44

PARTICLE PHYSICS AND COSMOLOGY

44.1. **IDENTIFY** and **SET UP:** By momentum conservation the two photons must have equal and opposite momenta. Then $E = pc$ says the photons must have equal energies. Their total energy must equal the rest mass energy $E = mc^2$ of the pion. Once we have found the photon energy we can use $E = hf$ to calculate the photon frequency and use $\lambda = c/f$ to calculate the wavelength.

EXECUTE: The mass of the pion is $270m_e$, so the rest energy of the pion is $270(0.511\text{ MeV}) = 138\text{ MeV}$. Each photon has half this energy, or 69 MeV.

$$E = hf \text{ so } f = \frac{E}{h} = \frac{(69\times10^6\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.7\times10^{22}\text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{2.998\times10^8\text{ m/s}}{1.7\times10^{22}\text{ Hz}} = 1.8\times10^{-14}\text{ m} = 18\text{ fm}.$$

EVALUATE: These photons are in the gamma ray part of the electromagnetic spectrum.

44.2. **IDENTIFY:** The energy (rest mass plus kinetic) of the muons is equal to the energy of the photons.

SET UP: $\gamma + \gamma \to \mu^+ + \mu^-$, $E = hc/\lambda$. $K = (\gamma - 1)mc^2$.

EXECUTE: **(a)** $\gamma + \gamma \to \mu^+ + \mu^-$. Each photon must have energy equal to the rest mass energy of a μ^+ or a μ^-: $\frac{hc}{\lambda} = 105.7\times10^6\text{ eV}$. $\lambda = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^8\text{ m/s})}{105.7\times10^6\text{ eV}} = 1.17\times10^{-14}\text{ m} = 0.0117\text{ pm}.$

Conservation of linear momentum requires that the μ^+ and μ^- move in opposite directions with equal speeds.

(b) $\lambda = \frac{0.0117\text{ pm}}{2}$ so each photon has energy $2(105.7\text{ MeV}) = 211.4\text{ MeV}$. The energy released in the reaction is $2(211.4\text{ MeV}) - 2(105.7\text{ MeV}) = 211.4\text{ MeV}$. The kinetic energy of each muon is half this, 105.7 MeV. Using $K = (\gamma - 1)mc^2$ gives $\gamma - 1 = \frac{K}{mc^2} = \frac{105.7\text{ MeV}}{105.7\text{ MeV}} = 1$. $\gamma = 2$. $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}. \quad v = \sqrt{\frac{3}{4}}c = 0.866c = 2.60\times10^8\text{ m/s}.$$

EVALUATE: The muon speeds are a substantial fraction of the speed of light, so special relativity must be used.

44.3. **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the decay.

SET UP: The mass of the pion is $m_{\pi^+} = 270m_e$ and the mass of the muon is $m_{\mu^+} = 207m_e$. The rest energy of an electron is 0.511 MeV.

EXECUTE: **(a)** $\Delta m = m_{\pi+} - m_{\mu+} = 270m_e - 207m_e = 63m_e \Rightarrow E = 63(0.511\text{ MeV}) = 32\text{ MeV}$.

EVALUATE: **(b)** A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.4. **IDENTIFY:** In the annihilation the total energy of the proton and antiproton is converted to the energy of the two photons.

SET UP: The rest energy of a proton or antiproton is 938.3 MeV. Conservation of linear momentum requires that the two photons have equal energies.

EXECUTE: **(a)** The energy will be the proton rest energy, 938.3 MeV, corresponding to a frequency of 2.27×10^{23} Hz and a wavelength of 1.32×10^{-15} m.

(b) The energy of each photon will be $938.3\text{ MeV}+830\text{ MeV}=1768\text{ MeV}$, with frequency 42.8×10^{22} Hz and wavelength 7.02×10^{-16} m.

EVALUATE: When the initial kinetic energy of the proton and antiproton increases, the wavelength of the photons decreases.

44.5. **IDENTIFY:** The kinetic energy of the alpha particle is due to the mass decrease.

SET UP and **EXECUTE:** ${}^{1}_{0}\text{n}+{}^{10}_{5}\text{B}\rightarrow{}^{7}_{3}\text{Li}+{}^{4}_{2}\text{He}$. The mass decrease in the reaction is

$m({}^{1}_{0}\text{n})+m({}^{10}_{5}\text{B})-m({}^{7}_{3}\text{Li})-m({}^{4}_{2}\text{He})=1.008665\text{ u}+10.012937\text{ u}-7.016004\text{ u}-4.002603\text{ u}=0.002995\text{ u}$

and the energy released is $E=(0.002995\text{ u})(931.5\text{ MeV/u})=2.79\text{ MeV}$. Assuming the initial momentum is zero, $m_{\text{Li}}v_{\text{Li}}=m_{\text{He}}v_{\text{He}}$ and $v_{\text{Li}}=\dfrac{m_{\text{He}}}{m_{\text{Li}}}v_{\text{He}}$. $\frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2+\frac{1}{2}m_{\text{He}}v_{\text{He}}^2=E$ becomes

$\frac{1}{2}m_{\text{Li}}\left(\dfrac{m_{\text{He}}}{m_{\text{Li}}}\right)^2 v_{\text{He}}^2+\frac{1}{2}m_{\text{He}}v_{\text{He}}^2=E$ and $v_{\text{He}}=\sqrt{\dfrac{2E}{m_{\text{He}}}\left(\dfrac{m_{\text{Li}}}{m_{\text{Li}}+m_{\text{He}}}\right)}$. $E=4.470\times10^{-13}$ J.

$m_{\text{He}}=4.002603\text{ u}-2(0.0005486\text{ u})=4.0015\text{ u}=6.645\times10^{-27}\text{ kg}$.

$m_{\text{Li}}=7.016004\text{ u}-3(0.0005486\text{ u})=7.0144\text{ u}$. This gives $v_{\text{He}}=9.26\times10^{6}$ m/s.

EVALUATE: The speed of the alpha particle is considerably less than the speed of light, so it is not necessary to use the more complicated relativistic formulas.

44.6. **IDENTIFY:** The range is limited by the lifetime of the particle, which itself is limited by the uncertainty principle.

SET UP: $\Delta E\Delta t=\hbar/2$.

EXECUTE: $\Delta t=\dfrac{\hbar}{2\Delta E}=\dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s}/2\pi)}{2(783\times10^{6}\text{ eV})}=4.20\times10^{-25}$ s. The range of the force is

$c\Delta t=(2.998\times10^{8}\text{ m/s})(4.20\times10^{-25}\text{ s})=1.26\times10^{-16}\text{ m}=0.126\text{ fm}$.

EVALUATE: This range is less than the diameter of an atomic nucleus.

44.7. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E=(\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E=(\Delta m)c^2=(400\text{ kg}+400\text{ kg})(3.00\times10^{8}\text{ m/s})^2=7.2\times10^{19}\text{ J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihiliation is not so easy to do!

44.8. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be leftover kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_{\text{a}}^2=2mc^2(E_m+mc^2)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy $(20.0\text{ GeV}=20{,}000\text{ MeV})$ is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to $E_{\text{a}}=\sqrt{2mc^2E_m}$.

EXECUTE: **(a)** Using the formula for available energy gives

$$E_{\text{a}}=\sqrt{2mc^2E_m}=\sqrt{2(0.511\text{ MeV})(20.0\text{ GeV})}=143\text{ MeV}$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 71.5 MeV. The *total* energy is twice this, or 143 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 20,000 MeV of energy to get the same available energy that we got with only 143 MeV of energy with the colliding beams.

44.9. **(a) IDENTIFY and SET UP:** Eq. (44.7) says $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes $B = 2\pi mf/|q|$.

