36

DIFFRACTION

36.1. **IDENTIFY:** Use $y = x\tan\theta$ to calculate the angular position θ of the first minimum. The minima are located by Eq. (36.2): $\sin\theta = \frac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots$ First minimum means $m = 1$ and $\sin\theta_1 = \lambda/a$ and $\lambda = a\sin\theta_1$. Use this equation to calculate λ.

SET UP: The central maximum is sketched in Figure 36.1.

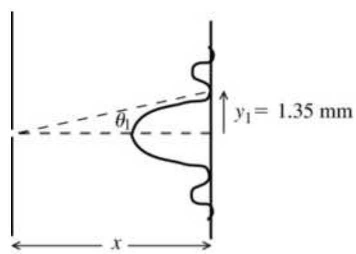


Figure 36.1

EXECUTE: $y_1 = x\tan\theta_1$

$\tan\theta_1 = \frac{y_1}{x} = \frac{1.35\times10^{-3}\text{ m}}{2.00\text{ m}} = 0.675\times10^{-3}$

$\theta_1 = 0.675\times10^{-3}$ rad

$\lambda = a\sin\theta_1 = (0.750\times10^{-3}\text{ m})\sin(0.675\times10^{-3}\text{ rad}) = 506\text{ nm}$

EVALUATE: θ_1 is small so the approximation used to obtain Eq. (36.3) is valid and this equation could have been used.

36.2. **IDENTIFY:** The angle is small, so $y_m = x\frac{m\lambda}{a}$.

SET UP: $y_1 = 10.2$ mm

EXECUTE: $y_1 = \frac{x\lambda}{a} \Rightarrow a = \frac{x\lambda}{y_1} = \frac{(0.600\text{ m})(5.46\times10^{-7}\text{ m})}{10.2\times10^{-3}\text{ m}} = 3.21\times10^{-5}\text{ m}.$

EVALUATE: The diffraction pattern is observed at a distance of 60.0 cm from the slit.

36.3. **IDENTIFY:** The dark fringes are located at angles θ that satisfy $\sin\theta = \frac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots$.

SET UP: The largest value of $|\sin\theta|$ is 1.00.

EXECUTE: **(a)** Solve for *m* that corresponds to $\sin\theta = 1$: $m = \frac{a}{\lambda} = \frac{0.0666\times10^{-3}\text{ m}}{585\times10^{-9}\text{ m}} = 113.8$. The largest value *m* can have is 113. $m = \pm1, \pm2, \ldots, \pm113$ gives 226 dark fringes.

(b) For $m = \pm113$, $\sin\theta = \pm113\left(\frac{585\times10^{-9}\text{ m}}{0.0666\times10^{-3}\text{ m}}\right) = \pm0.9926$ and $\theta = \pm83.0°$.

EVALUATE: When the slit width *a* is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

36.4. **IDENTIFY** and **SET UP:** λ/a is very small, so the approximate expression $y_m = R\dfrac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

EXECUTE: $y_1 = \dfrac{\lambda R}{a} = \dfrac{(633\times10^{-9}\ \text{m})(3.50\ \text{m})}{0.750\times10^{-3}\ \text{m}} = 2.95\times10^{-3}\ \text{m} = 2.95\ \text{mm}.$ $2y_1 = 5.90\ \text{mm}.$

EVALUATE: When a is decreased, the width $2y_1$ of the central maximum increases.

36.5. **IDENTIFY:** The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$.

SET UP: $a = 12.0$ cm. $x = 8.00$ m.

EXECUTE: The angle to the first minimum is $\theta = \arcsin\left(\dfrac{\lambda}{a}\right) = \arcsin\left(\dfrac{9.00\ \text{cm}}{12.00\ \text{cm}}\right) = 48.6°.$

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta = (8.00\ \text{m})\tan(48.6°) = \pm(9.07\ \text{m}).$

EVALUATE: $2\lambda/a$ is greater than 1, so only the $m = 1$ minimum is seen.

36.6. **IDENTIFY:** The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin\theta = \dfrac{\lambda}{a}$. The time between crests is the period T. $f = \dfrac{1}{T}$ and $\lambda = \dfrac{v}{f}$.

SET UP: The time between crests is the period, so $T = 1.0$ h.

