUATE: When f increases, λ decreases so k increases and the pressure amplitude increases.

16.4. **IDENTIFY:** Apply $p_{max} = BkA$. $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{v}$, so $p_{max} = \dfrac{2\pi fBA}{v}$.

SET UP: $v = 344$ m/s

EXECUTE: $f = \dfrac{vp_{max}}{2\pi BA} = \dfrac{(344\text{ m/s})(10.0\text{ Pa})}{2\pi(1.42\times10^5\text{ Pa})(1.00\times10^{-6}\text{ m})} = 3.86\times10^3\text{ Hz}$

EVALUATE: Audible frequencies range from about 20 Hz to about 20,000 Hz, so this frequency is audible.

16.5. **IDENTIFY** and **SET UP:** Use the relation $v = f\lambda$ to find the wavelength or frequency of various sounds.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{1531\text{ m/s}}{17\text{ Hz}} = 90\text{ m}.$

(b) $f = \dfrac{v}{\lambda} = \dfrac{1531\text{ m/s}}{0.015\text{ m}} = 102\text{ kHz}.$

(c) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{25\times10^3\text{ Hz}} = 1.4\text{ cm}.$

(d) For $f = 78$ kHz, $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{78\times10^3\text{ Hz}} = 4.4\text{ mm}.$ For $f = 39$ kHz, $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{39\times10^3\text{ Hz}} = 8.8\text{ mm}.$

The range of wavelengths is 4.4 mm to 8.8 mm.

(e) $\lambda = 0.25$ mm so $f = \dfrac{v}{\lambda} = \dfrac{1550\text{ m/s}}{0.25\times10^{-3}\text{m}} = 6.2\text{ MHz}.$

EVALUATE: Nonaudible (to human) sounds cover a wide range of frequencies and wavelengths.

16.6. **IDENTIFY:** $v = f\lambda$. Apply Eq. (16.7) for the waves in the liquid and Eq. (16.8) for the waves in the metal bar.

SET UP: In part (b) the wave speed is $v = \dfrac{d}{t} = \dfrac{1.50\text{ m}}{3.90\times10^{-4}\text{ s}}.$

EXECUTE: **(a)** Using Eq. (16.7), $B = v^2\rho = (\lambda f)^2\rho$, so

$B = [(8\text{ m})(400\text{ Hz})]^2(1300\text{ kg/m}^3) = 1.33\times10^{10}\text{ Pa}.$

(b) Using Eq. (16.8), $Y = v^2\rho = (L/t)^2\rho = [(1.50\text{ m})/(3.90\times10^{-4}\text{ s})]^2(6400\text{ kg/m}^3) = 9.47\times10^{10}\text{ Pa}.$

EVALUATE: In the liquid, $v = 3200$ m/s and in the metal, $v = 3850$ m/s. Both these speeds are much greater than the speed of sound in air.

16.7. **IDENTIFY:** $d = vt$ for the sound waves in air and in water.

SET UP: Use $v_{water} = 1482$ m/s at $20°$C, as given in Table 16.1. In air, $v = 344$ m/s.

EXECUTE: Since along the path to the diver the sound travels 1.2 m in air, the sound wave travels in water for the same time as the wave travels a distance $22.0\text{ m} - 1.20\text{ m} = 20.8\text{ m}$ in air. The depth of the diver is

$(20.8\text{ m})\dfrac{v_{water}}{v_{air}} = (20.8\text{ m})\dfrac{1482\text{ m/s}}{344\text{ m/s}} = 89.6\text{ m}.$ This is the depth of the diver; the distance from the horn is 90.8 m.

EVALUATE: The time it takes the sound to travel from the horn to the person on shore is

$t_1 = \dfrac{22.0\text{ m}}{344\text{ m/s}} = 0.0640\text{ s}.$ The time it takes the sound to travel from the horn to the diver is

$t_2 = \dfrac{1.2\text{ m}}{344\text{ m/s}} + \dfrac{89.6\text{ m}}{1482\text{ m/s}} = 0.0035\text{ s} + 0.0605\text{ s} = 0.0640\text{ s}.$ These times are indeed the same. For three figure accuracy the distance of the horn above the water can't be neglected.

16.8. **IDENTIFY:** Apply Eq. (16.10) to each gas.

SET UP: In each case, express M in units of kg/mol. For H_2, $\gamma = 1.41$. For He and Ar, $\gamma = 1.67$.

EXECUTE: **(a)** $v_{H_2} = \sqrt{\dfrac{(1.41)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(2.02\times10^{-3}\text{ kg/mol})}} = 1.32\times10^3\text{ m/s}$

(b) $v_{\text{He}} = \sqrt{\dfrac{(1.67)(8.3145\ \text{J/mol}\cdot\text{K})(300.15\ \text{K})}{(4.00\times 10^{-3}\ \text{kg/mol})}} = 1.02\times 10^{3}\ \text{m/s}$

(c) $v_{\text{Ar}} = \sqrt{\dfrac{(1.67)(8.3145\ \text{J/mol}\cdot\text{K})(300.15\ \text{K})}{(39.9\times 10^{-3}\ \text{kg/mol})}} = 323\ \text{m/s}.$

(d) Repeating the calculation of Example 16.4 at $T = 300.15\ \text{K}$ gives $v_{\text{air}} = 348\ \text{m/s}$, and so $v_{\text{H}_2} = 3.80 v_{\text{air}}$, $v_{\text{He}} = 2.94 v_{\text{air}}$ and $v_{\text{Ar}} = 0.928 v_{\text{air}}$.

EVALUATE: v is larger for gases with smaller M.

16.9. **IDENTIFY:** $v = f\lambda$. The relation of v to gas temperature is given by $v = \sqrt{\dfrac{\gamma RT}{M}}$.

SET UP: Let $T = 22.0°\text{C} = 295.15\ \text{K}$.

EXECUTE: At 22.0°C, $\lambda = \dfrac{v}{f} = \dfrac{325\ \text{m/s}}{1250\ \text{Hz}} = 0.260\ \text{m} = 26.0\ \text{cm}$. $\lambda = \dfrac{v}{f} = \dfrac{1}{f}\sqrt{\dfrac{\gamma RT}{M}}$. $\dfrac{\lambda}{\sqrt{T}} = \dfrac{1}{f}\sqrt{\dfrac{\gamma R}{M}}$, which is constant, so $\dfrac{\lambda_1}{\sqrt{T_1}} = \dfrac{\lambda_2}{\sqrt{T_2}}$. $T_2 = T_1\left(\dfrac{\lambda_2}{\lambda_1}\right)^2 = (295.15\ \text{K})\left(\dfrac{28.5\ \text{cm}}{26.0\ \text{cm}}\right)^2 = 354.6\ \text{K} = 81.4°\text{C}.$

EVALUATE: When T increases v increases and for fixed f, λ increases. Note that we did not need to know either γ or M for the gas.

16.10. **IDENTIFY:** $v = \sqrt{\dfrac{\gamma RT}{M}}$. Take the derivative of v with respect to T. In part (b) replace dv by Δv and dT by ΔT in the expression derived in part (a).

SET UP: $\dfrac{d(x^{1/2})}{dx} = \tfrac{1}{2}x^{-1/2}$. In Eq. (16.10), T must be in kelvins. $20°\text{C} = 293\ \text{K}$. $\Delta T = 1\ \text{C}° = 1\ \text{K}$.

EXECUTE: **(a)** $\dfrac{dv}{dT} = \sqrt{\dfrac{\gamma R}{M}}\dfrac{dT^{1/2}}{dT} = \sqrt{\dfrac{\gamma R}{M}}\tfrac{1}{2}T^{-1/2} = \dfrac{1}{2T}\sqrt{\dfrac{\gamma RT}{M}} = \dfrac{v}{2T}$. Rearranging gives $\dfrac{dv}{v} = \dfrac{1}{2}\dfrac{dT}{T}$, the desired result.

(b) $\dfrac{\Delta v}{v} = \dfrac{1}{2}\dfrac{\Delta T}{T}$. $\Delta v = \dfrac{v}{2}\dfrac{\Delta T}{T} = \left(\dfrac{344\ \text{m/s}}{2}\right)\left(\dfrac{1\ \text{K}}{293\ \text{K}}\right) = 0.59\ \text{m/s}.$

EVALUATE: Since $\dfrac{\Delta T}{T} = 3.4\times 10^{-3}$ and $\dfrac{\Delta v}{v}$ is one-half this, replacing dT by ΔT and dv by Δv is accurate. Using the result from part (a) is much simpler than calculating v for 20°C and for 21°C and subtracting, and is not subject to round-off errors.

16.11. **IDENTIFY** and **SET UP:** Use $t = \text{distance/speed}$. Calculate the time it takes each sound wave to travel the $L = 80.0\ \text{m}$ length of the pipe. Use Eq. (16.8) to calculate the speed of sound in the brass rod.

EXECUTE: wave in air: $t = 80.0\ \text{m}/(344\ \text{m/s}) = 0.2326\ \text{s}$

wave in the metal: $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{9.0\times 10^{10}\ \text{Pa}}{8600\ \text{kg/m}^3}} = 3235\ \text{m/s}$

$t = \dfrac{80.0\ \text{m}}{3235\ \text{m/s}} = 0.0247\ \text{s}$

The time interval between the two sounds is $\Delta t = 0.2326\ \text{s} - 0.0247\ \text{s} = 0.208\ \text{s}$

EVALUATE: The restoring forces that propagate the sound waves are much greater in solid brass than in air, so v is much larger in brass.

16.12. **IDENTIFY:** For transverse waves, $v_{\text{trans}} = \sqrt{\dfrac{F}{\mu}}$. For longitudinal waves, $v_{\text{long}} = \sqrt{\dfrac{Y}{\rho}}$.

SET UP: The mass per unit length μ is related to the density (assumed uniform) and the cross-section area A by $\mu = A\rho$.

EXECUTE: $v_{\text{long}} = 30v_{\text{trans}}$ gives $\sqrt{\dfrac{Y}{\rho}} = 30\sqrt{\dfrac{F}{\mu}}$ and $\dfrac{Y}{\rho} = 900\dfrac{F}{A\rho}$. Therefore, $F/A = \dfrac{Y}{900}$.

EVALUATE: Typical values of Y are on the order of 10^{11} Pa, so the stress must be about 10^{8} Pa. If A is on the order of $1\ \text{mm}^2 = 10^{-6}\ \text{m}^2$, this requires a force of about 100 N.

16.13. **IDENTIFY** and **SET UP:** Sound delivers energy (and hence power) to the ear. For a whisper, $I = 1\times10^{-10}\ \text{W/m}^2$. The area of the tympanic membrane is $A = \pi r^2$, with $r = 4.2\times10^{-3}$ m. Intensity is energy per unit time per unit area.

EXECUTE: **(a)** $E = IAt = (1\times10^{-10}\ \text{W/m}^2)\pi(4.2\times10^{-3}\ \text{m})^2(1\ \text{s}) = 5.5\times10^{-15}$ J.

(b) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.5\times10^{-15}\ \text{J})}{2.0\times10^{-6}\ \text{kg}}} = 7.4\times10^{-5}\ \text{m/s} = 0.074\ \text{mm/s}.$

EVALUATE: Compared to the energy of ordinary objects, it takes only a very small amount of energy for hearing. As part (b) shows, a mosquito carries a lot more energy than is needed for hearing.

16.14. **IDENTIFY:** The intensity I is given in terms of the displacement amplitude by Eq. (16.12) and in terms of the pressure amplitude by Eq. (16.14). $\omega = 2\pi f$. Intensity is energy per second per unit area.

SET UP: For part (a), $I = 10^{-12}\ \text{W/m}^2$. For part (b), $I = 3.2\times10^{-3}\ \text{W/m}^2$.

EXECUTE: **(a)** $I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$.

$$A = \frac{1}{\omega}\sqrt{\frac{2I}{\sqrt{\rho B}}} = \frac{1}{2\pi(1000\ \text{Hz})}\sqrt{\frac{2(1\times10^{-12}\ \text{W/m}^2)}{\sqrt{(1.20\ \text{kg/m}^3)(1.42\times10^5\ \text{Pa})}}} = 1.1\times10^{-11}\ \text{m}.\quad I = \frac{p_{\max}^2}{2\sqrt{\rho B}}.$$

$$p_{\max} = \sqrt{2I\sqrt{\rho B}} = \sqrt{2(1\times10^{-12}\ \text{W/m}^2)\sqrt{(1.20\ \text{kg/m}^3)(1.42\times10^5\ \text{Pa})}} = 2.9\times10^{-5}\ \text{Pa} = 2.8\times10^{-10}\ \text{atm}$$

(b) A is proportional to $\sqrt{I}$, so $A = (1.1\times10^{-11}\ \text{m})\sqrt{\dfrac{3.2\times10^{-3}\ \text{W/m}^2}{1\times10^{-12}\ \text{W/m}^2}} = 6.2\times10^{-7}$ m. $p_{\max}$ is also proportional to $\sqrt{I}$, so $p_{\max} = (2.9\times10^{-5}\ \text{Pa})\sqrt{\dfrac{3.2\times10^{-3}\ \text{W/m}^2}{1\times10^{-12}\ \text{W/m}^2}} = 1.6\ \text{Pa} = 1.6\times10^{-5}$ atm.