EXECUTE: A deuteron is a deuterium nucleus $({}^2_1\text{H})$. Its charge is $q = +e$. Its mass is the mass of the neutral ${}^2_1\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$m = 2.014102\ \text{u} - 0.0005486\ \text{u} = 2.013553\ \text{u}(1.66054\times10^{-27}\ \text{kg/1 u}) = 3.344\times10^{-27}\ \text{kg}$

$$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344\times10^{-27}\ \text{kg})(9.00\times10^{6}\ \text{Hz})}{1.602\times10^{-19}\ \text{C}} = 1.18\ \text{T}$$

(b) Eq. (44.8): $K = \dfrac{q^2B^2R^2}{2m} = \dfrac{[(1.602\times10^{-19}\ \text{C})(1.18\ \text{T})(0.320\ \text{m})]^2}{2(3.344\times10^{-27}\ \text{kg})}$.

$K = 5.471\times10^{-13}\ \text{J} = (5.471\times10^{-13}\ \text{J})(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.42\ \text{MeV}$

$$K = \tfrac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.471\times10^{-13}\ \text{J})}{3.344\times10^{-27}\ \text{kg}}} = 1.81\times10^{7}\ \text{m/s}$$

EVALUATE: $v/c = 0.06$, so it is ok to use the nonrelativistic expression for kinetic energy.

44.10. **IDENTIFY:** Apply Eqs. (44.6) and (44.7). $f = \dfrac{\omega}{2\pi}$. In part (c) apply conservation of energy.

SET UP: The relativistic form for the kinetic energy is $K = (\gamma - 1)mc^2$. A proton has mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $2f = \dfrac{\omega}{\pi} = \dfrac{eB}{m\pi} = 3.97\times10^{7}/\text{s}$.

(b) $v = \omega R = \dfrac{eBR}{m} = 3.12\times10^{7}\ \text{m/s}$

(c) For three-figure precision, the relativistic form of the kinetic energy must be used, $eV = (\gamma-1)mc^2$,

so $eV = (\gamma-1)mc^2$, so $V = \dfrac{(\gamma-1)mc^2}{e} = 5.11\times10^{6}\ \text{V}$.

EVALUATE: The kinetic energy of the protons in part (c) is 5.11 MeV. This is 0.5% of their rest energy. If the nonrelativistic expression for the kinetic energy is used, we obtain $V = 5.08\times10^{6}\ \text{V}$.

44.11. **(a) IDENTIFY and SET UP:** The masses of the target and projectile particles are equal, so Eq. (44.10) can be used. $E_\text{a}^2 = 2mc^2(E_m + mc^2)$. E_a is specified; solve for the energy E_m of the beam particles.

EXECUTE: $E_m = \dfrac{E_\text{a}^2}{2mc^2} - mc^2$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}^4_2\text{He}$ atomic mass:

$m = m_\alpha = 4.002603\ \text{u} - 2(0.0005486\ \text{u}) = 4.001506\ \text{u}$

Then $mc^2 = (4.001506\ \text{u})(931.5\ \text{MeV/u}) = 3.727\ \text{GeV}$.

$$E_m = \frac{E_\text{a}^2}{2mc^2} - mc^2 = \frac{(16.0\ \text{GeV})^2}{2(3.727\ \text{GeV})} - 3.727\ \text{GeV} = 30.6\ \text{GeV}.$$

(b) Each beam must have $\tfrac{1}{2}E_\text{a} = 8.0\ \text{GeV}$.

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

44.12. **IDENTIFY:** $E = \gamma mc^2$, where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$. The relativistic version of Eq. (44.7) is $\omega = \dfrac{|q|B}{m\gamma}$.

SET UP: A proton has rest energy $mc^2 = 938.3$ MeV.

EXECUTE: **(a)** $\gamma = \dfrac{E}{mc^2} = \dfrac{1000 \times 10^3 \text{ MeV}}{938.3 \text{ MeV}} = 1065.8$, so $v = 0.999999559c$.

(b) Nonrelativistic: $\omega = \dfrac{eB}{m} = 3.83 \times 10^8$ rad/s.

Relativistic: $\omega = \dfrac{eB}{m}\dfrac{1}{\gamma} = 3.59 \times 10^5$ rad/s.

EVALUATE: The relativistic expression gives a smaller value for ω.

44.13. **(a) IDENTIFY** and **SET UP:** For a proton beam on a stationary proton target and since E_a is much larger than the proton rest energy we can use Eq. (44.11): $E_a^2 = 2mc^2 E_m$.

EXECUTE: $E_m = \dfrac{E_a^2}{2mc^2} = \dfrac{(77.4 \text{ GeV})^2}{2(0.938 \text{ GeV})} = 3200$ GeV

(b) IDENTIFY and **SET UP:** For colliding beams the total momentum is zero and the available energy E_a is the total energy for the two colliding particles.

EXECUTE: For proton-proton collisions the colliding beams each have the same energy, so the total energy of each beam is $\frac{1}{2}E_a = 38.7$ GeV.

EVALUATE: For a stationary target less than 3% of the beam energy is available for conversion into mass. The beam energy for a colliding beam experiment is a factor of (1/83) times smaller than the required energy for a stationary target experiment.

44.14. **IDENTIFY:** Only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy.

SET UP: To create the η^0, the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy.

EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_a = 2m_p + m_\eta = 2(938.3 \text{ MeV}) + 547.3 \text{ MeV} = 2420 \text{ MeV}$$

Each incident proton has half of the rest mass energy, or $1210 \text{ MeV} = 1.21$ GeV.

EVALUATE: As we saw in Problem 44.13, we would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

44.15. **IDENTIFY:** The kinetic energy comes from the mass decrease.

SET UP: Table 44.3 gives $m(\text{K}^+) = 493.7 \text{ MeV}/c^2$, $m(\pi^0) = 135.0 \text{ MeV}/c^2$, and $m(\pi^\pm) = 139.6 \text{ MeV}/c^2$.

EXECUTE: **(a)** Charge must be conserved, so $\text{K}^+ \to \pi^0 + \pi^+$ is the only possible decay.

(b) The mass decrease is $m(\text{K}^+) - m(\pi^0) - m(\pi^+) = 493.7 \text{ MeV}/c^2 - 135.0 \text{ MeV}/c^2 - 139.6 \text{ MeV}/c^2 = 219.1 \text{ MeV}/c^2$. The energy released is 219.1 MeV.

EVALUATE: The π mesons do not share this energy equally since they do not have equal masses.

44.16. **IDENTIFY:** The energy is due to the mass difference.

SET UP: The energy released is the energy equivalent of the mass decrease. From Table 44.3, the μ^- has mass $105.7 \text{ MeV}/c^2$ and the e^- has mass $0.511 \text{ MeV}/c^2$.

EXECUTE: The mass decrease is $105.7 \text{ MeV}/c^2 - 0.511 \text{ MeV}/c^2 = 105.2 \text{ MeV}/c^2$ and the energy equivalent is 105.2 MeV.

EVALUATE: The electron does not get all of this energy; the neutrinos also get some of it.

44.17. **IDENTIFY:** Table 44.1 gives the mass in units of GeV/c^2. This is the value of mc^2 for the particle.

SET UP: $m(\text{Z}^0) = 91.2 \text{ GeV}/c^2$.

EXECUTE: $E = 91.2 \times 10^9 \text{ eV} = 1.461 \times 10^{-8} \text{ J}$; $m = E/c^2 = 1.63 \times 10^{-25} \text{ kg}$; $m(\text{Z}^0)/m(\text{p}) = 97.2$

EVALUATE: The rest energy of a proton is 938 MeV; the rest energy of the Z^0 is 97.2 times as great.

44.18. **IDENTIFY:** The energy of the photon equals the difference in the rest energies of the Σ^0 and Λ^0. For a photon, $p = E/c$.

SET UP: Table 44.3 gives the rest energies to be 1193 MeV for the Σ^0 and 1116 MeV for the Λ^0.