EXECUTE: **(a)** $f = \dfrac{1}{T} = \dfrac{1}{1.0\ \text{h}} = 1.0\ \text{h}^{-1}.$ $\lambda = \dfrac{v}{f} = \dfrac{800\ \text{km/h}}{1.0\ \text{h}^{-1}} = 800\ \text{km}.$

(b) Africa-Antarctica: $\sin\theta = \dfrac{800\ \text{km}}{4500\ \text{km}}$ and $\theta = 10.2°.$

Australia-Antarctica: $\sin\theta = \dfrac{800\ \text{km}}{3700\ \text{km}}$ and $\theta = 12.5°.$

EVALUATE: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

36.7. **IDENTIFY:** We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.
SET UP: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin\theta_m = m\lambda/a$, where $m = 1, 2, 3, \ldots$.

EXECUTE: **(a)** The frequency of the water waves is $f = 75.0\ \text{min}^{-1} = 1.25\ \text{s}^{-1} = 1.25\ \text{Hz},$ so their wavelength is $\lambda = v/f = (15.0\ \text{cm/s})/(1.25\ \text{Hz}) = 12.0\ \text{cm}.$

At the first point for which destructive interference occurs, we have $\tan\theta = (0.613\ \text{m})/(3.20\ \text{m}) \Rightarrow \theta = 10.84°.$ $a\sin\theta = \lambda$ and

$$a = \lambda/\sin\theta = (12.0\ \text{cm})/(\sin 10.84°) = 63.8\ \text{cm}.$$

(b) First find the angles at which destructive interference occurs.

$$\sin\theta_2 = 2\lambda/a = 2(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_2 = \pm 22.1°$$

$$\sin\theta_3 = 3\lambda/a = 3(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_3 = \pm 34.3°$$

$$\sin\theta_4 = 4\lambda/a = 4(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_4 = \pm 48.8°$$

$$\sin\theta_5 = 5\lambda/a = 5(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_5 = \pm 70.1°$$

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a$.

36.8. **IDENTIFY:** The angle is small, so $y_m = x\dfrac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1$, so $y_1 = 3.00$ mm.

EXECUTE: **(a)** $y_1 = \frac{x\lambda}{a} \Rightarrow a = \frac{x\lambda}{y_1} = \frac{(2.50\text{ m})(5.00\times10^{-7}\text{ m})}{3.00\times10^{-3}\text{ m}} = 4.17\times10^{-4}\text{ m}.$

(b) $a = \frac{x\lambda}{y_1} = \frac{(2.50\text{ m})(5.00\times10^{-5}\text{ m})}{3.00\times10^{-3}\text{ m}} = 4.17\times10^{-2}\text{ m} = 4.2\text{ cm}.$

(c) $a = \frac{x\lambda}{y_1} = \frac{(2.50\text{ m})(5.00\times10^{-10}\text{ m})}{3.00\times10^{-3}\text{ m}} = 4.17\times10^{-7}\text{ m}.$

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.9. **IDENTIFY** and **SET UP:** $v = f\lambda$ gives λ. The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin\theta = m\lambda/a$, $m = \pm1, \pm2, \ldots$ Solve for θ.

EXECUTE: $\lambda = v/f = (344\text{ m/s})/(1250\text{ Hz}) = 0.2752\text{ m}$; $a = 1.00\text{ m}$. $m = \pm1$, $\theta = \pm16.0°$; $m = \pm2$, $\theta = \pm33.4°$; $m = \pm3$, $\theta = \pm55.6°$; no solution for larger m

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a \ll 1$.

36.10. **IDENTIFY:** Compare E_y to the expression $E_y = E_{max}\sin(kx - \omega t)$ and determine k, and from that calculate λ. $f = c/\lambda$. The dark bands are located by $\sin\theta = \frac{m\lambda}{a}$.

SET UP: $c = 3.00\times10^8$ m/s. The first dark band corresponds to $m = 1$.

EXECUTE: **(a)** $E = E_{max}\sin(kx - \omega t)$. $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.20\times10^7\text{ m}^{-1}} = 5.24\times10^{-7}\text{ m}.$

$f\lambda = c \Rightarrow f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{5.24\times10^{-7}\text{ m}} = 5.73\times10^{14}\text{ Hz}.$

(b) $a\sin\theta = \lambda$. $a = \frac{\lambda}{\sin\theta} = \frac{5.24\times10^{-7}\text{ m}}{\sin 28.6°} = 1.09\times10^{-6}\text{ m}.$

(c) $a\sin\theta = m\lambda (m = 1, 2, 3, \ldots)$. $\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\frac{5.24\times10^{-7}\text{ m}}{1.09\times10^{-6}\text{ m}}$ and $\theta_2 = \pm74°$.