(c) area $= (5.00\ \text{mm})^2 = 2.5\times10^{-5}\ \text{m}^2$. Part (a): $(1\times10^{-12}\ \text{W/m}^2)(2.5\times10^{-5}\ \text{m}^2) = 2.5\times10^{-17}$ J/s.

Part (b): $(3.2\times10^{-3}\ \text{W/m}^2)(2.5\times10^{-5}\ \text{m}^2) = 8.0\times10^{-8}$ J/s.

EVALUATE: For faint sounds the displacement and pressure variation amplitudes are very small. Intensities for audible sounds vary over a very wide range.

16.15. **IDENTIFY:** Apply Eq. (16.12) and solve for A. $\lambda = v/f$, with $v = \sqrt{B/\rho}$.

SET UP: $\omega = 2\pi f$. For air, $B = 1.42\times10^5$ Pa.

EXECUTE: **(a)** The amplitude is

$$A = \sqrt{\frac{2\text{I}}{\sqrt{\rho B}\,\omega^2}} = \sqrt{\frac{2(3.00\times10^{-6}\ \text{W/m}^2)}{\sqrt{(1000\ \text{kg/m}^3)(2.18\times10^9\ \text{Pa})}\,(2\pi(3400\ \text{Hz}))^2}} = 9.44\times10^{-11}\ \text{m}.$$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{\sqrt{B/\rho}}{f} = \dfrac{\sqrt{(2.18\times10^9\ \text{Pa})/(1000\ \text{kg/m}^3)}}{3400\ \text{Hz}} = 0.434$ m.

(b) Repeating the above with $B = 1.42\times10^5\,\text{Pa}$ and the density of air gives $A = 5.66\times10^{-9}$ m and $\lambda = 0.100$ m.

EVALUATE: **(c)** The amplitude is larger in air, by a factor of about 60. For a given frequency, the much less dense air molecules must have a larger amplitude to transfer the same amount of energy.

16.16. **IDENTIFY:** Knowing the sound level in decibels, we can determine the rate at which energy is delivered to the eardrum.

SET UP: Intensity is energy per unit time per unit area. $\beta = (10\text{ dB})\log\left(\dfrac{I}{I_0}\right)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.

The area of the eardrum is $A = \pi r^2$, with $r = 4.2\times10^{-3}$ m. Part (b) of Problem 16.13 gave $v = 0.074$ mm/s.

EXECUTE: **(a)** $\beta = 110$ dB gives $11.0 = \log\left(\dfrac{I}{I_0}\right)$ and $I = (10^{11})I_0 = 0.100\text{ W/m}^2$.

$E = IAt = (0.100\text{ W/m}^2)\pi(4.2\times10^{-3}\text{ m})^2(1\text{ s}) = 5.5\ \mu\text{J}$.

(b) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.5\times10^{-6}\text{ J})}{2.0\times10^{-6}\text{ kg}}} = 2.3$ m/s. This is about 31,000 times faster than the speed in Problem 16.13b.

EVALUATE: Even though the sound wave intensity level is very high, the rate at which energy is delivered to the eardrum is very small, because the area of the eardrum is very small.

16.17. **IDENTIFY** and **SET UP:** Apply Eqs. (16.5), (16.11) and (16.15).

EXECUTE: **(a)** $\omega = 2\pi f = (2\pi\text{ rad})(150\text{ Hz}) = 942.5$ rad/s

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{942.5\text{ rad/s}}{344\text{ m/s}} = 2.74\text{ rad/m}$$

$B = 1.42\times10^5$ Pa (Example 16.1)

Then $p_{\text{max}} = BkA = (1.42\times10^5\text{ Pa})(2.74\text{ rad/m})(5.00\times10^{-6}\text{ m}) = 1.95$ Pa.

(b) Eq. (16.11): $I = \frac{1}{2}\omega BkA^2$

$I = \frac{1}{2}(942.5\text{ rad/s})(1.42\times10^5\text{ Pa})(2.74\text{ rad/m})(5.00\times10^{-6}\text{ m})^2 = 4.58\times10^{-3}\text{ W/m}^2$.

(c) Eq. (16.15): $\beta = (10\text{ dB})\log(I/I_0)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.

$\beta = (10\text{ dB})\log((4.58\times10^{-3}\text{ W/m}^2)/(1\times10^{-12}\text{ W/m}^2)) = 96.6$ dB.

EVALUATE: Even though the displacement amplitude is very small, this is a very intense sound. Compare the sound intensity level to the values in Table 16.2.

16.18. **IDENTIFY:** Changing the sound intensity level will decrease the rate at which energy reaches the ear.

SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\dfrac{I_2}{I_1}\right)$.

EXECUTE: **(a)** $\Delta\beta = -30$ dB so $\log\left(\dfrac{I_2}{I_1}\right) = -3$ and $\dfrac{I_2}{I_1} = 10^{-3} = 1/1000$.

(b) $I_2/I_1 = \frac{1}{2}$ so $\Delta\beta = 10\log\left(\frac{1}{2}\right) = -3.0$ dB

EVALUATE: Because of the logarithmic relationship between the intensity and intensity level of sound, a small change in the intensity level produces a large change in the intensity.

16.19. **IDENTIFY:** Use Eq. (16.13) to relate I and p_{max}. $\beta = (10\text{ dB})\log(I/I_0)$. Eq. (16.4) says the pressure amplitude and displacement amplitude are related by $p_{\text{max}} = BkA = B\left(\dfrac{2\pi f}{v}\right)A$.

SET UP: At 20°C the bulk modulus for air is 1.42×10^5 Pa and $v = 344$ m/s. $I_0 = 1\times10^{-12}\text{ W/m}^2$.

EXECUTE: **(a)** $I = \dfrac{vp_{\text{max}}^2}{2B} = \dfrac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})^2}{2(1.42\times10^5\text{ Pa})} = 4.4\times10^{-12}\text{ W/m}^2$

(b) $\beta = (10\text{ dB})\log\left(\dfrac{4.4\times10^{-12}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}\right) = 6.4$ dB

(c) $A = \dfrac{vp_{\text{max}}}{2\pi fB} = \dfrac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})}{2\pi(400\text{ Hz})(1.42\times10^5\text{ Pa})} = 5.8\times10^{-11}$ m

EVALUATE: This is a very faint sound and the displacement and pressure amplitudes are very small. Note that the displacement amplitude depends on the frequency but the pressure amplitude does not.

16.20. IDENTIFY and **SET UP:** Apply the relation $\beta_2 - \beta_1 = (10\ \text{dB})\log(I_2/I_1)$ that is derived in Example 16.9.

EXECUTE: **(a)** $\Delta\beta = (10\ \text{dB})\log\left(\dfrac{4I}{I}\right) = 6.0\ \text{dB}$

(b) The total number of crying babies must be multiplied by four, for an increase of 12 kids.

EVALUATE: For $I_2 = \alpha I_1$, where α is some factor, the increase in sound intensity level is $\Delta\beta = (10\ \text{dB})\log\alpha$. For $\alpha = 4$, $\Delta\beta = 6.0$ dB.

16.21. IDENTIFY and **SET UP:** Let 1 refer to the mother and 2 to the father. Use the result derived in Example 16.9 for the difference in sound intensity level for the two sounds. Relate intensity to distance from the source using Eq. (15.26).

EXECUTE: From Example 16.9, $\beta_2 - \beta_1 = (10\ \text{dB})\log(I_2/I_1)$

Eq. (15.26): $I_1/I_2 = r_2^2/r_1^2$ or $I_2/I_1 = r_1^2/r_2^2$

$\Delta\beta = \beta_2 - \beta_1 = (10\ \text{dB})\log(I_2/I_1) = (10\ \text{dB})\log(r_1/r_2)^2 = (20\ \text{dB})\log(r_1/r_2)$

$\Delta\beta = (20\ \text{dB})\log(1.50\ \text{m}/0.30\ \text{m}) = 14.0\ \text{dB}.$

EVALUATE: The father is 5 times closer so the intensity at his location is 25 times greater.

16.22. IDENTIFY: $\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$. $\beta_2 - \beta_1 = (10\ \text{dB})\log\dfrac{I_2}{I_1}$. Solve for $\dfrac{I_2}{I_1}$.

SET UP: If $\log y = x$ then $y = 10^x$. Let $\beta_2 = 70$ dB and $\beta_1 = 95$ dB.

EXECUTE: $70.0\ \text{dB} - 95.0\ \text{dB} = -25.0\ \text{dB} = (10\ \text{dB})\log\dfrac{I_2}{I_1}$. $\log\dfrac{I_2}{I_1} = -2.5$ and $\dfrac{I_2}{I_1} = 10^{-2.5} = 3.2\times10^{-3}$.

EVALUATE: $I_2 < I_1$ when $\beta_2 < \beta_1$.

16.23. IDENTIFY: The intensity of sound obeys an inverse square law.

SET UP: $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$. $\beta = (10\ \text{dB})\log\left(\dfrac{I}{I_0}\right)$, with $I_0 = 1\times10^{-12}\ \text{W/m}^2$.

EXECUTE: **(a)** $\beta = 53$ dB gives $5.3 = \log\left(\dfrac{I}{I_0}\right)$ and $I = (10^{5.3})I_0 = 2.0\times10^{-7}\ \text{W/m}^2$.

(b) $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (3.0\ \text{m})\sqrt{\dfrac{4}{1}} = 6.0\ \text{m}.$

(c) $\beta = \dfrac{53\ \text{dB}}{4} = 13.25$ dB gives $1.325 = \log\left(\dfrac{I}{I_0}\right)$ and $I = 2.1\times10^{-11}\ \text{W/m}^2$.

$r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (3.0\ \text{m})\sqrt{\dfrac{2.0\times10^{-7}\ \text{W/m}^2}{2.1\times10^{-11}\ \text{W/m}^2}} = 290\ \text{m}.$

EVALUATE: **(d)** Intensity obeys the inverse square law but noise level does not.

16.24. IDENTIFY: We must use the relationship between intensity and sound level.

SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\dfrac{I_2}{I_1}\right)$.

EXECUTE: **(a)** $\Delta\beta = 5.00$ dB gives $\log\left(\dfrac{I_2}{I_1}\right) = 0.5$ and $\dfrac{I_2}{I_1} = 10^{0.5} = 3.16$.

(b) $\dfrac{I_2}{I_1} = 100$ gives $\Delta\beta = 10\log(100) = 20$ dB.

(c) $\dfrac{I_2}{I_1} = 2$ gives $\Delta\beta = 10\log 2 = 3.0$ dB.

EVALUATE: Every doubling of the intensity increases the decibel level by 3.0 dB.

16.25. **IDENTIFY** and **SET UP:** An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.
EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.

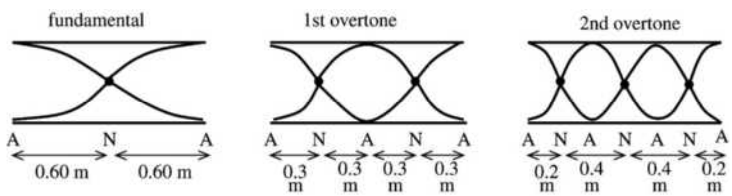


Figure 16.25a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m
(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.

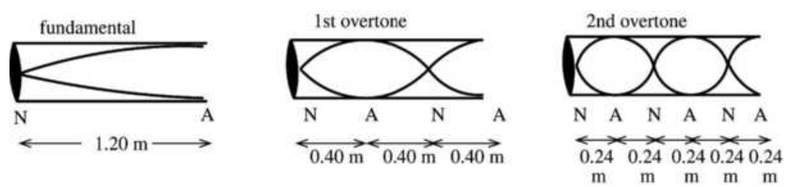


Figure 16.25b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m
EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

16.26. **IDENTIFY:** For an open pipe, $f_1 = \dfrac{v}{2L}$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$. $v = f\lambda$.

SET UP: $v = 344$ m/s. For a pipe, there must be a displacement node at a closed end and an antinode at the open end.

EXECUTE: **(a)** $L = \dfrac{v}{2f_1} = \dfrac{344 \text{ m/s}}{2(594 \text{ Hz})} = 0.290 \text{ m}.$

(b) There is a node at one end, an antinode at the other end and no other nodes or antinodes in between, so $\frac{\lambda_1}{4} = L$ and $\lambda_1 = 4L = 4(0.290\text{ m}) = 1.16\text{ m}$.