EXECUTE: **(a)** We shall assume that the kinetic energy of the Λ^0 is negligible. In that case we can set the value of the photon's energy equal to Q:

$$Q = (1193 - 1116) \text{ MeV} = 77 \text{ MeV} = E_{\text{photon}}.$$

(b) The momentum of this photon is

$$p = \frac{E_{\text{photon}}}{c} = \frac{(77 \times 10^6 \text{ eV})(1.60 \times 10^{-18} \text{ J/eV})}{(3.00 \times 10^8 \text{ m/s})} = 4.1 \times 10^{-20} \text{ kg} \cdot \text{m/s}$$

EVALUATE: To justify our original assumption, we can calculate the kinetic energy of a Λ^0 that has this value of momentum

$$K_{\Lambda^0} = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(77 \text{ MeV})^2}{2(1116 \text{ MeV})} = 2.7 \text{ MeV} \ll Q = 77 \text{ MeV}.$$

Thus, we can ignore the momentum of the Λ^0 without introducing a large error.

44.19. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: The mass decrease is $m(\Sigma^+) - m(\text{p}) - m(\pi^0)$ and the energy released is $mc^2(\Sigma^+) - mc^2(\text{p}) - mc^2(\pi^0) = 1189 \text{ MeV} - 938.3 \text{ MeV} - 135.0 \text{ MeV} = 116 \text{ MeV}$. (The mc^2 values for each particle were taken from Table 44.3.)

EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no leftover energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.

SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{\text{K}^-}$.

EXECUTE: Using Table 44.3, the energy difference is

$$E = (\Delta m)c^2 = 1672 \text{ MeV} - 1116 \text{ MeV} - 494 \text{ MeV} = 62 \text{ MeV}$$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

44.21. **IDENTIFY** and **SET UP:** The lepton numbers for the particles are given in Table 44.2.

EXECUTE: **(a)** $\mu^- \to \text{e}^- + \nu_\text{e} + \overline{\nu}_\mu \Rightarrow L_\mu: +1 \neq -1, L_\text{e}: 0 \neq +1 + 1$, so lepton numbers are not conserved.

(b) $\tau^- \to \text{e}^- + \overline{\nu}_\text{e} + \nu_\tau \Rightarrow L_\text{e}: 0 = +1 - 1$; $L_\tau: +1 = +1$, so lepton numbers are conserved.

(c) $\pi^+ \to \text{e}^+ + \gamma$. Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

(d) $\text{n} \to \text{p} + \text{e}^- + \overline{\nu}_\text{e} \Rightarrow L_\text{e}: 0 = +1 - 1$, so the lepton numbers are conserved.

EVALUATE: The decays where lepton numbers are conserved are among those listed in Tables 44.2 and 44.3.

44.22. **IDENTIFY and SET UP:** p and n have baryon number $+1$ and $\overline{\text{p}}$ has baryon number -1. e^+, e^-, $\overline{\nu}_e$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.

EXECUTE: **(a)** reactants: $B = 1 + 1 = 2$. Products: $B = 1 + 0 = 1$. Not conserved.

(b) reactants: $B = 1 + 1 = 2$. Products: $B = 0 + 0 = 0$. Not conserved.

(c) reactants: $B = +1$. Products: $B = 1 + 0 + 0 = +1$. Conserved.

(d) reactants: $B = 1 - 1 = 0$. Products: $B = 0$. Conserved.

EVALUATE: Even though a reaction obeys conservation of baryon number it may still not occur spontaneously, if it is not energetically allowed or if other conservation laws are violated.

44.23. **IDENTIFY and SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given in Table 44.3.

EXECUTE: **(a)** $\text{K}^+ \to \mu^+ + \nu_\mu$; $S_{\text{K}^+} = +1$, $S_{\mu+} = 0$, $S_{\nu_\mu} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(b) $\text{n} + \text{K}^+ \to \text{p} + \pi^0$; $S_\text{n} = 0$, $S_{\text{K}^+} = +1$, $S_\text{p} = 0$, $S_{\pi^0} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(c) $\text{K}^+ + \text{K}^- \to \pi^0 + \pi^0$; $S_{\text{K}^+} = +1$; $S_{\text{K}^-} = -1$; $S_{\pi^0} = 0$

$S = +1 - 1 = 0$ initially; $S = 0$ for the products; S is <u>conserved</u>

(d) $\text{p} + \text{K}^- \to \Lambda^0 + \pi^0$; $S_\text{p} = 0$, $S_{\text{K}^-} = -1$, $S_{\Lambda^0} = -1$, $S_{\pi^0} = 0$.

$S = -1$ initially; $S = -1$ for the products; S is <u>conserved</u>

EVALUATE: Strangeness is not a conserved quantity in weak interactions, and strangeness nonconserving reactions or decays can occur.

44.24. **IDENTIFY and SET UP:** Numerical values for the fundamental physical constant are given in Appendix F.

EXECUTE: **(a)** Using the values of the constants from Appendix F,

$$\frac{e^2}{4\pi\epsilon_0\hbar c} = 7.29660475 \times 10^{-3} = \frac{1}{137.050044},\ \text{or } 1/137 \text{ to three figures.}$$

(b) From Section 39.3, $v_1 = \dfrac{e^2}{2\epsilon_0 h}$. But notice this is just $\left(\dfrac{e^2}{4\pi\epsilon_0\hbar c}\right)c$, as claimed.

EVALUATE: $U = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$, so $\dfrac{e^2}{4\pi\epsilon_0}$ has units of $\text{J}\cdot\text{m}$. $\hbar c$ has units of $(\text{J}\cdot\text{s})(\text{m/s}) = \text{J}\cdot\text{m}$, so $\dfrac{e^2}{4\pi\epsilon_0\hbar c}$ is indeed dimensionless.

44.25. **IDENTIFY and SET UP:** f^2 has units of energy times distance. $\hbar$ has units of $\text{J}\cdot\text{s}$ and c has units of m/s.

EXECUTE: $\left[\dfrac{f^2}{\hbar c}\right] = \dfrac{(\text{J}\cdot\text{m})}{(\text{J}\cdot\text{s})(\text{m}\cdot\text{s}^{-1})} = 1$ and thus $\dfrac{f^2}{\hbar c}$ is dimensionless.

EVALUATE: Since $\dfrac{f^2}{\hbar c}$ is dimensionless, it has the same numerical value in all system of units.

44.26. **IDENTIFY and SET UP:** Construct the diagram as specified in the problem. In part (b), use quark charges $\boldsymbol{u} = +\dfrac{2}{3}$, $\boldsymbol{d} = \dfrac{-1}{3}$, and $\boldsymbol{s} = \dfrac{-1}{3}$ as a guide.

EXECUTE: **(a)** The diagram is given in Figure 44.26. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole" in an otherwise regular lattice in the $S - Q$ plane.

(b) The quark composition of each particle is shown in the figure.

EVALUATE: The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω^-!

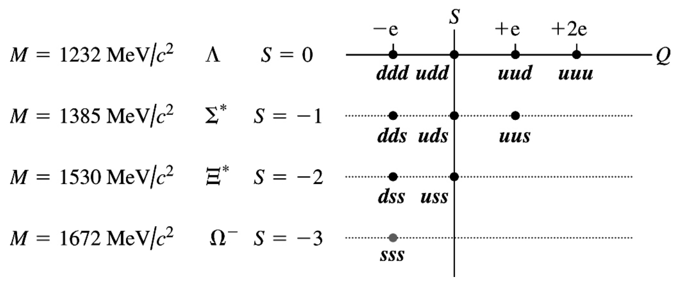

Figure 44.26

44.27. **IDENTIFY** and **SET UP:** Each value for the combination is the sum of the values for each quark. Use Table 44.4.

EXECUTE: **(a)** ***uds***

$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$S = 0 + 0 - 1 = -1$

$C = 0 + 0 + 0 = 0$

(b) ***c$\bar{u}$***

The values for $\bar{u}$ are the negative for those for u.