EVALUATE: For $m = 3$, $\frac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. **IDENTIFY** and **SET UP:** $\sin\theta = \lambda/a$ locates the first minimum. $y = x\tan\theta$.

EXECUTE: $\tan\theta = y/x = (36.5\text{ cm})/(40.0\text{ cm})$ and $\theta = 42.38°$.

$a = \lambda/\sin\theta = (620\times10^{-9}\text{ m})/(\sin 42.38°) = 0.920\ \mu\text{m}$

EVALUATE: $\theta = 0.74$ rad and $\sin\theta = 0.67$, so the approximation $\sin\theta \approx \theta$ would not be accurate.

36.12. **IDENTIFY:** Calculate the angular positions of the minima and use $y = x\tan\theta$ to calculate the distance on the screen between them.

(a) SET UP: The central bright fringe is shown in Figure 36.12a.

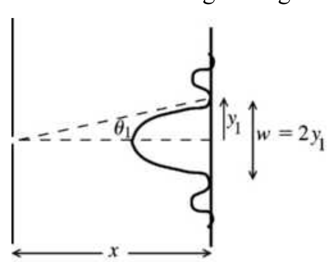


Figure 36.12a

EXECUTE: The first minimum is located by

$\sin\theta_1 = \frac{\lambda}{a} = \frac{633\times10^{-9}\text{ m}}{0.350\times10^{-3}\text{ m}} = 1.809\times10^{-3}$

$\theta_1 = 1.809\times10^{-3}$ rad

$y_1 = x\tan\theta_1 = (3.00\text{ m})\tan(1.809\times10^{-3}\text{ rad}) = 5.427\times10^{-3}\text{ m}$

$w = 2y_1 = 2(5.427\times10^{-3}\text{ m}) = 1.09\times10^{-2}\text{ m} = 10.9\text{ mm}$

(b) SET UP: The first bright fringe on one side of the central maximum is shown in Figure 36.12b.

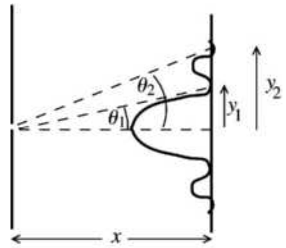


Figure 36.12b

EXECUTE: $w = y_2 - y_1$

$y_1 = 5.427\times10^{-3}$ m (part (a))

$\sin\theta_2 = \dfrac{2\lambda}{a} = 3.618\times10^{-3}$

$\theta_2 = 3.618\times10^{-3}$ rad

$y_2 = x\tan\theta_2 = 1.085\times10^{-2}$ m

$w = y_2 - y_1 = 1.085\times10^{-2}\text{ m} - 5.427\times10^{-3}\text{ m} = 5.4\text{ mm}$

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

36.13. IDENTIFY: The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$. For part (b) apply Eq. (36.7).

SET UP: For the first minimum, $m = 1$. The intensity at $\theta = 0$ is I_0.

EXECUTE: **(a)** $\sin\theta = \dfrac{m\lambda}{a} = \sin 90.0° = 1 = \dfrac{m\lambda}{a} = \dfrac{\lambda}{a}$. Thus $a = \lambda = 580\text{ nm} = 5.80\times10^{-4}$ mm.

(b) According to Eq. (36.7),

$$\frac{I}{I_0} = \left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 = \left\{\frac{\sin[\pi(\sin\pi/4)]}{\pi(\sin\pi/4)}\right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45°$, $\dfrac{I}{I_0} = \left\{\dfrac{\sin[(\pi/2)(\sin\pi/4)]}{(\pi/2)(\sin\pi/4)}\right\}^2 = 0.65$. As a/λ decreases, the screen becomes more uniformly illuminated.

36.14. IDENTIFY: $I = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2$. $\beta = \dfrac{2\pi}{\lambda}a\sin\theta$.

SET UP: The angle θ is small, so $\sin\theta \approx \tan\theta \approx y/x$.