(c) $f_1 = \frac{v}{4L} = \frac{1}{2}\left(\frac{v}{2L}\right) = \frac{1}{2}(594\text{ Hz}) = 297\text{ Hz}$.

EVALUATE: We could also calculate f_1 for the stopped pipe as $f_1 = \frac{v}{\lambda_1} = \frac{344\text{ m/s}}{1.16\text{ m}} = 297\text{ Hz}$, which agrees with our result in part (c).

16.27. **IDENTIFY:** For a stopped pipe, the standing wave frequencies are given by Eq. (16.22).

SET UP: The first three standing wave frequencies correspond to $n = 1$, 3 and 5.

EXECUTE: $f_1 = \frac{(344\text{ m/s})}{4(0.17\text{ m})} = 506\text{ Hz}$, $f_3 = 3f_1 = 1517\text{ Hz}$, $f_5 = 5f_1 = 2529\text{ Hz}$.

EVALUATE: All three of these frequencies are in the audible range, which is about 20 Hz to 20,000 Hz.

16.28. **IDENTIFY:** The vocal tract is modeled as a stopped pipe, open at one end and closed at the other end, so we know the wavelength of standing waves in the tract.

SET UP: For a stopped pipe, $\lambda_n = 4L/n$ $(n = 1, 3, 5, \ldots)$ and $v = f\lambda$, so $f_1 = \frac{v}{4L}$ with $f_1 = 220\text{ Hz}$.

EXECUTE: $L = \frac{v}{4f_1} = \frac{344\text{ m/s}}{4(220\text{ Hz})} = 39.1\text{ cm}$. This result is a reasonable value for the mouth to diaphragm distance for a typical adult.

EVALUATE: 1244 Hz is not an integer multiple of the fundamental frequency of 220 Hz; it is 5.65 times the fundamental. The production of sung notes is more complicated than harmonics of an air column of fixed length.

16.29. **IDENTIFY:** For either type of pipe, stopped or open, the fundamental frequency is proportional to the wave speed v. The wave speed is given in turn by Eq. (16.10).

SET UP: For He, $\gamma = 5/3$ and for air, $\gamma = 7/5$.

EXECUTE: **(a)** The fundamental frequency is proportional to the square root of the ratio $\frac{\gamma}{M}$, so

$$f_{\text{He}} = f_{\text{air}}\sqrt{\frac{\gamma_{\text{He}}}{\gamma_{\text{air}}} \cdot \frac{M_{\text{air}}}{M_{\text{He}}}} = (262\text{ Hz})\sqrt{\frac{(5/3)}{(7/5)} \cdot \frac{28.8}{4.00}} = 767\text{ Hz}.$$

(b) No. In either case the frequency is proportional to the speed of sound in the gas.

EVALUATE: The frequency is much higher for helium, since the rms speed is greater for helium.

16.30. **IDENTIFY:** There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node. $v = f\lambda$.

SET UP: $v = 344\text{ m/s}$. The node to node distance is $\lambda/2$.

EXECUTE: **(a)** $\frac{\lambda_1}{2} = L$ so $\lambda_1 = 2L$. Each successive overtone adds an additional $\lambda/2$ along the pipe, so

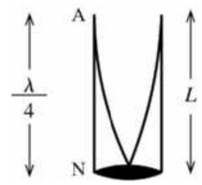
$n\left(\frac{\lambda_n}{2}\right) = L$ and $\lambda_n = \frac{2L}{n}$, where $n = 1, 2, 3, \ldots$ $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$.

(b) $f_1 = \frac{v}{2L} = \frac{344\text{ m/s}}{2(2.50\text{ m})} = 68.8\text{ Hz}$. $f_2 = 2f_1 = 138\text{ Hz}$. $f_3 = 3f_1 = 206\text{ Hz}$. All three of these frequencies are audible.

EVALUATE: A pipe of length L closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length L that is fixed at both ends.

16.31. **IDENTIFY and SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use Eq. (15.1) to calculate f. There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31.

EXECUTE: (a)



$$\lambda/4 = L$$
$$\lambda = 4L = 4(0.140\text{ m}) = 0.560\text{ m}$$
$$f = \frac{v}{\lambda} = \frac{344\text{ m/s}}{0.560\text{ m}} = 614\text{ Hz}$$

Figure 16.31

(b) Now the length L of the air column becomes $\frac{1}{2}(0.140\text{ m}) = 0.070\text{ m}$ and $\lambda = 4L = 0.280\text{ m}$.

$$f = \frac{v}{\lambda} = \frac{344\text{ m/s}}{0.280\text{ m}} = 1230\text{ Hz}$$

EVALUATE: Smaller L means smaller λ which in turn corresponds to larger f.

16.32. **IDENTIFY:** The wire will vibrate in its second overtone with frequency f_3^{wire} when $f_3^{\text{wire}} = f_1^{\text{pipe}}$. For a stopped pipe, $f_1^{\text{pipe}} = \dfrac{v}{4L_{\text{pipe}}}$. The second overtone standing wave frequency for a wire fixed at both ends is $f_3^{\text{wire}} = 3\left(\dfrac{v_{\text{wire}}}{2L_{\text{wire}}}\right)$. $v_{\text{wire}} = \sqrt{F/\mu}$.

SET UP: The wire has $\mu = \dfrac{m}{L_{\text{wire}}} = \dfrac{7.25\times10^{-3}\text{ kg}}{0.850\text{ m}} = 8.53\times10^{-3}\text{ kg/m}$. The speed of sound in air is $v = 344\text{ m/s}$.

EXECUTE: $v_{\text{wire}} = \sqrt{\dfrac{4110\text{ N}}{8.53\times10^{-3}\text{ kg/m}}} = 694\text{ m/s}$. $f_3^{\text{wire}} = f_1^{\text{pipe}}$ gives $3\dfrac{v_{\text{wire}}}{2L_{\text{wire}}} = \dfrac{v}{4L_{\text{pipe}}}$.

$$L_{\text{pipe}} = \frac{2L_{\text{wire}}v}{12v_{\text{wire}}} = \frac{2(0.850\text{ m})(344\text{ m/s})}{12(694\text{ m/s})} = 0.0702\text{ m} = 7.02\text{ cm}.$$

EVALUATE: The fundamental for the pipe has the same frequency as the third harmonic of the wire. But the wave speeds for the two objects are different and the two standing waves have different wavelengths.

16.33.

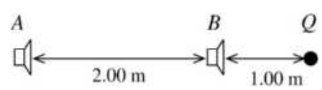


Figure 16.33

(a) IDENTIFY and SET UP: Path difference from points A and B to point Q is $3.00\text{ m} - 1.00\text{ m} = 2.00\text{ m}$, as shown in Figure 16.33. Constructive interference implies path difference $= n\lambda$, $n = 1, 2, 3, \ldots$

EXECUTE: $2.00\text{ m} = n\lambda$ so $\lambda = 2.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{2.00\text{ m}} = \frac{n(344\text{ m/s})}{2.00\text{ m}} = n(172\text{ Hz}),\quad n = 1, 2, 3, \ldots$$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) IDENTIFY and SET UP: Destructive interference implies path difference $= (n/2)\lambda$, $n = 1, 3, 5, \ldots$

EXECUTE: $2.00\text{ m} = (n/2)\lambda$ so $\lambda = 4.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{4.00\text{ m}} = \frac{n(344\text{ m/s})}{(4.00\text{ m})} = n(86\text{ Hz}),\quad n = 1, 3, 5, \ldots.$$

The lowest frequency for which destructive interference occurs is 86 Hz.

EVALUATE: As the frequency is slowly increased, the intensity at Q will fluctuate, as the interference changes between destructive and constructive.

16.34. **IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point P is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

SET UP: The wavelength is $\lambda = v/f = (344\text{ m/s})/(206\text{ Hz}) = 1.67\text{ m}$. Since P is between the speakers, x must be in the range 0 to L, where $L = 2.00\text{ m}$ is the distance between the speakers.

EXECUTE: The difference in path length is $\Delta l = (L - x) - x = L - 2x$, or $x = (L - \Delta l)/2$. For destructive interference, $\Delta l = (n + (1/2))\lambda$, and for constructive interference, $\Delta l = n\lambda$.

(a) Destructive interference: $n = 0$ gives $\Delta l = 0.835\text{ m}$ and $x = 0.58\text{ m}$. $n = -1$ gives $\Delta l = -0.835\text{ m}$ and $x = 1.42\text{ m}$. No other values of n place P between the speakers.

(b) Constructive interference: $n = 0$ gives $\Delta l = 0$ and $x = 1.00\text{ m}$. $n = 1$ gives $\Delta l = 1.67\text{ m}$ and $x = 0.17\text{ m}$. $n = -1$ gives $\Delta l = -1.67\text{ m}$ and $x = 1.83\text{ m}$. No other values of n place P between the speakers.

(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling and floor.

EVALUATE: Points of constructive interference are a distance $\lambda/2$ apart, and the same is true for the points of destructive interference.

16.35. **IDENTIFY:** For constructive interference the path difference is an integer number of wavelengths and for destructive interference the path difference is a half-integer number of wavelengths.

SET UP: $\lambda = v/f = (344\text{ m/s})/(688\text{ Hz}) = 0.500\text{ m}$

EXECUTE: To move from constructive interference to destructive interference, the path difference must change by $\lambda/2$. If you move a distance x toward speaker B, the distance to B gets shorter by x and the distance to A gets longer by x so the path difference changes by $2x$. $2x = \lambda/2$ and $x = \lambda/4 = 0.125\text{ m}$.

EVALUATE: If you walk an additional distance of 0.125 m farther, the interference again becomes constructive.

16.36. **IDENTIFY:** Destructive interference occurs when the path difference is a half integer number of wavelengths.

SET UP: $v = 344\text{ m/s}$, so $\lambda = v/f = (344\text{ m/s})/(172\text{ Hz}) = 2.00\text{ m}$. If $r_A = 8.00\text{ m}$ and r_B are the distances of the person from each speaker, the condition for destructive interference is $r_B - r_A = \left(n + \tfrac{1}{2}\right)\lambda$, where n is any integer.

EXECUTE: Requiring $r_B = r_A + \left(n + \tfrac{1}{2}\right)\lambda > 0$ gives $n + \tfrac{1}{2} > -r_A/\lambda = 0 - (8.00\text{ m})/(2.00\text{ m}) = -4$, so the smallest value of r_B occurs when $n = -4$, and the closest distance to B is

$$r_B = 8.00\text{ m} + \left(-4 + \tfrac{1}{2}\right)(2.00\text{ m}) = 1.00\text{ m}.$$

EVALUATE: For $r_B = 1.00\text{ m}$, the path difference is $r_A - r_B = 7.00\text{ m}$. This is 3.5λ.

16.37. **IDENTIFY:** Compare the path difference to the wavelength.

SET UP: $\lambda = v/f = (344\text{ m/s})/(860\text{ Hz}) = 0.400\text{ m}$

EXECUTE: The path difference is $13.4\text{ m} - 12.0\text{ m} = 1.4\text{ m}$. $\dfrac{\text{path difference}}{\lambda} = 3.5$. The path difference is a half-integer number of wavelengths, so the interference is destructive.

EVALUATE: The interference is destructive at any point where the path difference is a half-integer number of wavelengths.

16.38. **IDENTIFY:** For constructive interference, the path difference is an integer number of wavelengths. For destructive interference, the path difference is a half-integer number of wavelengths.

SET UP: One speaker is 4.50 m from the microphone and the other is 4.03 m from the microphone, so the path difference is 0.42 m. $f = v/\lambda$.

EXECUTE: **(a)** $\lambda = 0.42\text{ m}$ gives $f = \dfrac{v}{\lambda} = 820\text{ Hz}$; $2\lambda = 0.42\text{ m}$ gives $\lambda = 0.21\text{ m}$ and $f = \dfrac{v}{\lambda} = 1640\text{ Hz}$; $3\lambda = 0.42\text{ m}$ gives $\lambda = 0.14\text{ m}$ and $f = \dfrac{v}{\lambda} = 2460\text{ Hz}$, and so on. The frequencies for constructive interference are $n(820\text{ Hz})$, $n = 1, 2, 3, \ldots$.

(b) $\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.84\text{ m}$ and $f = \dfrac{v}{\lambda} = 410\text{ Hz}$; $3\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.28\text{ m}$ and $f = \dfrac{v}{\lambda} = 1230\text{ Hz}$; $5\lambda/2 = 0.42\text{ m}$ gives $\lambda = 0.168\text{ m}$ and $f = \dfrac{v}{\lambda} = 2050\text{ Hz}$, and so on. The frequencies for destructive interference are $(2n+1)(410\text{ Hz})$, $n = 0, 1, 2, \ldots.$

EVALUATE: The frequencies for constructive interference lie midway between the frequencies for destructive interference.