$Q = \frac{2}{3}e - \frac{2}{3}e = 0$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = +1 + 0 = +1$

(c) ***ddd***

$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$

$S = 0 + 0 + 0 = 0$

$C = 0 + 0 + 0 = 0$

(d) ***d$\bar{c}$***

$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = 0 - 1 = -1$

EVALUATE: The charge, baryon number, strangeness and charm quantum numbers of a particle are determined by the particle's quark composition.

44.28. **IDENTIFY:** Quark combination produce various particles.

SET UP: The properties of the quarks are given in Table 44.5. An antiquark has charge and quantum numbers of opposite sign from the corresponding quark.

EXECUTE: **(a)** $Q/e = \frac{2}{3} + \frac{2}{3} + \left(-\frac{1}{3}\right) = +1.$ $B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.$ $S = 0 + 0 + (-1) = -1.$ $C = 0 + 0 + 0 = 0.$

(b) $Q/e = \frac{2}{3} + \frac{1}{3} = +1.$ $B = \frac{1}{3} + \left(-\frac{1}{3}\right) = 0.$ $S = 0 + 1 = 1.$ $C = 1 + 0 = 1.$

(c) $Q/e = \frac{1}{3} + \frac{1}{3} + \left(-\frac{2}{3}\right) = 0.$ $B = -\frac{1}{3} + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) = -1.$ $S = 0 + 0 + 0 = 0.$ $C = 0 + 0 + 0 = 0.$

(d) $Q/e = -\frac{2}{3} + \left(-\frac{1}{3}\right) = -1.$ $B = -\frac{1}{3} + \frac{1}{3} = 0.$ $S = 0 + 0 = 0.$ $C = -1 + 0 = -1.$

EVALUATE: The charge must always come out to be a whole number.

44.29. **IDENTIFY:** A proton is made up of ***uud*** quarks and a neutron consists of ***udd*** quarks.

SET UP and **EXECUTE:** If a proton decays by β^+ decay, we have $\mathrm{p} \to \mathrm{e}^+ + \mathrm{n} + \nu_e$ (both charge and lepton number are conserved).

EVALUATE: Since a proton consists of ***uud*** quarks and a neutron is ***udd*** quarks, it follows that in β^+ decay a ***u*** quark changes to a ***d*** quark.

44.30. **IDENTIFY:** The decrease in the rest energy of the particles that exist before and after the decay equals the energy that is released.

SET UP: The upsilon has rest energy 9460 MeV and each tau has rest energy 1777 MeV.

EXECUTE: $(m_\Upsilon - 2m_\tau)c^2 = (9460\text{ MeV} - 2(1777\text{ MeV})) = 5906\text{ MeV}$

EVALUATE: Over half of the rest energy of the upsilon is released in the decay.

44.31. **IDENTIFY** and **SET UP:** To obtain the quark content of an antiparticle, replace quarks by antiquarks and antiquarks by quarks in the quark composition of the particle.

EXECUTE: **(a)** The antiparticle must consist of the antiquarks so $\overline{\mathrm{n}} = \overline{\boldsymbol{u}}\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}$.

(b) $\mathrm{n} = \boldsymbol{udd}$ is not its own antiparticle, since n and $\overline{\mathrm{n}}$ have different quark content.

(c) $\psi = \boldsymbol{c}\overline{\boldsymbol{c}}$ so $\overline{\psi} = \overline{\boldsymbol{c}}\boldsymbol{c} = \psi$ so the ψ is its own antiparticle.

EVALUATE: We can see from Table 44.3 that none of the baryons are their own antiparticles and that none of the charged mesons are their own antiparticles. The ψ is a neutral meson and all the neutral mesons are their own antiparticles.

44.32. **IDENTIFY:** The charge, baryon number and strangeness of the particles are the sums of these values for their constituent quarks.

SET UP: The properties of the six quarks are given in Table 44.5.

EXECUTE: **(a)** $S = 1$ indicates the presence of one $\overline{\boldsymbol{s}}$ antiquark and no ***s*** quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a ***u***, and the quark content is $\boldsymbol{u}\overline{\boldsymbol{s}}$.

(b) The particle has an $\overline{\boldsymbol{s}}$ antiquark, and for a baryon number of -1 the particle must consist of three antiquarks. For a net charge of $-e$, the quark content must be $\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}\overline{\boldsymbol{s}}$.

(c) $S = -2$ means that there are two ***s*** quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a ***u*** quark and the quark content is ***uss***.

EVALUATE: The particles with baryon number zero are mesons and consist of a quark-antiquark pair. Particles with baryon number 1 consist of three quarks and are baryons. Particles with baryon number -1 consist of three antiquarks and are antibaryons.

44.33. **(a) IDENTIFY** and **SET UP:** Use Eq. (44.14) to calculate v.

EXECUTE: $v = \left[\dfrac{(\lambda_0/\lambda_S)^2 - 1}{(\lambda_0/\lambda_S)^2 + 1}\right]c = \left[\dfrac{(658.5\text{ nm}/590\text{ nm})^2 - 1}{(658.5\text{ nm}/590\text{ nm})^2 + 1}\right]c = 0.1094c$

$v = (0.1094)(2.998 \times 10^8\text{ m/s}) = 3.28 \times 10^7\text{ m/s}$

(b) IDENTIFY and **SET UP:** Use Eq. (44.15) to calculate r.

EXECUTE: $r = \dfrac{v}{H_0} = \dfrac{3.28 \times 10^4\text{ km/s}}{(71(\text{km/s})/\text{Mpc})(1\text{ Mpc}/3.26\text{ Mly})} = 1510\text{ Mly}$

EVALUATE: The red shift $\lambda_0/\lambda_S - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.8 and 44.9, that had a red shift of 0.053.

44.34. **IDENTIFY:** In Example 44.8, z is defined as $z = \dfrac{\lambda_0 - \lambda_S}{\lambda_S}$. Apply Eq. (44.13) to solve for v. Hubble's law is given by Eq. (44.15).

SET UP: The Hubble constant has a value of $H_0 = 7.1 \times 10^4\ \dfrac{\text{m/s}}{\text{Mpc}}$.

EXECUTE: **(a)** $1 + z = 1 + \dfrac{(\lambda_0 - \lambda_S)}{\lambda_S} = \dfrac{\lambda_0}{\lambda_S}$. Now we use Eq. (44.13) to obtain

$1 + z = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}} = \sqrt{\dfrac{1+\beta}{1-\beta}}$.

(b) Solving the above equation for β we obtain $\beta = \dfrac{(1+z)^2 - 1}{(1+z)^2 + 1} = \dfrac{1.5^2 - 1}{1.5^2 + 1} = 0.3846.$ Thus,

$v = 0.3846c = 1.15 \times 10^8$ m/s.

(c) We can use Eq. (44.15) to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.15 \times 10^8 \text{ m/s})}{(7.1 \times 10^4 \text{ (m/s)/Mpc})} = 1.6 \times 10^3 \text{ Mpc}$$

EVALUATE: $1 \text{ pc} = 3.26$ ly, so the distance in part (c) is 5.2×10^9 ly.

44.35. **(a) IDENTIFY and SET UP:** Hubble's law is Eq. (44.15), with $H_0 = 71$ (km/s)/(Mpc). 1 Mpc = 3.26 Mly.

EXECUTE: $r = 5210$ Mly so $v = H_0 r = ((71 \text{ km/s})/\text{Mpc})(1 \text{ Mpc}/3.26 \text{ Mly})(5210 \text{ Mly}) = 1.1 \times 10^5$ km/s

(b) IDENTIFY and SET UP: Use v from part (a) in Eq. (44.13).