EXECUTE: $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \approx \dfrac{2\pi a}{\lambda}\dfrac{y}{x} = \dfrac{2\pi(4.50\times10^{-4}\text{ m})}{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}y = (1520\text{ m}^{-1})y.$

(a) $y = 1.00\times10^{-3}$ m: $\dfrac{\beta}{2} = \dfrac{(1520\text{ m}^{-1})(1.00\times10^{-3}\text{ m})}{2} = 0.760.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(0.760)}{0.760}\right)^2 = 0.822I_0$$

(b) $y = 3.00\times10^{-3}$ m: $\dfrac{\beta}{2} = \dfrac{(1520\text{ m}^{-1})(3.00\times10^{-3}\text{ m})}{2} = 2.28.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(2.28)}{2.28}\right)^2 = 0.111I_0.$$

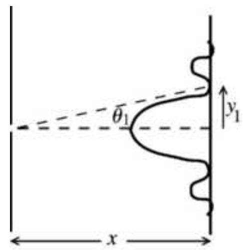
(c) $y = 5.00\times10^{-3}$ m: $\dfrac{\beta}{2} = \dfrac{(1520\text{ m}^{-1})(5.00\times10^{-3}\text{ m})}{2} = 3.80.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(3.80)}{3.80}\right)^2 = 0.0259I_0.$$

EVALUATE: The first minimum occurs at $y_1 = \dfrac{\lambda x}{a} = \dfrac{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}{4.50\times10^{-4}\text{ m}} = 4.1$ mm. The distances in parts (a) and (b) are within the central maximum. $y = 5.00$ mm is within the first secondary maximum.

36.15. **(a) IDENTIFY:** Use Eq. (36.2) with $m = 1$ to locate the angular position of the first minimum and then use $y = x\tan\theta$ to find its distance from the center of the screen.

SET UP: The diffraction pattern is sketched in Figure 36.15.



$$\sin\theta_1 = \frac{\lambda}{a} = \frac{540\times10^{-9}\text{ m}}{0.240\times10^{-3}\text{ m}} = 2.25\times10^{-3}$$

$$\theta_1 = 2.25\times10^{-3}\text{ rad}$$

Figure 36.15

$$y_1 = x\tan\theta_1 = (3.00\text{ m})\tan(2.25\times10^{-3}\text{ rad}) = 6.75\times10^{-3}\text{ m} = 6.75\text{ mm}$$

(b) IDENTIFY and **SET UP:** Use Eqs. (36.5) and (36.6) to calculate the intensity at this point.

EXECUTE: Midway between the center of the central maximum and the first minimum implies

$$y = \frac{1}{2}(6.75\text{ mm}) = 3.375\times10^{-3}\text{ m}.$$

$$\tan\theta = \frac{y}{x} = \frac{3.375\times10^{-3}\text{ m}}{3.00\text{ m}} = 1.125\times10^{-3};\ \theta = 1.125\times10^{-3}\text{ rad}$$

The phase angle β at this point on the screen is

$$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \frac{2\pi}{540\times10^{-9}\text{ m}}(0.240\times10^{-3}\text{ m})\sin(1.125\times10^{-3}\text{ rad}) = \pi.$$

Then $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2 = (6.00\times10^{-6}\text{ W/m}^2)\left(\dfrac{\sin\pi/2}{\pi/2}\right)^2.$

$$I = \left(\frac{4}{\pi^2}\right)(6.00\times10^{-6}\text{ W/m}^2) = 2.43\times10^{-6}\text{ W/m}^2.$$

EVALUATE: The intensity at this point midway between the center of the central maximum and the first minimum is less than half the maximum intensity. Compare this result to the corresponding one for the two-slit pattern, Exercise 35.22.

36.16. **IDENTIFY:** The intensity on the screen varies as the light spreads out (diffracts) after passing through the single slit.

SET UP: $I = I_0\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta = \dfrac{2\pi}{\lambda}a\sin\theta.$

EXECUTE: $\beta = \dfrac{2\pi}{\lambda}a\sin\theta = \left(\dfrac{2\pi}{486\times10^{-9}\text{ m}}\right)(0.0290\times10^{-3}\text{ m})\sin 1.20^\circ = 7.852\text{ rad}.$

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = (4.00\times10^{-5}\text{ W/m}^2)\left[\frac{\sin(3.926\text{ rad})}{3.926\text{ rad}}\right]^2 = 1.29\times10^{-6}\text{ W/m}^2.$$

EVALUATE: The intensity is less than 1/30 of the intensity of the light at the slit.

36.17. **IDENTIFY** and **SET UP:** Use Eq. (36.6) to calculate λ and use Eq. (36.5) to calculate I. $\theta = 3.25^\circ$, $\beta = 56.0$ rad, $a = 0.105\times10^{-3}$ m.