16.39. **IDENTIFY:** The beat is due to a difference in the frequencies of the two sounds.

SET UP: $f_{\text{beat}} = f_1 - f_2$. Tightening the string increases the wave speed for transverse waves on the string and this in turn increases the frequency.

EXECUTE: **(a)** If the beat frequency increases when she raises her frequency by tightening the string, it must be that her frequency is 433 Hz, 3 Hz above concert *A*.

(b) She needs to lower her frequency by loosening her string.

EVALUATE: The beat would only be audible if the two sounds are quite close in frequency. A musician with a good sense of pitch can come very close to the correct frequency just from hearing the tone.

16.40. **IDENTIFY:** $f_{\text{beat}} = |f_1 - f_2|$. $v = f\lambda$.

SET UP: $v = 344\text{ m/s}$. Let $\lambda_1 = 6.50\text{ cm}$ and $\lambda_2 = 6.52\text{ cm}$. $\lambda_2 > \lambda_1$ so $f_1 > f_2$.

EXECUTE: $f_1 - f_2 = v\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right) = \dfrac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = \dfrac{(344\text{ m/s})(0.02\times10^{-2}\text{ m})}{(6.50\times10^{-2}\text{ m})(6.52\times10^{-2}\text{ m})} = 16\text{ Hz}$. There are 16 beats per second.

EVALUATE: We could have calculated f_1 and f_2 and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

16.41. **IDENTIFY:** $f_{\text{beat}} = |f_a - f_b|$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$.

SET UP: $v = 344\text{ m/s}$. Let $L_a = 1.14\text{ m}$ and $L_b = 1.16\text{ m}$. $L_b > L_a$ so $f_{1a} > f_{1b}$.

EXECUTE: $f_{1a} - f_{1b} = \dfrac{v}{4}\left(\dfrac{1}{L_a} - \dfrac{1}{L_b}\right) = \dfrac{v(L_b - L_a)}{4L_aL_b} = \dfrac{(344\text{ m/s})(2.00\times10^{-2}\text{ m})}{4(1.14\text{ m})(1.16\text{ m})} = 1.3\text{ Hz}$. There are 1.3 beats per second.

EVALUATE: Increasing the length of the pipe increases the wavelength of the fundamental and decreases the frequency.

16.42. **IDENTIFY:** The motors produce sound having the same frequency as the motor. If the motors are almost, but not quite, the same, a beat will result.

SET UP: $f_{\text{beat}} = f_1 - f_2$. 1 rpm = 60 Hz.

EXECUTE: **(a)** 575 rpm = 9.58 Hz. The frequency of the other propeller differs by 2.0 Hz, so the frequency of the other propeller is either 11.6 Hz or 7.6 Hz. These frequencies correspond to 696 rpm or 456 rpm.

(b) When the speed and rpm of the second propeller is increased the beat frequency increases, so the frequency of the second propeller moves farther from the frequency of the first and the second propeller is turning at 696 rpm.

EVALUATE: If the frequency of the second propeller was 7.6 Hz then it would have moved close to the frequency of the first when its frequency was increased and the beat frequency would have decreased.

16.43. **IDENTIFY:** Apply the Doppler shift equation $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right) f_{\text{S}}$.

SET UP: The positive direction is from listener to source. $f_{\text{S}} = 1200\text{ Hz}$. $f_{\text{L}} = 1240\text{ Hz}$.

EXECUTE: $v_{\text{L}} = 0$. $v_{\text{S}} = -25.0\text{ m/s}$. $f_{\text{L}} = \left(\dfrac{v}{v + v_{\text{S}}}\right) f_{\text{S}}$ gives

$$v = \frac{v_{\text{S}} f_{\text{L}}}{f_{\text{S}} - f_{\text{L}}} = \frac{(-25\text{ m/s})(1240\text{ Hz})}{1200\text{ Hz} - 1240\text{ Hz}} = 780\text{ m/s}.$$

EVALUATE: $f_{\text{L}} > f_{\text{S}}$ since the source is approaching the listener.

16.44. **IDENTIFY:** Follow the steps of Example 16.18.

SET UP: In the first step, $v_S = +20.0\text{ m/s}$ instead of -30.0 m/s. In the second step, $v_L = -20.0\text{ m/s}$ instead of $+30.0$ m/s.

EXECUTE: $f_W = \left(\dfrac{v}{v+v_S}\right)f_S = \left(\dfrac{340\text{ m/s}}{340\text{ m/s}+20.0\text{ m/s}}\right)(300\text{ Hz}) = 283\text{ Hz}$. Then

$f_L = \left(\dfrac{v+v_L}{v}\right)f_W = \left(\dfrac{340\text{ m/s}-20.0\text{ m/s}}{340\text{ m/s}}\right)(283\text{ Hz}) = 266\text{ Hz}.$

EVALUATE: When the car is moving toward the reflecting surface, the received frequency back at the source is higher than the emitted frequency. When the car is moving away from the reflecting surface, as is the case here, the received frequency back at the source is lower than the emitted frequency.

16.45. **IDENTIFY:** Apply the Doppler shift equation $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392\text{ Hz}$.

(a) $v_S = 0$. $v_L = -15.0\text{ m/s}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s}-15.0\text{ m/s}}{344\text{ m/s}}\right)(392\text{ Hz}) = 375\text{ Hz}$

(b) $v_S = +35.0\text{ m/s}$. $v_L = +15.0\text{ m/s}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s}+15.0\text{ m/s}}{344\text{ m/s}+35.0\text{ m/s}}\right)(392\text{ Hz}) = 371\text{ Hz}$

(c) $f_{beat} = f_1 - f_2 = 4\text{ Hz}$

EVALUATE: The distance between whistle *A* and the listener is increasing, and for whistle *A* $f_L < f_S$. The distance between whistle *B* and the listener is also increasing, and for whistle *B* $f_L < f_S$.

16.46. **IDENTIFY** and **SET UP:** Apply Eqs. (16.27) and (16.28) for the wavelengths in front of and behind the source. Then $f = v/\lambda$. When the source is at rest $\lambda = \dfrac{v}{f_S} = \dfrac{344\text{ m/s}}{400\text{ Hz}} = 0.860\text{ m}$.

EXECUTE: **(a)** Eq. (16.27): $\lambda = \dfrac{v-v_S}{f_S} = \dfrac{344\text{ m/s}-25.0\text{ m/s}}{400\text{ Hz}} = 0.798\text{ m}$

(b) Eq. (16.28): $\lambda = \dfrac{v+v_S}{f_S} = \dfrac{344\text{ m/s}+25.0\text{ m/s}}{400\text{ Hz}} = 0.922\text{ m}$

(c) $f_L = v/\lambda$ (since $v_L = 0$), so $f_L = (344\text{ m/s})/0.798\text{ m} = 431\text{ Hz}$

(d) $f_L = v/\lambda = (344\text{ m/s})/0.922\text{ m} = 373\text{ Hz}$

EVALUATE: In front of the source (source moving toward listener) the wavelength is decreased and the frequency is increased. Behind the source (source moving away from listener) the wavelength is increased and the frequency is decreased.

16.47. **IDENTIFY:** The distance between crests is λ. In front of the source $\lambda = \dfrac{v-v_S}{f_S}$ and behind the source $\lambda = \dfrac{v+v_S}{f_S}$. $f_S = 1/T$.

SET UP: $T = 1.6$ s. $v = 0.32$ m/s. The crest to crest distance is the wavelength, so $\lambda = 0.12$ m.

EXECUTE: **(a)** $f_S = 1/T = 0.625\text{ Hz}$. $\lambda = \dfrac{v-v_S}{f_S}$ gives

$v_S = v - \lambda f_S = 0.32\text{ m/s} - (0.12\text{ m})(0.625\text{ Hz}) = 0.25\text{ m/s}.$

(b) $\lambda = \dfrac{v+v_S}{f_S} = \dfrac{0.32\text{ m/s}+0.25\text{ m/s}}{0.625\text{ Hz}} = 0.91\text{ m}$

EVALUATE: If the duck was held at rest but still paddled its feet, it would produce waves of wavelength $\lambda = \dfrac{0.32\text{ m/s}}{0.625\text{ Hz}} = 0.51\text{ m}$. In front of the duck the wavelength is decreased and behind the duck the wavelength is increased. The speed of the duck is 78% of the wave speed, so the Doppler effects are large.

16.48. **IDENTIFY:** Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$.

SET UP: $f_S = 1000\text{ Hz}$. The positive direction is from the listener to the source. $v = 344\text{ m/s}$.

EXECUTE: **(a)** $v_S = -(344\text{ m/s})/2 = -172\text{ m/s}$, $v_L = 0$.

$$f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344\text{ m/s}}{344\text{ m/s} - 172\text{ m/s}}\right)(1000\text{ Hz}) = 2000\text{ Hz}$$

(b) $v_S = 0$, $v_L = +172\text{ m/s}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s} + 172\text{ m/s}}{344\text{ m/s}}\right)(1000\text{ Hz}) = 1500\text{ Hz}$

EVALUATE: The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

16.49. **IDENTIFY:** Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$.

SET UP: The positive direction is from the motorcycle toward the car. The car is stationary, so $v_S = 0$.

EXECUTE: $f_L = \dfrac{v+v_L}{v+v_S}f_S = (1 + v_L/v)f_S$, which gives

$$v_L = v\left(\frac{f_L}{f_S} - 1\right) = (344\text{ m/s})\left(\frac{490\text{ Hz}}{520\text{ Hz}} - 1\right) = -19.8\text{ m/s}.$$ You must be traveling at 19.8 m/s.

EVALUATE: $v_L < 0$ means that the listener is moving away from the source.

16.50. **IDENTIFY:** Apply the Doppler effect formula, Eq. (16.29).

(a) SET UP: The positive direction is from the listener toward the source, as shown in Figure 16.50a.

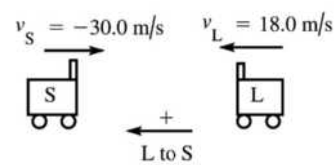


Figure 16.50a

EXECUTE: $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s} + 18.0\text{ m/s}}{344\text{ m/s} - 30.0\text{ m/s}}\right)(262\text{ Hz}) = 302\text{ Hz}$

EVALUATE: Listener and source are approaching and $f_L > f_S$.

(b) SET UP: See Figure 16.50b.

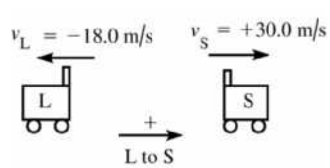


Figure 16.50b

EXECUTE: $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s} - 18.0\text{ m/s}}{344\text{ m/s} + 30.3\text{ m/s}}\right)(262\text{ Hz}) = 288\text{ Hz}$

EVALUATE: Listener and source are moving away from each other and $f_L < f_S$.

16.51. **IDENTIFY:** Each bird is a moving source of sound and a moving observer, so each will experience a Doppler shift.

SET UP: Let one bird be the listener and the other be the source. Use coordinates as shown in Figure 16.51, with the positive direction from listener to source. $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$.

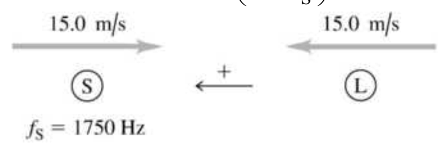


Figure 16.51

EXECUTE: **(a)** $f_S = 1750\text{ Hz}$, $v_S = -15.0\text{ m/s}$, and $v_L = +15.0\text{ m/s}$.

$f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344\text{ m/s} + 15.0\text{ m/s}}{344\text{ m/s} - 15.0\text{ m/s}}\right)(1750\text{ Hz}) = 1910\text{ Hz}.$

(b) One canary hears a frequency of 1910 Hz and the waves move past it at $344\text{ m/s} + 15\text{ m/s}$, so the wavelength it detects is $\lambda = \dfrac{344\text{ m/s} + 15\text{ m/s}}{1910\text{ Hz}} = 0.188\text{ m}$. For a stationary bird, $\lambda = \dfrac{344\text{ m/s}}{1750\text{ Hz}} = 0.197\text{ m}$.

EVALUATE: The approach of the two birds raises the frequency, and the motion of the source toward the listener decreases the wavelength.

16.52. **IDENTIFY:** There is a Doppler shift due to the motion of the fire engine as well as due to the motion of the truck, which reflects the sound waves.

SET UP: We use the Doppler shift equation $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$.

EXECUTE: **(a)** First consider the truck as the listener, as shown in Figure 16.52a.