EXECUTE: $\dfrac{\lambda_0}{\lambda_S} = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$

$\dfrac{v}{c} = \dfrac{1.1 \times 10^8 \text{ m/s}}{2.9980 \times 10^8 \text{ m/s}} = 0.367$ so $\dfrac{\lambda_0}{\lambda_S} = \sqrt{\dfrac{1+0.367}{1-0.367}} = 1.5$

EVALUATE: The galaxy in Examples 44.8 and 44.9 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.36. **IDENTIFY:** Set $v = c$ in Eq. (44.15).

SET UP: $H_0 = 71 \dfrac{\text{km/s}}{\text{Mpc}}$. 1 Mpc = 3.26 Mly, so $H_0 = 22 \dfrac{\text{km/s}}{\text{Mly}}$.

EXECUTE: **(a)** From Eq. (44.15), $r = \dfrac{c}{H_0} = \dfrac{3.00 \times 10^5 \text{ km/s}}{22 \text{ (km/s)/Mly}} = 1.4 \times 10^4$ Mly.

EVALUATE: **(b)** This distance represents looking back in time so far that the light has not been able to reach us.

44.37. **IDENTIFY and SET UP:** $m_H = 1.67 \times 10^{-27}$ kg. The ideal gas law says $pV = nRT$. Normal pressure is 1.013×10^5 Pa and normal temperature is about $27\ °\text{C} = 300$ K. 1 mole is 6.02×10^{23} atoms.

EXECUTE: **(a)** $\dfrac{6.3 \times 10^{-27} \text{ kg/m}^3}{1.67 \times 10^{-27} \text{ kg/atom}} = 3.8 \text{ atoms/m}^3$

(b) $V = (4 \text{ m})(7 \text{ m})(3 \text{ m}) = 84 \text{ m}^3$ and $(3.8 \text{ atoms/m}^3)(84 \text{ m}^3) = 320$ atoms

(c) With $p = 1.013 \times 10^5$ pa, $V = 84 \text{ m}^3$, $T = 300$ K the ideal gas law gives the number of moles to be

$n = \dfrac{pV}{RT} = \dfrac{(1.013 \times 10^5 \text{ Pa})(84 \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 3.4 \times 10^3$ moles.

$(3.4 \times 10^3 \text{ moles})(6.02 \times 10^{23} \text{ atoms/mol}) = 2.0 \times 10^{27}$ atoms

EVALUATE: The average density of the universe is very small. Interstellar space contains a very small number of atoms per cubic meter, compared to the number of atoms per cubit meter in ordinary material on the earth, such as air.

44.38. **IDENTIFY and SET UP:** The dimensions of $\hbar$ are energy times time, the dimensions of G are energy times length per mass squared. The numerical values of the physical constants are given in Appendix F.

EXECUTE: **(a)** The dimensions of $\sqrt{\hbar G/c^3}$ are

$$\left[\frac{(\text{E} \cdot \text{T})(\text{E} \cdot \text{L/M}^2)}{(\text{L/T})^3}\right]^{1/2} = \left[\frac{\text{E}}{\text{M}}\right]\left[\frac{\text{T}^2}{\text{L}}\right] = \left[\frac{\text{L}}{\text{T}}\right]^2\left[\frac{\text{T}^2}{\text{L}}\right] = \text{L}.$$

(b) $\left(\dfrac{\hbar G}{c^3}\right)^{1/2} = \left(\dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)}{2\pi (3.00 \times 10^8 \text{ m/s})^3}\right)^{1/2} = 1.616 \times 10^{-35}$ m.

EVALUATE: Both the dimensional analysis and the numerical calculation agree that the units of this quantity are meters.

44.39. IDENTIFY and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: (a) $p + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$ or can write as ${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$

If neutral atom masses are used then the masses of the two atomic electrons on each side of the reaction will cancel.

Taking the atomic masses from Table 43.2, the mass decrease is $m({}^1_1\text{H}) + m({}^2_1\text{H}) - m({}^3_2\text{He}) = 1.007825\text{ u} +$ $2.014102\text{ u} - 3.016029\text{ u} = 0.005898\text{ u}$. The energy released is the energy equivalent of this mass decrease: $(0.005898\text{ u})(931.5\text{ MeV/u}) = 5.494\text{ MeV}$.

(b) ${}^1_0\text{n} + {}^3_2\text{He} \rightarrow {}^4_2\text{He}$

If neutral helium masses are used then the masses of the two atomic electrons on each side of the reaction equation will cancel. The mass decrease is $m({}^1_0\text{n}) + m({}^3_2\text{He}) - m({}^4_2\text{He}) = 1.008665\text{ u} +$ $3.016029\text{ u} - 4.002603\text{ u} = 0.022091\text{ u}$. The energy released is the energy equivalent of this mass decrease: $(0.022091\text{ u})(931.15\text{ MeV/u}) = 20.58\text{ MeV}$.

EVALUATE: These are important nucleosynthesis reactions, discussed in Section 44.7.

44.40. IDENTIFY: The energy released in the reaction is the energy equivalent of the mass decrease that occurs in the reaction.

SET UP: 1 u is equivalent to 931.5 MeV. The neutral atom masses are given in Table 43.2.

EXECUTE: $3m({}^4\text{He}) - m({}^{12}\text{C}) = 7.80 \times 10^{-3}\text{ u}$, or 7.27 MeV.

EVALUATE: The neutral atom masses include 6 electrons on each side of the reaction equation. The electron masses cancel and we obtain the same mass change as would be calculated using nuclear masses.

44.41. IDENTIFY: The reaction energy Q is defined in Eq. (43.23) and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.

SET UP: Use the particle masses given in Section 43.1. 1 u is equivalent to 931.5 MeV.

EXECUTE: $\Delta m = m_e + m_p - m_n - m_{\nu_e}$ so assuming $m_{\nu_e} \approx 0$,

$$\Delta m = 0.0005486\text{ u} + 1.007276\text{ u} - 1.008665\text{ u} = -8.40 \times 10^{-4}\text{ u}$$

$\Rightarrow E = (\Delta m)c^2 = (-8.40 \times 10^{-4}\text{ u})(931.5\text{ MeV/u}) = -0.783\text{ MeV}$ and is endoergic.

EVALUATE: The energy consumed in the reaction would have to come from the initial kinetic energy of the reactants.

44.42. IDENTIFY: The reaction energy Q is defined in Eq. (43.23) and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.

SET UP: 1 u is equivalent to 931.5 MeV. Use the neutral atom masses that are given in Table 43.2.

EXECUTE: $m_{{}^{12}_6\text{C}} + m_{{}^4_2\text{He}} - m_{{}^{16}_8\text{O}} = 7.69 \times 10^{-3}\text{u}$, or 7.16 MeV, an exoergic reaction.

EVALUATE: 7.16 MeV of energy is released in the reaction.

44.43. IDENTIFY and **SET UP:** The Wien displacement law (Eq. 39.21) sys $\lambda_m T$ equals a constant. Use this to relate $\lambda_{m,1}$ at T_1 to $\lambda_{m,2}$ at T_2.

EXECUTE: $\lambda_{m,1}T_1 = \lambda_{m,2}T_2$

$$\lambda_{m,1} = \lambda_{m,2}\left(\frac{T_2}{T_1}\right) = 1.062 \times 10^{-3}\text{ m}\left(\frac{2.728\text{ K}}{3000\text{ K}}\right) = 966\text{ nm}$$

EVALUATE: The peak wavelength was much less when the temperature was much higher.

44.44. IDENTIFY: Use the Bohr model to calculate the ionization energy of positronium.

SET UP and **EXECUTE:** The reduced mass is $m_r = \dfrac{mm}{m+m} = m/2$. For a hydrogen with an infinitely massive nucleus, the ground state energy is $E_1 = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2} = -13.6\text{ eV}$. For positronium,

$$E_1 = -\frac{1}{\epsilon_0^2}\frac{m_r e^4}{8n^2h^2} = \frac{1}{2}\left(-\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}\right) = -(13.6\text{ eV})/2 = -6.80\text{ eV}.$$ The ionization energy is 6.80 eV.