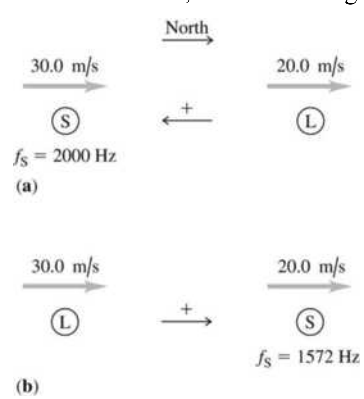


Figure 16.52

$f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344\text{ m/s} - 20.0\text{ m/s}}{344\text{ m/s} - 30.0\text{ m/s}}\right)(2000\text{ Hz}) = 2064\text{ Hz}$. Now consider the truck as a source, with $f_S = 2064\text{ Hz}$, and the fire engine driver as the listener (Figure 16.52b).

$f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344\text{ m/s} + 30.0\text{ m/s}}{344\text{ m/s} + 20.0\text{ m/s}}\right)(2064\text{ Hz}) = 2120\text{ Hz}$. The objects are getting closer together so the frequency is increased.

(b) The driver detects a frequency of 2120 Hz and the waves returning from the truck move past him at $344\text{ m/s} + 30.0\text{ m/s}$, so the wavelength he measures is $\lambda = \dfrac{344\text{ m/s} + 30\text{ m/s}}{2120\text{ Hz}} = 0.176\text{ m}$. The wavelength of waves emitted by the fire engine when it is stationary is $\lambda = \dfrac{344\text{ m/s}}{2000\text{ Hz}} = 0.172\text{ m}$.

EVALUATE: In (a) the objects are getting closer together so the frequency is increased. In (b), the quantities to use in the equation $v = f\lambda$ are measured *relative to the observer*.

16.53. **IDENTIFY:** Apply Eq. (16.30).

SET UP: Require $f_R = 1.100 f_S$. Since $f_R > f_S$ the star would be moving toward us and $v < 0$, so $v = -|v|$. $c = 3.00\times10^8$ m/s.

EXECUTE: $f_R = \sqrt{\dfrac{c+|v|}{c-|v|}} f_S$. $f_R = 1.100 f_S$ gives $\dfrac{c+|v|}{c-|v|} = (1.100)^2$. Solving for $|v|$ gives

$$|v| = \frac{[(1.100)^2 - 1]c}{1+(1.100)^2} = 0.0950c = 2.85\times10^7 \text{ m/s}.$$

EVALUATE: $\dfrac{v}{c} = 9.5\%$. $\dfrac{\Delta f}{f_S} = \dfrac{f_R - f_S}{f_S} = 10.0\%$. $\dfrac{v}{c}$ and $\dfrac{\Delta f}{f_S}$ are approximately equal.

16.54. **IDENTIFY:** Apply Eq. (16.30). The source is moving away, so v is positive.

SET UP: $c = 3.00\times10^8$ m/s. $v = +50.0\times10^3$ m/s.

EXECUTE: $f_R = \sqrt{\dfrac{c-v}{c+v}} f_S = \sqrt{\dfrac{3.00\times10^8 \text{ m/s} - 50.0\times10^3 \text{ m/s}}{3.00\times10^8 \text{ m/s} + 50.0\times10^3 \text{ m/s}}}(3.330\times10^{14} \text{ Hz}) = 3.329\times10^{14}$ Hz

EVALUATE: $f_R < f_S$ since the source is moving away. The difference between f_R and f_S is very small since $v \ll c$.

16.55. **IDENTIFY:** Apply Eq. (16.31) to calculate α. Use the method of Example 16.19 to calculate t.

SET UP: Mach 1.70 means $v_S/v = 1.70$.

EXECUTE: **(a)** In Eq. (16.31), $v/v_S = 1/1.70 = 0.588$ and $\alpha = \arcsin(0.588) = 36.0°$.

(b) As in Example 16.19, $t = \dfrac{(950 \text{ m})}{(1.70)(344 \text{ m/s})(\tan(36.0°))} = 2.23$ s.

EVALUATE: The angle α decreases when the speed v_S of the plane increases.

16.56. **IDENTIFY:** Apply Eq. (16.31).

SET UP: The Mach number is the value of v_S/v, where v_S is the speed of the shuttle and v is the speed of sound at the altitude of the shuttle.

EXECUTE: **(a)** $\dfrac{v}{v_S} = \sin\alpha = \sin 58.0° = 0.848$. The Mach number is $\dfrac{v_S}{v} = \dfrac{1}{0.848} = 1.18$.

(b) $v_S = \dfrac{v}{\sin\alpha} = \dfrac{331 \text{ m/s}}{\sin 58.0°} = 390$ m/s

(c) $\dfrac{v_S}{v} = \dfrac{390 \text{ m/s}}{344 \text{ m/s}} = 1.13$. The Mach number would be 1.13. $\sin\alpha = \dfrac{v}{v_S} = \dfrac{344 \text{ m/s}}{390 \text{ m/s}}$ and $\alpha = 61.9°$.

EVALUATE: The smaller the Mach number, the larger the angle of the shock-wave cone.

16.57. **IDENTIFY:** $f_{beat} = |f - f_0|$. $f = \dfrac{v}{2L}$. Changing the tension changes the wave speed and this alters the frequency.

SET UP: $v = \sqrt{\dfrac{FL}{m}}$ so $f = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$, where $F = F_0 + \Delta F$. Let $f_0 = \dfrac{1}{2}\sqrt{\dfrac{F_0}{mL}}$. We can assume that $\Delta F/F_0$ is very small. Increasing the tension increases the frequency, so $f_{beat} = f - f_0$.

EXECUTE: **(a)** $f_{beat} = f - f_0 = \dfrac{1}{2\sqrt{mL}}\left(\sqrt{F_0 + \Delta F} - \sqrt{F_0}\right) = \dfrac{1}{2}\sqrt{\dfrac{F_0}{mL}}\left(\left[1 + \dfrac{\Delta F}{F_0}\right]^{1/2} - 1\right)$.

$\left[1 + \dfrac{\Delta F}{F_0}\right]^{1/2} = 1 + \dfrac{\Delta F}{2F_0}$ when $\Delta F/F_0$ is small. This gives that $f_{beat} = f_0\left(\dfrac{\Delta F}{2F_0}\right)$.

(b) $\dfrac{\Delta F}{F_0} = \dfrac{2f_{\text{beat}}}{f_0} = \dfrac{2(1.5\text{ Hz})}{440\text{ Hz}} = 0.68\%.$

EVALUATE: The fractional change in frequency is one-half the fractional change in tension.

16.58. **IDENTIFY:** The displacement $y(x,t)$ is given in Eq. (16.1) and the pressure variation is given in Eq. (16.4). The pressure variation is related to the displacement by Eq. (16.3).

SET UP: $k = 2\pi/\lambda$

EXECUTE: **(a)** Mathematically, the waves given by Eq. (16.1) and Eq. (16.4) are out of phase. Physically, at a displacement node, the air is most compressed or rarefied on either side of the node, and the pressure gradient is zero. Thus, displacement nodes are pressure antinodes.

(b) The graphs have the same form as in Figure 16.3 in the textbook.

(c) $p(x,t) = -B\dfrac{\partial y(x,t)}{\partial x}$. When $y(x,t)$ versus x is a straight line with positive slope, $p(x,t)$ is constant and negative. When $y(x,t)$ versus x is a straight line with negative slope, $p(x,t)$ is constant and positive. The graph of $p(x,0)$ is given in Figure 16.58. The slope of the straightline segments for $y(x,0)$ is 1.6×10^{-4}, so for the wave in Figure P16.58 in the textbook, $p_{\text{max-non}} = (1.6\times10^{-4})B$. The sinusoidal wave has amplitude $p_{\text{max}} = BkA = (2.5\times10^{-4})B$. The difference in the pressure amplitudes is because the two $y(x,0)$ functions have different slopes.

EVALUATE: **(d)** $p(x,t)$ has its largest magnitude where $y(x,t)$ has the greatest slope. This is where $y(x,t) = 0$ for a sinusoidal wave but it is not true in general.

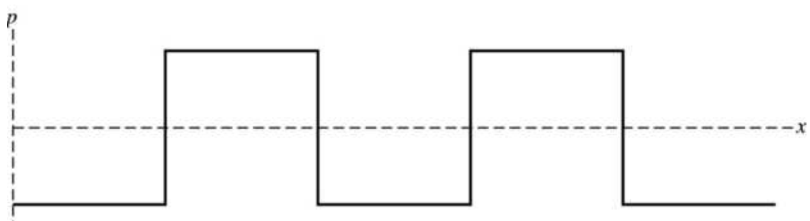


Figure 16.58

16.59. **IDENTIFY:** The sound intensity level is $\beta = (10\text{ dB})\log(I/I_0)$, so the same sound intensity level β means the same intensity I. The intensity is related to pressure amplitude by Eq. (16.13) and to the displacement amplitude by Eq. (16.12).

SET UP: $v = 344$ m/s. $\omega = 2\pi f$. Each octave higher corresponds to a doubling of frequency, so the note sung by the bass has frequency $(932\text{ Hz})/8 = 116.5$ Hz. Let 1 refer to the note sung by the soprano and 2 refer to the note sung by the bass. $I_0 = 1\times10^{-12}\text{ W/m}^2$.

EXECUTE: **(a)** $I = \dfrac{vp_{\text{max}}^2}{2B}$ and $I_1 = I_2$ gives $p_{\text{max},1} = p_{\text{max},2}$; the ratio is 1.00.

(b) $I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 = \frac{1}{2}\sqrt{\rho B}4\pi^2 f^2 A^2$. $I_1 = I_2$ gives $f_1 A_1 = f_2 A_2$. $\dfrac{A_2}{A_1} = \dfrac{f_1}{f_2} = 8.00.$

(c) $\beta = 72.0$ dB gives $\log(I/I_0) = 7.2$. $\dfrac{I}{I_0} = 10^{7.2}$ and $I = 1.585\times10^{-5}\text{ W/m}^2$. $I = \frac{1}{2}\sqrt{\rho B}4\pi^2 f^2 A^2$.

$$A = \frac{1}{2\pi f}\sqrt{\frac{2I}{\sqrt{\rho B}}} = \frac{1}{2\pi(932\text{ Hz})}\sqrt{\frac{2(1.585\times10^{-5}\text{ W/m}^2)}{\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}}} = 4.73\times10^{-8}\text{ m} = 47.3\text{ nm}.$$

EVALUATE: Even for this loud note the displacement amplitude is very small. For a given intensity, the displacement amplitude depends on the frequency of the sound wave but the pressure amplitude does not.

16.60. **IDENTIFY:** Use the equations that relate intensity level and intensity, intensity and pressure amplitude, pressure amplitude and displacement amplitude, and intensity and distance.

(a) SET UP: Use the intensity level β to calculate I at this distance. $\beta = (10 \text{ dB})\log(I/I_0)$

EXECUTE: $52.0 \text{ dB} = (10 \text{ dB})\log(I/(10^{-12} \text{ W/m}^2))$

$\log(I/(10^{-12} \text{ W/m}^2)) = 5.20$ implies $I = 1.585\times10^{-7} \text{ W/m}^2$

SET UP: Then use Eq. (16.14) to calculate $p_{\max}$:

$I = \dfrac{p_{\max}^2}{2\rho v}$ so $p_{\max} = \sqrt{2\rho v I}$

From Example 16.5, $\rho = 1.20 \text{ kg/m}^3$ for air at $20°\text{C}$.

EXECUTE: $p_{\max} = \sqrt{2\rho v I} = \sqrt{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})(1.585\times10^{-7} \text{ W/m}^2)} = 0.0114 \text{ Pa}$

(b) SET UP: Eq. (16.5): $p_{\max} = BkA$ so $A = \dfrac{p_{\max}}{Bk}$

For air $B = 1.42\times10^5$ Pa (Example 16.1).

EXECUTE: $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{v} = \dfrac{(2\pi \text{ rad})(587 \text{ Hz})}{344 \text{ m/s}} = 10.72 \text{ rad/m}$

$A = \dfrac{p_{\max}}{Bk} = \dfrac{0.0114 \text{ Pa}}{(1.42\times10^5 \text{ Pa})(10.72 \text{ rad/m})} = 7.49\times10^{-9} \text{m}$

(c) SET UP: $\beta_2 - \beta_1 = (10 \text{ dB})\log(I_2/I_1)$ (Example 16.9).

Eq. (15.26): $I_1/I_2 = r_2^2/r_1^2$ so $I_2/I_1 = r_1^2/r_2^2$

EXECUTE: $\beta_2 - \beta_1 = (10 \text{ dB})\log(r_1/r_2)^2 = (20 \text{ dB})\log(r_1/r_2)$.

$\beta_2 = 52.0 \text{ dB}$ and $r_2 = 5.00$ m. Then $\beta_1 = 30.0$ dB and we need to calculate r_1.

$52.0 \text{ dB} - 30.0 \text{ dB} = (20 \text{ dB})\log(r_1/r_2)$

$22.0 \text{ dB} = (20 \text{ dB})\log(r_1/r_2)$

$\log(r_1/r_2) = 1.10$ so $r_1 = 12.6 r_2 = 63.0$ m.

EVALUATE: The decrease in intensity level corresponds to a decrease in intensity, and this means an increase in distance. The intensity level uses a logarithmic scale, so simple proportionality between r and β doesn't apply.

16.61. **IDENTIFY:** The sound is first loud when the frequency f_0 of the speaker equals the frequency f_1 of the fundamental standing wave for the gas in the tube. The tube is a stopped pipe, and $f_1 = \dfrac{v}{4L}$. $v = \sqrt{\dfrac{\gamma RT}{M}}$.

The sound is next loud when the speaker frequency equals the first overtone frequency for the tube.

SET UP: A stopped pipe has only odd harmonics, so the frequency of the first overtone is $f_3 = 3f_1$.

EXECUTE: **(a)** $f_0 = f_1 = \dfrac{v}{4L} = \dfrac{1}{4L}\sqrt{\dfrac{\gamma RT}{M}}$. This gives $T = \dfrac{16L^2 M f_0{}^2}{\gamma R}$.

(b) $3f_0$.

EVALUATE: **(c)** Measure f_0 and L. Then $f_0 = \dfrac{v}{4L}$ gives $v = 4Lf_0$.

16.62. **IDENTIFY:** $f_{\text{beat}} = |f_A - f_B|$. $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{FL}{m}}$ gives $f_1 = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. Apply $\Sigma\tau_z = 0$ to the bar to find the tension in each wire.

SET UP: For $\Sigma\tau_z = 0$ take the pivot at wire A and let counterclockwise torques be positive. The free-body diagram for the bar is given in Figure 16.62. Let L be the length of the bar.

EXECUTE: $\Sigma\tau_z = 0$ gives $F_B L - w_{\text{lead}}(3L/4) - w_{\text{bar}}(L/2) = 0$.

$F_B = 3w_{\text{lead}}/4 + w_{\text{bar}}/2 = 3(185 \text{ N})/4 + (165 \text{ N})/2 = 221 \text{ N}$. $F_A + F_B = w_{\text{bar}} + w_{\text{lead}}$ so

$F_A = w_{bar} + w_{lead} - F_B = 165\text{ N} + 185\text{ N} - 221\text{ N} = 129\text{ N}$. $f_{1A} = \dfrac{1}{2}\sqrt{\dfrac{129\text{ N}}{(5.50\times10^{-3}\text{ kg})(0.750\text{ m})}} = 88.4\text{ Hz}$.

$f_{1B} = f_{1A}\sqrt{\dfrac{221\text{ N}}{129\text{ N}}} = 115.7\text{ Hz}$. $f_{beat} = f_{1B} - f_{1A} = 27.3\text{ Hz}$.

EVALUATE: The frequency increases when the tension in the wire increases.

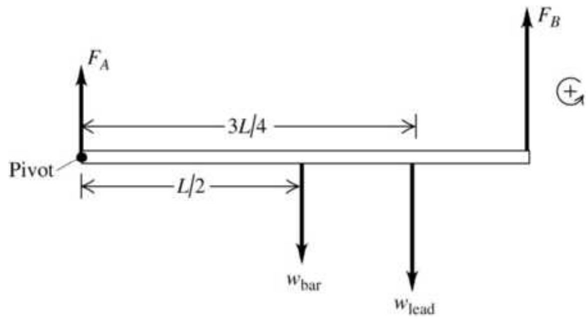


Figure 16.62

16.63. **IDENTIFY:** The flute acts as a stopped pipe and its harmonic frequencies are given by Eq. (16.23). The resonant frequencies of the string are $f_n = nf_1, n = 1, 2, 3, \ldots$ The string resonates when the string frequency equals the flute frequency.

SET UP: For the string $f_{1s} = 600.0\text{ Hz}$. For the flute, the fundamental frequency is $f_{1f} = \dfrac{v}{4L} = \dfrac{344.0\text{ m/s}}{4(0.1075\text{ m})} = 800.0\text{ Hz}$. Let n_f label the harmonics of the flute and let n_s label the harmonics of the string.

EXECUTE: For the flute and string to be in resonance, $n_f f_{1f} = n_s f_{1s}$, where $f_{1s} = 600.0\text{ Hz}$ is the fundamental frequency for the string. $n_s = n_f(f_{1f}/f_{1s}) = \frac{4}{3}n_f$. n_s is an integer when $n_f = 3N, N = 1, 3, 5, \ldots$ (the flute has only odd harmonics). $n_f = 3N$ gives $n_s = 4N$.

Flute harmonic $3N$ resonates with string harmonic $4N, N = 1, 3, 5, \ldots$

EVALUATE: We can check our results for some specific values of N. For $N = 1$, $n_f = 3$ and $f_{3f} = 2400\text{ Hz}$. For this N, $n_s = 4$ and $f_{4s} = 2400\text{ Hz}$. For $N = 3$, $n_f = 9$ and $f_{9f} = 7200\text{ Hz}$, and $n_s = 12$, $f_{12s} = 7200\text{ Hz}$. Our general results do give equal frequencies for the two objects.

16.64. **IDENTIFY:** The harmonics of the string are $f_n = nf_1 = n\left(\dfrac{v}{2l}\right)$, where l is the length of the string. The tube is a stopped pipe and its standing wave frequencies are given by Eq. (16.22). For the string, $v = \sqrt{F/\mu}$, where F is the tension in the string.

SET UP: The length of the string is $d = L/10$, so its third harmonic has frequency $f_3^{string} = 3\dfrac{1}{2d}\sqrt{F/\mu}$.

The stopped pipe has length L, so its first harmonic has frequency $f_1^{pipe} = \dfrac{v_s}{4L}$.

EXECUTE: **(a)** Equating f_1^{string} and f_1^{pipe} and using $d = L/10$ gives $F = \dfrac{1}{3600}\mu v_s^2$.

(b) If the tension is doubled, all the frequencies of the string will increase by a factor of $\sqrt{2}$. In particular, the third harmonic of the string will no longer be in resonance with the first harmonic of the pipe because the frequencies will no longer match, so the sound produced by the instrument will be diminished.

(c) The string will be in resonance with a standing wave in the pipe when their frequencies are equal. Using $f_1^{pipe} = 3f_1^{string}$, the frequencies of the pipe are $nf_1^{pipe} = 3nf_1^{string}$ (where $n = 1, 3, 5, \ldots$). Setting this equal to the frequencies of the string $n'f_1^{\text{string}}$, the harmonics of the string are $n' = 3n = 3, 9, 15, \ldots$ The *n*th harmonic of the pipe is in resonance with the 3*n*th harmonic of the string.

EVALUATE: Each standing wave for the air column is in resonance with a standing wave on the string. But the reverse is not true; not all standing waves of the string are in resonance with a harmonic of the pipe.

16.65. **IDENTIFY** and **SET UP:** The frequency of any harmonic is an integer multiple of the fundamental. For a stopped pipe only odd harmonics are present. For an open pipe, all harmonics are present. See which pattern of harmonics fits to the observed values in order to determine which type of pipe it is. Then solve for the fundamental frequency and relate that to the length of the pipe.

EXECUTE: **(a)** For an open pipe the successive harmonics are $f_n = nf_1$, $n = 1, 2, 3, \ldots$ For a stopped pipe the successive harmonics are $f_n = nf_1$, $n = 1, 3, 5, \ldots.$ If the pipe is open and these harmonics are successive, then $f_n = nf_1 = 1372$ Hz and $f_{n+1} = (n+1)f_1 = 1764$ Hz. Subtract the first equation from the second: $(n+1)f_1 - nf_1 = 1764 \text{ Hz} - 1372 \text{ Hz}$. This gives $f_1 = 392$ Hz. Then $n = \dfrac{1372 \text{ Hz}}{392 \text{ Hz}} = 3.5$. But *n* must be an integer, so the pipe can't be open. If the pipe is stopped and these harmonics are successive, then $f_n = nf_1 = 1372$ Hz and $f_{n+2} = (n+2)f_1 = 1764$ Hz (in this case successive harmonics differ in *n* by 2). Subtracting one equation from the other gives $2f_1 = 392$ Hz and $f_1 = 196$ Hz. Then $n = 1372 \text{ Hz}/f_1 = 7$ so $1372 \text{ Hz} = 7f_1$ and $1764 \text{ Hz} = 9f_1$. The solution gives integer *n* as it should; the pipe is stopped.

(b) From part (a) these are the 7th and 9th harmonics.

(c) From part (a) $f_1 = 196$ Hz.

For a stopped pipe $f_1 = \dfrac{v}{4L}$ and $L = \dfrac{v}{4f_1} = \dfrac{344 \text{ m/s}}{4(196 \text{ Hz})} = 0.439$ m.

EVALUATE: It is essential to know that these are successive harmonics and to realize that 1372 Hz is not the fundamental. There are other lower frequency standing waves; these are just two successive ones.

16.66. **IDENTIFY:** The steel rod has standing waves much like a pipe open at both ends, since the ends are both displacement antinodes. An integral number of half wavelengths must fit on the rod, that is, $f_n = \dfrac{nv}{2L}$, with $n = 1, 2, 3, \ldots$

SET UP: Table 16.1 gives $v = 5941$ m/s for longitudinal waves in steel.

EXECUTE: **(a)** The ends of the rod are antinodes because the ends of the rod are free to oscillate.

(b) The fundamental can be produced when the rod is held at the middle because a node is located there.

(c) $f_1 = \dfrac{(1)(5941 \text{ m/s})}{2(1.50 \text{ m})} = 1980$ Hz

(d) The next harmonic is $n = 2$, or $f_2 = 3961$ Hz. We would need to hold the rod at an $n = 2$ node, which is located at $L/4 = 0.375$ m from either end.

EVALUATE: For the 1.50 m long rod the wavelength of the fundamental is $x = 2L = 3.00$ m. The node to antinode distance is $\lambda/4 = 0.75$ m. For the second harmonic $\lambda = L = 1.50$ m and the node to antinode distance is 0.375 m. There is a node at the middle of the rod, but forcing a node at 0.375 m from one end, by holding the rod there, prevents the rod from vibrating in the fundamental.

16.67. **IDENTIFY** and **SET UP:** There is a node at the piston, so the distance the piston moves is the node to node distance, $\lambda/2$. Use Eq. (15.1) to calculate *v* and Eq. (16.10) to calculate γ from *v*.

EXECUTE: **(a)** $\lambda/2 = 37.5$ cm, so $\lambda = 2(37.5 \text{ cm}) = 75.0 \text{ cm} = 0.750$ m.

$$v = f\lambda = (500 \text{ Hz})(0.750 \text{ m}) = 375 \text{ m/s}$$

(b) $v = \sqrt{\gamma RT/M}$ (Eq. 16.10)

$$\gamma = \frac{Mv^2}{RT} = \frac{(28.8\times10^{-3} \text{ kg/mol})(375 \text{ m/s})^2}{(8.3145 \text{ J/mol}\cdot\text{K})(350 \text{ K})} = 1.39.$$

(c) EVALUATE: There is a node at the piston so when the piston is 18.0 cm from the open end the node is inside the pipe, 18.0 cm from the open end. The node to antinode distance is $\lambda/4 = 18.8$ cm, so the antinode is 0.8 cm beyond the open end of the pipe.
The value of γ we calculated agrees with the value given for air in Example 16.4.

16.68. **IDENTIFY:** For a stopped pipe the frequency of the fundamental is $f_1 = \dfrac{v}{4L}$. The speed of sound in air depends on temperature, as shown by Eq. (16.10).
SET UP: Example 16.4 shows that the speed of sound in air at 20°C is 344 m/s.

EXECUTE: **(a)** $L = \dfrac{v}{4f} = \dfrac{344\text{ m/s}}{4(349\text{ Hz})} = 0.246\text{ m}$

(b) The frequency will be proportional to the speed, and hence to the square root of the Kelvin temperature. The temperature necessary to have the frequency be higher is $(293.15\text{ K})([370\text{ Hz}]/[349\text{ Hz}])^2 = 329.5\text{ K}$, which is 56.3°C.
EVALUATE: $56.3°\text{C} = 133°\text{F}$, so this extreme rise in pitch won't occur in practical situations. But changes in temperature can have noticeable effects on the pitch of the organ notes.

16.69. **IDENTIFY:** $v = f\lambda$. $v = \sqrt{\dfrac{\gamma RT}{M}}$. Solve for γ.

SET UP: The wavelength is twice the separation of the nodes, so $\lambda = 2L$, where $L = 0.200$ m.

EXECUTE: $v = \lambda f = 2Lf = \sqrt{\dfrac{\gamma RT}{M}}$. Solving for γ,

$$\gamma = \frac{M}{RT}(2Lf)^2 = \frac{(16.0\times10^{-3}\text{ kg/mol})}{(8.3145\text{ J/mol}\cdot\text{K})\ (293.15\text{ K})}(2(0.200\text{ m})(1100\text{ Hz}))^2 = 1.27.$$

EVALUATE: This value of γ is smaller than that of air. We will see in Chapter 19 that this value of γ is a typical value for polyatomic gases.