EVALUATE: This is half the ionization energy of hydrogen.

44.45. **IDENTIFY** and **SET UP:** For colliding beams the available energy is twice the beam energy. For a fixed-target experiment only a portion of the beam energy is available energy (Eqs. 44.9 and 44.10).

EXECUTE: **(a)** $E_a = 2(7.0 \text{ TeV}) = 14.0 \text{ TeV}$

(b) Need $E_a = 14.0 \text{ TeV} = 14.0\times10^6 \text{ MeV}$. Since the target and projectile particles are both protons Eq. (44.10) can be used: $E_a^2 = 2mc^2(E_m + mc^2)$

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(14.0\times10^6 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 938.3 \text{ MeV} = 1.0\times10^{11} \text{ MeV} = 1.0\times10^5 \text{ TeV}.$$

EVALUATE: This shows the great advantage of colliding beams at relativistic energies.

44.46. **IDENTIFY:** The initial total energy of the colliding proton and antiproton equals the total energy of the two photons.

SET UP: For a particle with mass, $E = K + mc^2$. For a proton, $m_p c^2 = 938 \text{ MeV}$.

EXECUTE: $K + m_p c^2 = \dfrac{hc}{\lambda}, K = \dfrac{hc}{\lambda} - m_p c^2 = 652 \text{ MeV}.$

EVALUATE: If the kinetic energies of the colliding particles increase, then the wavelength of each photon decreases.

44.47. **IDENTIFY:** The energy comes from a mass decrease.

SET UP: A charged pion decays into a muon plus a neutrino. The muon in turn decays into an electron or positron plus two neutrinos.

EXECUTE: **(a)** $\pi^- \to \mu^- + \text{neutrino} \to e^- + \text{three neutrinos}.$

(b) If we neglect the mass of the neutrinos, the mass decrease is

$m(\pi^-) - m(e^-) = 273m_e - m_e = 272m_e = 2.480\times10^{-28} \text{ kg}.$

$E = mc^2 = 2.23\times10^{-11} \text{ J} = 139 \text{ MeV}.$

(c) The total energy delivered to the tissue is $(50.0 \text{ J/kg})(10.0\times10^{-3} \text{ kg}) = 0.500 \text{ J}$. The number of π^- mesons required is $\dfrac{0.500 \text{ J}}{2.23\times10^{-11} \text{ J}} = 2.24\times10^{10}.$

(d) The RBE for the electrons that are produced is 1.0, so the equivalent dose is

$1.0(50.0 \text{ Gy}) = 50.0 \text{ Sv} = 5.0\times10^3 \text{ rem}.$

EVALUATE: The π are heavier than electrons and therefore behave differently as they hit the tissue.

44.48. **IDENTIFY:** Apply Eq. (44.9).

SET UP: In Eq. (44.9), $E_a = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_p, m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_a^2 - (m_{\pi^-}c^2)^2 - (m_p c^2)^2}{2m_p c^2} - (m_{\pi^-})c^2.$$

EXECUTE: $$K = \frac{(1193 \text{ MeV} + 497.7 \text{ MeV})^2 - (139.6 \text{ MeV})^2 - (938.3 \text{ MeV})^2}{2(938.3 \text{ MeV})} - 139.6 \text{ MeV} = 904 \text{ MeV}.$$

EVALUATE: The increase in rest energy is

$(m_{\Sigma^0} + m_{K^0} - m_{\pi^-} - m_p)c^2 = 1193 \text{ MeV} + 497.7 \text{ MeV} - 139.6 \text{ MeV} - 938.3 \text{ MeV} = 613 \text{ MeV}$. The threshold kinetic energy is larger than this because not all the kinetic energy of the beam is available to form new particle states.

44.49. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving proton is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be leftover kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2(E_m + mc^2)$ for two particles of equal mass when one is initially stationary. The *minimum* available energy must be equal to the rest mass energies of the products, which in this case is two protons, a K^+ and a K^-. The available energy must be at least the sum of the final rest masses.

EXECUTE: The minimum amount of available energy must be

$E_a = 2m_p + m_{K^+} + m_{K^-} = 2(938.3\text{ MeV}) + 493.7\text{ MeV} + 493.7\text{ MeV} = 2864\text{ MeV} = 2.864\text{ GeV}$

Solving the available energy formula for E_m gives $E_a^2 = 2mc^2(E_m + mc^2)$ and

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(2864\text{ MeV})^2}{2(938.3\text{ MeV})} - 938.3\text{ MeV} = 3432.6\text{ MeV}$$

Recalling that E_m is the *total* energy of the proton, including its rest mass energy (RME), we have

$$K = E_m - \text{RME} = 3432.6\text{ MeV} - 938.3\text{ MeV} = 2494\text{ MeV} = 2.494\text{ GeV}$$

Therefore the threshold kinetic energy is $K = 2494\text{ MeV} = 2.494\text{ GeV}$.

EVALUATE: Considerably less energy would be needed if the experiment were done using colliding beams of protons.

44.50. **IDENTIFY:** Charge must be conserved. The energy released equals the decrease in rest energy that occurs in the decay.

SET UP: The rest energies are given in Table 44.3.

EXECUTE: **(a)** The decay products must be neutral, so the only possible combinations are

$\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$.

(b) $m_{\eta_0} - 3m_{\pi^0} = 142.3\text{ Me V}/c^2$, so the kinetic energy of the π^0 mesons is 142.3 MeV. For the other reaction, $K = (m_{\eta_0} - m_{\pi^0} - m_{\pi^+} - m_{\pi^-})c^2 = 133.1\text{ MeV}$.

EVALUATE: The total momentum of the decay products must be zero. This imposes a correlation between the directions of the velocities of the decay products.

44.51. **IDENTIFY:** Baryon number, charge, strangeness and lepton numbers are all conserved in the reactions.

SET UP: Use Table 44.3 to identify the missing particle, once its properties have been determined.

EXECUTE: **(a)** The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+.

(b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero and the particle is π^-.

(c) The baryon number is -1, the charge is 0, the strangeness is zero, all lepton numbers are 0 and the particle is an antineutron.

(d) The baryon number is 0 the charge is $+e$, the strangeness is 0, the muonic lepton number is -1, all other lepton numbers are 0 and the particle is μ^+.

EVALUATE: Rest energy considerations would determine if each reaction is endoergic or exoergic.

44.52. **IDENTIFY:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let Δt be the mean lifetime.

SET UP: The rest energy of the ψ is 3097 MeV.

EXECUTE: $\Delta t = 7.6\times10^{-21}\text{ s} \Rightarrow \Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{1.054\times10^{-34}\text{ J}\cdot\text{s}}{2(7.6\times10^{-21}\text{ s})} = 6.93\times10^{-15}\text{ J} = 43\text{ keV}.$

$$\frac{\Delta E}{m_\psi c^2} = \frac{0.043\text{ MeV}}{3097\text{ MeV}} = 1.4\times10^{-5}.$$

EVALUATE: The energy width due to the lifetime of the particle is a small fraction of its rest energy.

44.53. **IDENTIFY** and **SET UP:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let ΔE be the energy width and let Δt be the lifetime.

EXECUTE: $\dfrac{\hbar}{2\Delta E} = \dfrac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{2(4.4\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 7.5\times10^{-23}\text{ s}.$

EVALUATE: The shorter the lifetime, the greater the energy width.

44.54. **IDENTIFY** and **SET UP:** $\phi \to K^+ + K^-$. The total energy released is the energy equivalent of the mass decrease.

(a) EXECUTE: The mass decrease is $m(\phi) - m(K^+) - m(K^-)$. The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(K^+) - mc^2(K^-)$. The rest mass energy mc^2 for the ϕ meson is given Problem 44.53, and the values for K^+ and K^- are given in Table 44.3. The energy released then is $1019.4\text{ MeV} - 2(493.7\text{ MeV}) = 32.0\text{ MeV}$. The K^+ gets half this, 16.0 Mev.