16.70. **IDENTIFY:** Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

SET UP: $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}$

EXECUTE: **(a)** If the separation of the speakers is denoted h, the condition for destructive interference is $\sqrt{x^2 + h^2} - x = \beta\lambda$, where β is an odd multiple of one-half. Adding x to both sides, squaring, canceling the x^2 term from both sides and solving for x gives $x = \dfrac{h^2}{2\beta\lambda} - \dfrac{\beta}{2}\lambda$. Using $\lambda = 0.439$ m and $h = 2.00$ m yields 9.01 m for $\beta = \frac{1}{2}$, 2.71 m for $\beta = \frac{3}{2}$, 1.27 m for $\beta = \frac{5}{2}$, 0.53 m for $\beta = \frac{7}{2}$, and 0.026 m for $\beta = \frac{9}{2}$. These are the only allowable values of β that give positive solutions for x.
(b) Repeating the above for integral values of β, constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).
(c) If $h = \lambda/2$, there will be destructive interference at speaker B. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344\text{ m/s})/(4.0\text{ m}) = 86\text{ Hz}$.
EVALUATE: When f increases, λ is smaller and there are more occurrences of points of constructive and destructive interference.

16.71. **IDENTIFY:** Apply $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$.

SET UP: The positive direction is from the listener to the source. (a) The wall is the listener. $v_S = -30$ m/s. $v_L = 0$. $f_L = 600$ Hz. (b) The wall is the source and the car is the listener. $v_S = 0$. $v_L = +30$ m/s. $f_S = 600$ Hz.

EXECUTE: **(a)** $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. $f_S = \left(\dfrac{v+v_S}{v+v_L}\right)f_L = \left(\dfrac{344\text{ m/s} - 30\text{ m/s}}{344\text{ m/s}}\right)(600\text{ Hz}) = 548\text{ Hz}$

(b) $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s}+30\text{ m/s}}{344\text{ m/s}}\right)(600\text{ Hz}) = 652\text{ Hz}$

EVALUATE: Since the singer and wall are moving toward each other the frequency received by the wall is greater than the frequency sung by the soprano, and the frequency she hears from the reflected sound is larger still.

16.72. **IDENTIFY:** Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. The wall first acts as a listener and then as a source.

SET UP: The positive direction is from listener to source. The bat is moving toward the wall so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the original source frequency, f_{S1}. $f_{S1} = 1700$ Hz. $f_{L2} - f_{S1} = 10.0$ Hz.

EXECUTE: The wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = -v_{bat}$ and $v_L = 0$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives $f_{L1} = \left(\dfrac{v}{v-v_{bat}}\right)f_{S1}$. The wall receives the sound: $f_{S2} = f_{L1}$. $v_S = 0$ and $v_L = +v_{bat}$.

$$f_{L2} = \left(\frac{v+v_{bat}}{v}\right)f_{S2} = \left(\frac{v+v_{bat}}{v}\right)\left(\frac{v}{v-v_{bat}}\right)f_{S1} = \left(\frac{v+v_{bat}}{v-v_{bat}}\right)f_{S1}.$$

$$f_{L2} - f_{S1} = \Delta f = \left(\frac{v+v_{bat}}{v-v_{bat}} - 1\right)f_{S1} = \left(\frac{2v_{bat}}{v-v_{bat}}\right)f_{S1}.\quad v_{bat} = \frac{v\Delta f}{2f_{S1}+\Delta f} = \frac{(344\text{ m/s})(10.0\text{ Hz})}{2(1700\text{ Hz})+10.0\text{ Hz}} = 1.01\text{ m/s}.$$

EVALUATE: $f_{S1} < \Delta f$, so we can write our result as the approximate but accurate expression $\Delta f = \left(\dfrac{2v_{bat}}{v}\right)f_{S1}$.

16.73. **IDENTIFY:** For the sound coming directly to the observer at the top of the well, the source is moving away from the listener. For the reflected sound, the water at the bottom of the well is the "listener" so the source is moving toward the listener. The water reflects the same frequency sound it receives.

SET UP: $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. Take the positive direction to be from the listener to the source. For reflection off the bottom of the well the water surface first serves as a listener and then as a source. The falling siren has constant downward acceleration of g and obeys the equation $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$.

EXECUTE: For the falling siren, $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$, so the speed of the siren just before it hits the water is $\sqrt{2(9.80\text{ m/s}^2)(125\text{ m})} = 49.5$ m/s.

(a) The situation is shown in Figure 16.73a.

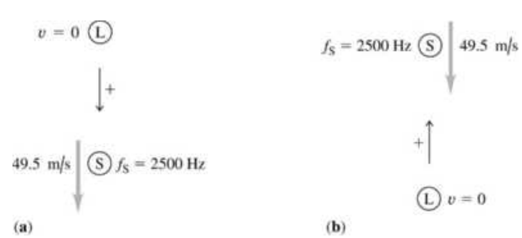


Figure 16.73

$$f_L = \left(\frac{v}{v+49.5\text{ m/s}}\right)f_S = \left(\frac{344\text{ m/s}}{344\text{ m/s}+49.5\text{ m/s}}\right)(2500\text{ Hz}) = 2186\text{ Hz}.\quad \lambda_L = \frac{v}{f_L} = \frac{344\text{ m/s}}{2186\text{ Hz}} = 0.157\text{ m}.$$

(b) The water serves as a listener (Figure 16.73b). $f_L = \left(\dfrac{v}{v - 49.5\text{ m/s}}\right) f_S = 2920\text{ Hz}$. The source and listener are approaching and the frequency is raised. $\lambda_L = \dfrac{v}{f_L} = 0.118\text{ m}$. Both the person and the water are at rest so there is no Doppler effect when the water serves as a source and the person is the listener. The person detects sound with frequency 2920 Hz and wavelength 0.118 m.

(c) $f_{\text{beat}} = f_1 - f_2 = 2920\text{ Hz} - 2186\text{ Hz} = 734\text{ Hz}$.

EVALUATE: In (a), the source is moving away from the listener and the frequency is lowered. In (b) the source is moving toward the "listener" so the frequency is increased.

16.74. **IDENTIFY:** Apply $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$. The heart wall first acts as the listener and then as the source.

SET UP: The positive direction is from listener to source. The heart wall is moving toward the receiver so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the source frequency, f_{S1}. $f_{L2} - f_{S1} = 72\text{ Hz}$.

EXECUTE: Heart wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = 0$. $v_L = -v_{\text{wall}}$. $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$ gives $f_{L1} = \left(\dfrac{v - v_{\text{wall}}}{v}\right) f_{S1}$.

Heart wall emits the sound: $f_{S2} = f_{L1}$. $v_S = +v_{\text{wall}}$. $v_L = 0$.

$$f_{L2} = \left(\frac{v}{v + v_{\text{wall}}}\right) f_{S2} = \left(\frac{v}{v + v_{\text{wall}}}\right)\left(\frac{v - v_{\text{wall}}}{v}\right) f_{S1} = \left(\frac{v - v_{\text{wall}}}{v + v_{\text{wall}}}\right) f_{S1}.$$

$f_{L2} - f_{S1} = \left(1 - \dfrac{v - v_{\text{wall}}}{v + v_{\text{wall}}}\right) f_{S1} = \left(\dfrac{2v_{\text{wall}}}{v + v_{\text{wall}}}\right) f_{S1}$. $v_{\text{wall}} = \dfrac{(f_{L2} - f_{S1})v}{2f_{S1} - (f_{L2} - f_{S1})}$. $f_{S1} \gg f_{L2} - f_{S1}$ and

$$v_{\text{wall}} = \frac{(f_{L2} - f_{S1})v}{2f_{S1}} = \frac{(72\text{ Hz})(1500\text{ m/s})}{2(2.00\times 10^6\text{ Hz})} = 0.0270\text{ m/s} = 2.70\text{ cm/s}.$$

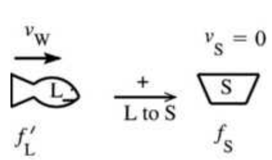
EVALUATE: $f_{S1} = 2.00\times 10^6\text{ Hz}$ and $f_{L2} - f_{S1} = 72\text{ Hz}$, so the approximation we made is very accurate. Within this approximation, the frequency difference between the reflected and transmitted waves is directly proportional to the speed of the heart wall.

16.75. **(a) IDENTIFY** and **SET UP:** Use Eq. (15.1) to calculate λ.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{1482\text{ m/s}}{22.0\times 10^3\text{ Hz}} = 0.0674\text{ m}$

(b) IDENTIFY: Apply the Doppler effect equation, Eq. (16.29). The Problem-Solving Strategy in the text (Section 16.8) describes how to do this problem. The frequency of the directly radiated waves is $f_S = 22{,}000\text{ Hz}$. The moving whale first plays the role of a moving listener, receiving waves with frequency f_L'. The whale then acts as a moving source, emitting waves with the same frequency, $f_S' = f_L'$ with which they are received. Let the speed of the whale be v_W.

SET UP: <u>whale receives waves</u> (Figure 16.75a)

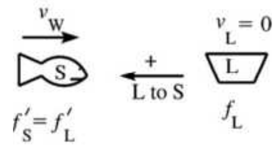


EXECUTE: $v_L = +v_W$

$$f_L' = f_S\left(\frac{v + v_L}{v + v_S}\right) = f_S\left(\frac{v + v_W}{v}\right)$$

Figure 16.75a

SET UP: whale re-emits the waves (Figure 16.75b)



EXECUTE: $v_S = -v_W$

$$f_L = f_S\left(\frac{v+v_L}{v+v_S}\right) = f'_S\left(\frac{v}{v-v_W}\right)$$

Figure 16.75b

But $f'_S = f'_L$ so $f_L = f_S\left(\frac{v+v_W}{v}\right)\left(\frac{v}{v-v_W}\right) = f_S\left(\frac{v+v_W}{v-v_W}\right)$.

Then $\Delta f = f_S - f_L = f_S\left(1-\frac{v+v_W}{v-v_W}\right) = f_S\left(\frac{v-v_W-v-v_W}{v-v_W}\right) = \frac{-2f_S v_W}{v-v_W}$.

$$\Delta f = \frac{-2(2.20\times10^4\ \text{Hz})(4.95\ \text{m/s})}{1482\ \text{m/s} - 4.95\ \text{m/s}} = 147\ \text{Hz}.$$

EVALUATE: Listener and source are moving toward each other so frequency is raised.

16.76. **IDENTIFY:** Apply the Doppler effect formula $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S$. In the SHM the source moves toward and away from the listener, with maximum speed $\omega_p A_p$.

SET UP: The direction from listener to source is positive.

EXECUTE: **(a)** The maximum velocity of the siren is $\omega_P A_P = 2\pi f_P A_P$. You hear a sound with frequency $f_L = f_{siren}v/(v+v_S)$, where v_S varies between $+2\pi f_P A_P$ and $-2\pi f_P A_P$. $f_{L-max} = f_{siren}v/(v-2\pi f_P A_P)$ and $f_{L-min} = f_{siren}v/(v+2\pi f_P A_P)$.

(b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

EVALUATE: When the platform is moving upward the frequency you hear is greater than f_{siren} and when it is moving downward the frequency you hear is less than f_{siren}. When the platform is at its maximum displacement from equilibrium its speed is zero and the frequency you hear is f_{siren}.

16.77. **IDENTIFY:** Follow the method of Example 16.18 and apply the Doppler shift formula twice, once with the insect as the listener and again with the insect as the source.

SET UP: Let v_{bat} be the speed of the bat, v_{insect} be the speed of the insect, and f_i be the frequency with which the sound waves both strike and are reflected from the insect. The positive direction in each application of the Doppler shift formula is from the listener to the source.

EXECUTE: The frequencies at which the bat sends and receives the signals are related by

$f_L = f_i\left(\frac{v+v_{bat}}{v-v_{insect}}\right) = f_S\left(\frac{v+v_{insect}}{v-v_{bat}}\right)\left(\frac{v+v_{bat}}{v-v_{insect}}\right)$. Solving for v_{insect},

$$v_{insect} = v\left[\frac{1-\frac{f_S}{f_L}\left(\frac{v+v_{bat}}{v-v_{bat}}\right)}{1+\frac{f_S}{f_L}\left(\frac{v+v_{bat}}{v-v_{bat}}\right)}\right] = v\left[\frac{f_L(v-v_{bat})-f_S(v+v_{bat})}{f_L(v-v_{bat})+f_S(v+v_{bat})}\right].$$

Letting $f_L = f_{refl}$ and $f_S = f_{bat}$ gives the result.

(b) If $f_{bat} = 80.7$ kHz, $f_{refl} = 83.5$ kHz, and $v_{bat} = 3.9$ m/s, then $v_{insect} = 2.0$ m/s.

EVALUATE: $f_{refl} > f_{bat}$ because the bat and insect are approaching each other.

16.78. **IDENTIFY:** Follow the steps specified in the problem. v is positive when the source is moving away from the receiver and v is negative when the source is moving toward the receiver. $|f_L - f_R|$ is the beat frequency.

SET UP: The source and receiver are approaching, so $f_R > f_S$ and $f_R - f_S = 46.0$ Hz.

EXECUTE: **(a)** $f_R = f_S\sqrt{\dfrac{c-v}{c+v}} = f_S\dfrac{\sqrt{1-v/c}}{\sqrt{1+v/c}} = f_S\left(1-\dfrac{v}{c}\right)^{1/2}\left(1+\dfrac{v}{c}\right)^{-1/2}.$

(b) For small *x*, the binomial theorem (see Appendix B) gives $(1-x)^{1/2} \approx 1-x/2,\ \ (1+x)^{-1/2} \approx 1-x/2.$

Therefore $f_L \approx f_S\left(1-\dfrac{v}{2c}\right)^2 \approx f_S\left(1-\dfrac{v}{c}\right),$ where the binomial theorem has been used to approximate $(1-x/2)^2 \approx 1-x.$

(c) For an airplane, the approximation $v \ll c$ is certainly valid. Solving the expression found in part (b) for v, $v = c\dfrac{f_S - f_R}{f_S} = c\dfrac{f_{\text{beat}}}{f_S} = (3.00\times10^8\ \text{m/s})\dfrac{-46.0\ \text{Hz}}{2.43\times10^8\ \text{Hz}} = -56.8\ \text{m/s}.$ The speed of the aircraft is 56.8 m/s.

EVALUATE: The approximation $v \ll c$ is seen to be valid. *v* is negative because the source and receiver are approaching. Since $v \ll c,$ the fractional shift in frequency, $\dfrac{\Delta f}{f},$ is very small.

16.79. **IDENTIFY:** Apply the result derived in part (b) of Problem 16.78. The radius of the nebula is $R = vt,$ where *t* is the time since the supernova explosion.

SET UP: When the source and receiver are moving toward each other, *v* is negative and $f_R > f_S.$ The light from the explosion reached earth 952 years ago, so that is the amount of time the nebula has expanded. $1\ \text{ly} = 9.46\times10^{15}$ m.

EXECUTE: **(a)** $v = c\dfrac{f_S - f_R}{f_S} = (3.00\times10^8\,\text{m/s})\dfrac{-0.018\times10^{14}\ \text{Hz}}{4.568\times10^{14}\ \text{Hz}} = -1.2\times10^6\,\text{m/s},$ with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S.$

(b) The radius is $(952\ \text{yr})(3.156\times10^7\ \text{s/yr})(1.2\times10^6\ \text{m/s}) = 3.6\times10^{16}\ \text{m} = 3.8\ \text{ly}.$

(c) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then $\left(\dfrac{2}{2\pi}\right)(3.75\ \text{ly})\dfrac{(60)(360)}{5} = 5.2\times10^3$ ly. The time it takes light to travel this distance is 5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

EVALUATE: $\left|\dfrac{v}{c}\right| = 4.0\times10^{-3},$ so even though $|v|$ is very large the approximation required for $v = c\dfrac{\Delta f}{f}$ is accurate.

16.80. **IDENTIFY:** The sound from the speaker moving toward the listener will have an increased frequency, while the sound from the speaker moving away from the listener will have a decreased frequency. The difference in these frequencies will produce a beat.

SET UP: The greatest frequency shift from the Doppler effect occurs when one speaker is moving away and one is moving toward the person. The speakers have speed $v_0 = r\omega,$ where $r = 0.75$ m.

$f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S,$ with the positive direction from the listener to the source. $v = 344$ m/s.

EXECUTE: **(a)** $f = \dfrac{v}{\lambda} = \dfrac{344\ \text{m/s}}{0.313\ \text{m}} = 1100\ \text{Hz}.$ $\omega = (75\ \text{rpm})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 7.85\ \text{rad/s}$ and $v_0 = (0.75\ \text{m})(7.85\ \text{rad/s}) = 5.89\ \text{m/s}.$

For speaker *A*, moving toward the listener: $f_{LA} = \left(\dfrac{v}{v-5.89\ \text{m/s}}\right)(1100\ \text{Hz}) = 1119\ \text{Hz}.$

For speaker B, moving toward the listener: $f_{\text{LB}} = \left(\dfrac{v}{v + 5.89\ \text{m/s}}\right)(1100\ \text{Hz}) = 1081\ \text{Hz}$.

$f_{\text{beat}} = f_1 - f_2 = 1119\ \text{Hz} - 1081\ \text{Hz} = 38\ \text{Hz}$.

(b) A person can hear individual beats only up to about 7 Hz and this beat frequency is much larger than that.

EVALUATE: As the turntable rotates faster the beat frequency at this position of the speakers increases.

16.81. **IDENTIFY:** Follow the method of Example 16.18 and apply the Doppler shift formula twice, once for the wall as a listener and then again with the wall as a source.

SET UP: In each application of the Doppler formula, the positive direction is from the listener to the source

EXECUTE: **(a)** The wall will receive and reflect pulses at a frequency $\dfrac{v}{v - v_{\text{w}}} f_0$, and the woman will hear this reflected wave at a frequency $\dfrac{v + v_{\text{w}}}{v} \dfrac{v}{v - v_{\text{w}}} f_0 = \dfrac{v + v_{\text{w}}}{v - v_{\text{w}}} f_0$. The beat frequency is

$$f_{\text{beat}} = f_0\left(\frac{v + v_{\text{w}}}{v - v_{\text{w}}} - 1\right) = f_0\left(\frac{2v_{\text{w}}}{v - v_{\text{w}}}\right).$$

(b) In this case, the sound reflected from the wall will have a lower frequency, and using $f_0(v - v_{\text{w}})/(v + v_{\text{w}})$ as the detected frequency, v_{w} is replaced by $-v_{\text{w}}$ in the calculation of part (a) and

$$f_{\text{beat}} = f_0\left(1 - \frac{v - v_{\text{w}}}{v + v_{\text{w}}}\right) = f_0\left(\frac{2v_{\text{w}}}{v + v_{\text{w}}}\right).$$

EVALUATE: The beat frequency is larger when she runs toward the wall, even though her speed is the same in both cases.

16.82. **IDENTIFY** and **SET UP:** Use Figure (16.37) to relate α and T.
Use this in Eq. (16.31) to eliminate $\sin\alpha$.

EXECUTE: Eq. (16.31): $\sin\alpha = v/v_{\text{S}}$ From Figure 16.37 $\tan\alpha = h/v_{\text{S}}T$. And $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin\alpha}{\sqrt{1 - \sin^2\alpha}}$.

Combining these equations we get $\dfrac{h}{v_{\text{S}}T} = \dfrac{v/v_{\text{S}}}{\sqrt{1 - (v/v_{\text{S}})^2}}$ and $\dfrac{h}{T} = \dfrac{v}{\sqrt{1 - (v/v_{\text{S}})^2}}$.

$$1 - (v/v_{\text{S}})^2 = \frac{v^2T^2}{h^2} \text{ and } v_{\text{S}}^2 = \frac{v^2}{1 - v^2T^2/h^2}$$

$$v_{\text{S}} = \frac{hv}{\sqrt{h^2 - v^2T^2}}$$ as was to be shown.

EVALUATE: For a given h, the faster the speed v_{S} of the plane, the greater is the delay time T. The maximum delay time is h/v, and T approaches this value as $v_{\text{S}} \to \infty$. $T \to 0$ as $v \to v_{\text{S}}$.

16.83. **IDENTIFY:** The phase of the wave is determined by the value of $x - vt$, so t increasing is equivalent to x decreasing with t constant. The pressure fluctuation and displacement are related by Eq. (16.3).

SET UP: $y(x,t) = -\dfrac{1}{B}\int p(x,t)dx$. If $p(x,t)$ versus x is a straight line, then $y(x,t)$ versus x is a parabola.

For air, $B = 1.42 \times 10^5$ Pa.

EXECUTE: **(a)** The graph is sketched in Figure 16.83a.

(b) From Eq. (16.4), the function that has the given $p(x,0)$ at $t = 0$ is given graphically in Figure 16.83b. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\ \text{m})/(344\ \text{m/s}) = 5.81 \times 10^{-4}$ s and the amplitude is equal to the area under the p versus x curve between $x = 0$ and $x = 0.0500$ m divided by B, or 7.04×10^{-6} m.

(c) Assuming a wave moving in the $+x$-direction, $y(0,t)$ is as shown in Figure 16.83c.

(d) The maximum velocity of a particle occurs when a particle is moving through the origin, and the particle speed is $v_y = -\dfrac{\partial y}{\partial x} v = \dfrac{pv}{B}$. The maximum velocity is found from the maximum pressure, and $v_{y\text{max}} = (40\ \text{Pa})(344\ \text{m/s})/(1.42\times10^5\ \text{Pa}) = 9.69\ \text{cm/s}$. The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\text{max}} = \frac{(80.0\ \text{Pa})/(0.100\ \text{m})}{(1.20\ \text{kg/m}^3)} = 6.67\times10^2\ \text{m/s}^2.$$

(e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude $667\ \text{m/s}^2$ and frequency $f = v/\lambda = (344\ \text{m/s})/(0.200\ \text{m}) = 1.72\ \text{kHz}$.

EVALUATE: We can verify that $p(x,t)$ versus *x* has a shape proportional to the slope of the graph of $y(x,t)$ versus *x*. The same is also true of the graphs versus *t*.

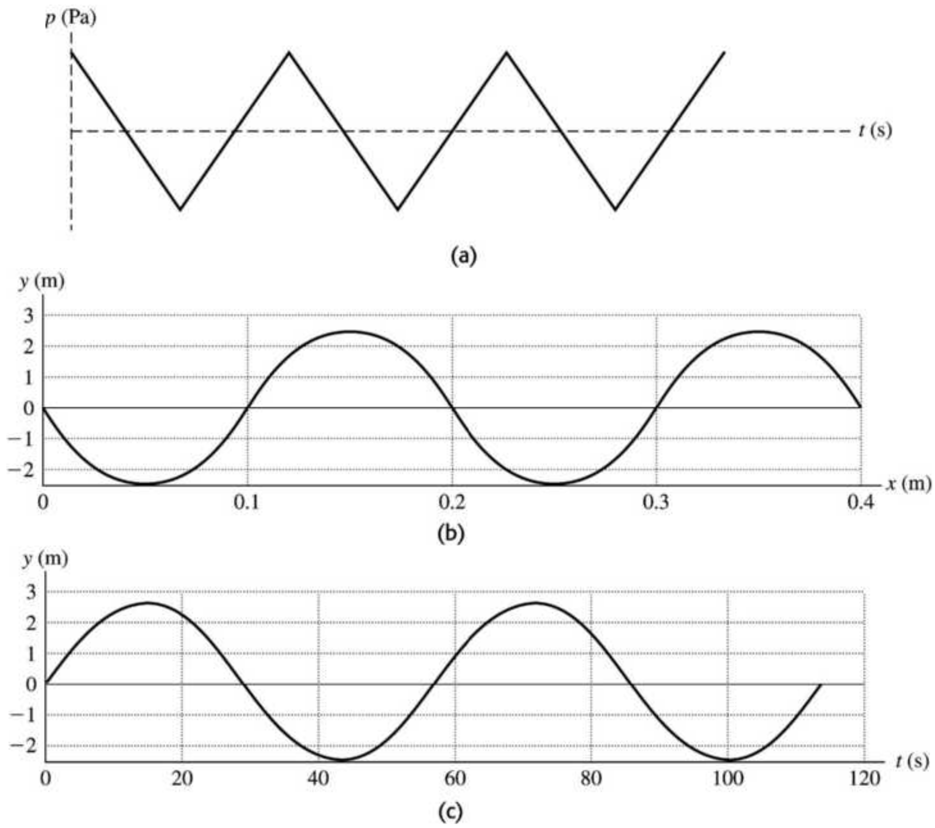


Figure 16.83

16.84. **IDENTIFY** and **SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 16.2.

EXECUTE: **(a)** The quantity of interest is the change in force per fractional length change. The force constant k' is the change in force per length change, so the force change per fractional length change is $k'L$, the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time *t is* $k'Ltv_y/v$. The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of *t* and solving for *v* gives $v^2 = L^2k'/m$.

(b) $v = (2.00\ \text{m})\sqrt{(1.50\ \text{N/m})/(0.250\ \text{kg})} = 4.90\ \text{m/s}$

EVALUATE: *A* larger k' corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.