EVALUATE: **(b)** Does the decay $\phi \to \mathrm{K}^+ + \mathrm{K}^- + \pi^0$ occur? The energy equivalent of the $\mathrm{K}^+ + \mathrm{K}^- + \pi^0$ mass is $493.7\text{ MeV} + 493.7\text{ MeV} + 135.0\text{ MeV} = 1122\text{ MeV}$. This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \to \mathrm{K}^+ + \pi^-$ occur? The reaction $\phi \to \mathrm{K}^+ + \mathrm{K}^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness -1, so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $\mathrm{K}^+ + \pi^-$ has strangeness -1. The decay $\phi \to \mathrm{K}^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \to \mathrm{K}^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

44.55. **IDENTIFY:** Apply $\left|\dfrac{dN}{dt}\right| = \lambda N$ to find the number of decays in one year.

SET UP: Water has a molecular mass of 18.0×10^{-3} kg/mol.

EXECUTE: **(a)** The number of protons in a kilogram is

$(1.00\text{ kg})\left(\dfrac{6.022\times10^{23}\text{ molecules/mol}}{18.0\times10^{-3}\text{ kg/mol}}\right)(2\text{ protons/molecule}) = 6.7\times10^{25}$. Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_\mathrm{p}c^2 = 938.3\text{ MeV} = 1.503\times10^{-10}\text{ J}$, and so the energy deposited in a year, per kilogram, is

$(6.7\times10^{25})\left(\dfrac{\ln(2)}{1.0\times10^{18}\text{ y}}\right)(1\text{ y})(1.50\times10^{-10}\text{ J}) = 7.0\times10^{-3}\text{ Gy} = 0.70\text{ rad}.$

(b) For an RBE of unity, the equivalent dose is $(1)(0.70\text{ rad}) = 0.70\text{ rem}$.

EVALUATE: The equivalent dose is much larger than that due to the natural background. It is not feasible for the proton lifetime to be as short as 1.0×10^{18} y.

44.56. **IDENTIFY:** The energy comes from the mass difference.

SET UP: $\Xi^- \to \Lambda^0 + \pi^-$. $p_\Lambda = p_\pi = p$. $E_\Xi = E_\Lambda + E_\pi$. $m_\Xi c^2 = 1321\text{ MeV}$. $m_\Lambda c^2 = 1116\text{ MeV}$. $m_\pi c^2 = 139.6\text{ MeV}$. $m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2c^2}$

EXECUTE: **(a)** The total energy released is

$m_\Xi c^2 - m_\pi c^2 - m_\Lambda c^2 = 1321\text{ MeV} - 139.6\text{ MeV} - 1116\text{ MeV} = 65.4\text{ MeV}$.

(b) $m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2c^2}$. $m_\Xi c^2 - \sqrt{m_\Lambda^2 c^4 + p^2c^2} = \sqrt{m_\pi^2 c^4 + p^2c^2}$. Square both sides:

$m_\Xi^2 c^4 + m_\Lambda^2 c^4 + p^2c^2 - 2m_\Xi c^2 E_\Lambda = m_\pi^2 c^4 + p^2c^2$. $E_\Lambda = \dfrac{m_\Xi^2 c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}$.

$K_\Lambda = \dfrac{m_\Xi^2 c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2} - m_\Lambda c^2$. $E_\pi = E_\Xi - E_\Lambda = m_\Xi c^2 - \dfrac{m_\Xi^2 c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}$.

$E_\pi = \dfrac{m_\Xi^2 c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2}$. $K_\pi = \dfrac{m_\Xi^2 c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2} - m_\pi c^2$. Putting in numbers gives

$$K_\Lambda = \frac{(1321\text{ MeV})^2 + (1116\text{ MeV})^2 - (139.6\text{ MeV})^2}{2(1321\text{ MeV})} - 1116\text{ MeV} = 8.5\text{ MeV}\ \ (13\%\text{ of total}).$$

$$K_\pi = \frac{(1321\text{ MeV})^2 - (1116\text{ MeV})^2 + (139.6\text{ MeV})^2}{2(1321\text{ MeV})} - 139.6\text{ MeV} = 56.9\text{ MeV}\ \ (87\%\text{ of total}).$$

EVALUATE: The two particles do not have equal kinetic energies because they have different masses.

44.57. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** For this model, $\frac{dR}{dt} = HR$, so $\frac{dR/dt}{R} = \frac{HR}{R} = H$, presumed to be the same for all points on the surface.

(b) For constant $\theta, \frac{dr}{dt} = \frac{dR}{dt}\theta = HR\theta = Hr.$

(c) See part (a), $H_0 = \frac{dR/dt}{R}.$

(d) The equation $\frac{dR}{dt} = H_0 R$ is a differential equation, the solution to which, for constant H_0, is $R(t) = R_0 e^{H_0 t}$, where R_0 is the value of R at $t = 0.$ This equation may be solved by separation of variables, as $\frac{dR/dt}{R} = \frac{d}{dt}\ln(R) = H_0$ and integrating both sides with respect to time.

EVALUATE: **(e)** A constant H_0 would mean a constant critical density, which is inconsistent with uniform expansion.

44.58. **IDENTIFY:** $H = \frac{1}{R}\frac{dR}{dt}.$

SET UP: From Problem 44.57, $r = R\theta \Rightarrow R = \frac{r}{\theta}.$

EXECUTE: $\frac{dR}{dt} = \frac{1}{\theta}\frac{dr}{dt} - \frac{r}{\theta^2}\frac{d\theta}{dt} = \frac{1}{\theta}\frac{dr}{dt}$ since $\frac{d\theta}{dt} = 0.$ So

$\frac{1}{R}\frac{dR}{dt} = \frac{1}{R\theta}\frac{dr}{dt} = \frac{1}{r}\frac{dr}{dt} \Rightarrow v = \frac{dr}{dt} = \left(\frac{1}{R}\frac{dR}{dt}\right)r = H_0 r.$ Now $\frac{dv}{d\theta} = 0 = \frac{d}{d\theta}\left(\frac{r}{R}\frac{dR}{dt}\right) = \frac{d}{d\theta}\left(\theta\frac{dR}{dt}\right)$

$\Rightarrow \theta\frac{dR}{dt} = K$ where K is a constant. $\Rightarrow \frac{dR}{dt} = \frac{K}{\theta} \Rightarrow R = \left(\frac{K}{\theta}\right)t$ since $\frac{d\theta}{dt} = 0 \Rightarrow H_0 = \frac{1}{R}\frac{dR}{dt} = \frac{\theta}{Kt}\frac{K}{\theta} = \frac{1}{t}.$

So the current value of the Hubble constant is $\frac{1}{T}$ where T is the present age of the universe.

EVALUATE: The current experimental value of H_0 is $2.3\times10^{-18}\ \text{s}^{-1}$, so $T = 4.4\times10^{17}\ \text{s} = 1.4\times10^{10}$ y.

44.59. **IDENTIFY:** The matter density is proportional to $1/R^3.$

SET UP and **EXECUTE:** **(a)** When the matter density was large enough compared to the dark energy density, the slowing due to gravitational attraction would have dominated over the cosmic repulsion due to dark energy.

(b) Matter density is proportional to $1/R^3$, so $R \propto \frac{1}{\rho^{1/3}}.$ Therefore $\frac{R}{R_0} = \left(\frac{1/\rho_{\text{past}}}{1/\rho_{\text{now}}}\right)^{1/3} = \left(\frac{\rho_{\text{now}}}{\rho_{\text{past}}}\right)^{1/3}$. If ρ_{m} and ρ_{DE} are the present-day densities of matter of all kinds and of dark energy, we have $\rho_{\text{DE}} = 0.726\rho_{\text{crit}}$ and $\rho_{\text{m}} = 0.274\rho_{\text{crit}}$ at the present time. Putting this into the above equation for R/R_0 gives

$$\frac{R}{R_0} = \left(\frac{\frac{0.274}{0.726}\rho_{\text{DE}}}{2\rho_{\text{DE}}}\right)^{1/3} = 0.574.$$

EVALUATE: **(c)** 300 My: speeding up $(R/R_0 = 0.98)$; 10.2 Gy: slowing down $(R/R_0 = 0.35)$.

44.60. **IDENTIFY:** The kinetic energy comes from the mass difference, and momentum is conserved.

SET UP: $\left|p_{\pi^+ y}\right| = \left|p_{\pi^- y}\right|.$ $p_{\pi^+}\sin\theta = p_{\pi^-}\sin\theta$ and $p_{\pi^+} = p_{\pi^-} = p_\pi.$ $m_{\text{K}}c^2 = 497.7$ MeV. $m_\pi c^2 = 139.6$ MeV.

EXECUTE: Conservation of momentum for the decay gives $p_K = 2p_{\pi x}$ and $p_K^2 = 4p_{\pi x}^2$.
$p_K^2 c^2 = E_K^2 - m_K^2 c^2$. $E_K = 497.7 \text{ MeV} + 225 \text{ MeV} = 722.7 \text{ MeV}$ so
$p_K^2 c^2 = (722.7 \text{ MeV})^2 - (497.7 \text{ MeV})^2 = 2.746 \times 10^5 \text{ (MeV)}^2$ and
$p_{\pi x}^2 c^2 = [2.746 \times 10^5 \text{ (MeV)}^2]/4 = 6.865 \times 10^4 \text{ (MeV)}^2$. Conservation of energy says $E_K = 2E_\pi$.
$E_\pi = \dfrac{E_K}{2} = 361.4 \text{ MeV}$.
$K_\pi = E_\pi - m_\pi c^2 = 361.4 \text{ MeV} - 139.6 \text{ MeV} = 222 \text{ MeV}$.
$p_\pi^2 c^2 = E_\pi^2 - (m_\pi c^2)^2 = (361.4 \text{ MeV})^2 - (139.6 \text{ MeV})^2 = 1.11 \times 10^5 \text{ (MeV)}^2$. The angle θ that the velocity of the π^+ particle makes with the $+x$-axis is given by $\cos\theta = \sqrt{\dfrac{p_{\pi x}^2 c^2}{p_\pi^2 c^2}} = \sqrt{\dfrac{6.865 \times 10^4}{1.11 \times 10^5}}$, which gives

$\theta = 38.2^\circ$.

EVALUATE: The pions have the same energy and go off at the same angle because they have equal masses.

44.61. **IDENTIFY:** The kinetic energy comes from the mass difference.

SET UP and **EXECUTE:** $K_\Sigma = 180 \text{ MeV}$. $m_\Sigma c^2 = 1197 \text{ MeV}$. $m_n c^2 = 939.6 \text{ MeV}$. $m_\pi c^2 = 139.6 \text{ MeV}$.
$E_\Sigma = K_\Sigma + m_\Sigma c^2 = 180 \text{ MeV} + 1197 \text{ MeV} = 1377 \text{ MeV}$. Conservation of the x-component of momentum gives $p_\Sigma = p_{nx}$. Then $p_{nx}^2 c^2 = p_\Sigma^2 c^2 = E_\Sigma^2 - (m_\Sigma c)^2 = (1377 \text{ MeV})^2 - (1197 \text{ MeV})^2 = 4.633 \times 10^5 \text{ (MeV)}^2$.
Conservation of energy gives $E_\Sigma = E_\pi + E_n$. $E_\Sigma = \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2} + \sqrt{m_n^2 c^4 + p_n^2 c^2}$.
$E_\Sigma - \sqrt{m_n^2 c^4 + p_n^2 c^2} = \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2}$. Square both sides:
$E_\Sigma^2 + m_n^2 c^4 + p_{nx}^2 c^2 + p_{ny}^2 c^2 - 2E_\Sigma E_n = m_\pi^2 c^4 + p_\pi^2 c^2$. $p_\pi = p_{ny}$ so
$E_\Sigma^2 + m_n^2 c^4 + p_{nx}^2 c^2 - 2E_\Sigma E_n = m_\pi^2 c^4$ and $E_n = \dfrac{E_\Sigma^2 + m_n^2 c^4 - m_\pi^2 c^4 + p_{nx}^2 c^2}{2E_\Sigma}$.

$$E_n = \frac{(1377 \text{ MeV})^2 + (939.6 \text{ MeV})^2 - (139.6 \text{ MeV})^2 + 4.633 \times 10^5 \text{ (MeV)}^2}{2(1377 \text{ MeV})} = 1170 \text{ MeV}.$$

$K_n = E_n - m_n c^2 = 1170 \text{ MeV} - 939.6 \text{ MeV} = 230 \text{ MeV}$.
$E_\pi = E_\Sigma - E_n = 1377 \text{ MeV} - 1170 \text{ MeV} = 207 \text{ MeV}$.
$K_\pi = E_\pi - m_\pi c^2 = 207 \text{ MeV} - 139.6 \text{ MeV} = 67 \text{ MeV}$.
$p_n^2 c^2 = E_n^2 - m_n^2 c^2 = (1170 \text{ MeV})^2 - (939.6 \text{ MeV})^2 = 4.861 \times 10^5 \text{ (MeV)}^2$. The angle θ the velocity of the neutron makes with the $+x$-axis is given by $\cos\theta = \dfrac{p_{nx}}{p_n} = \sqrt{\dfrac{4.633 \times 10^5}{4.861 \times 10^5}}$ and $\theta = 12.5^\circ$ below the $+x$-axis.

EVALUATE: The decay particles do not have equal energy because they have different masses.

44.62. **IDENTIFY:** Follow the steps specified in the problem. The Lorentz velocity transformation is given in Eq. (37.23).

SET UP: Let the $+x$-direction be the direction of the initial velocity of the bombarding particle.

EXECUTE: **(a)** For mass m, in Eq. (37.23) $u = -v_{cm}$, $v' = v_0$, and so $v_m = \dfrac{v_0 - v_{cm}}{1 - v_0 v_{cm}/c^2}$. For mass M, $u = -v_{cm}$, $v' = 0$, so $v_M = -v_{cm}$.

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0$, where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to

$\beta_m \gamma_m = (\beta_0 - \beta')\gamma_0 \gamma_M$, where $\beta_0 = \dfrac{v_0}{c}$, $\beta' = \dfrac{v_{cm}}{c}$, and the condition for no net momentum becomes

$m(\beta_0 - \beta')\gamma_0 \gamma_M = M\beta' \gamma_M$, or $\beta' = \dfrac{\beta_0}{1 + \dfrac{M}{m\gamma_0}} = \beta_0 \dfrac{m}{m + M\sqrt{1-\beta_0^2}}$. $v_{cm} = \dfrac{mv_0}{m + M\sqrt{1-(v_0/c)^2}}$.

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0 \gamma_0 \dfrac{M}{m + M\gamma_0}$, $v_M = -v_0 \gamma_0 \dfrac{m}{m\gamma_0 + M}$. After some more algebra,

$\gamma_m = \dfrac{m + M\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}$, $\gamma_M = \dfrac{M + m\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}$, from which

$m\gamma_m + M\gamma_M = \sqrt{m^2 + M^2 + 2mM\gamma_0}$. This last expression, multiplied by c^2, is the available energy E_a in the center of mass frame, so that

$E_a^2 = (m^2 + M^2 + 2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0 c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2 E_m$, which is Eq. (44.9).

EVALUATE: The energy E_a in the center-of-momentum frame is the energy that is available to form new particle states.