(a) EXECUTE: $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$ so

$$\lambda = \frac{2\pi a\sin\theta}{\beta} = \frac{2\pi(0.105\times10^{-3}\text{ m})\sin 3.25^\circ}{56.0\text{ rad}} = 668\text{ nm}$$

(b) $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2 = I_0\left(\dfrac{4}{\beta^2}\right)(\sin(\beta/2))^2 = I_0\dfrac{4}{(56.0\text{ rad})^2}[\sin(28.0\text{ rad})]^2 = 9.36\times10^{-5}\ I_0$

EVALUATE: At the first minimum $\beta = 2\pi$ rad and at the point considered in the problem $\beta = 17.8\pi$ rad, so the point is well outside the central maximum. Since β is close to $m\pi$ with $m = 18,$ this point is near one of the minima. The intensity here is much less than I_0.

36.18. IDENTIFY: Use $\beta = \dfrac{2\pi a}{\lambda}\sin\theta$ to calculate β.

SET UP: The total intensity is given by drawing an arc of a circle that has length E_0 and finding the length of the chord which connects the starting and ending points of the curve.

EXECUTE: **(a)** $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{\lambda}{2a} = \pi.$ From Figure 36.18a, $\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{\pi}E_0.$

The intensity is $I = \left(\dfrac{2}{\pi}\right)^2 I_0 = \dfrac{4I_0}{\pi^2} = 0.405I_0.$ This agrees with Eq. (36.5).

(b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{\lambda}{a} = 2\pi.$ From Figure 36.18b, it is clear that the total amplitude is zero, as is the intensity. This also agrees with Eq. (36.5).

(c) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{3\lambda}{2a} = 3\pi.$ From Figure 36.18c, $3\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{3\pi}E_0.$ The intensity is

$I = \left(\dfrac{2}{3\pi}\right)^2 I_0 = \dfrac{4}{9\pi^2}I_0.$ This agrees with Eq. (36.5).

EVALUATE: In part (a) the point is midway between the center of the central maximum and the first minimum. In part (b) the point is at the first maximum and in (c) the point is approximately at the location of the first secondary maximum. The phasor diagrams help illustrate the rapid decrease in intensity at successive maxima.

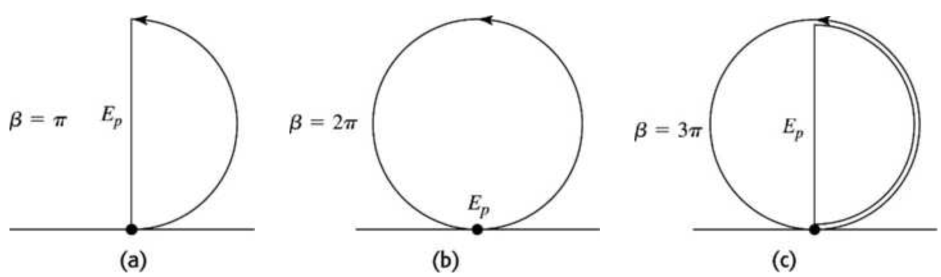


Figure 36.18

36.19. IDENTIFY: The space between the skyscrapers behaves like a single slit and diffracts the radio waves.
SET UP: Cancellation of the waves occurs when $a\ \sin\theta = m\lambda,\ m = 1, 2, 3, \ldots,$ and the intensity of the waves is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2,$ where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda}.$

EXECUTE: **(a)** First find the wavelength of the waves:
$\lambda = c/f = (3.00\times10^8\text{ m/s})/(88.9\text{ MHz}) = 3.375\text{ m}$
For no signal, $a\ \sin\theta = m\lambda.$
$m = 1$: $\sin\theta_1 = (1)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_1 = \pm13.0°$
$m = 2$: $\sin\theta_2 = (2)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_2 = \pm26.7°$
$m = 3$: $\sin\theta_3 = (3)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_3 = \pm42.4°$
$m = 4$: $\sin\theta_4 = (4)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_4 = \pm64.1°$

(b) $I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(15.0\text{ m})\sin(5.00°)}{3.375\text{ m}} = 1.217\text{ rad}$

$$I = (3.50\text{ W/m}^2)\left[\frac{\sin(1.217\text{ rad})}{1.217\text{ rad}}\right]^2 = 2.08\text{ W/m}^2$$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.20. **IDENTIFY:** The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{\text{DS}} = I_0\left(\cos^2\frac{\phi}{2}\right)$ and the single-slit factor is $I_{\text{SS}} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2$.

EXECUTE: **(a)** $d\sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d$.
$\sin\theta_1 = \lambda/d, \sin\theta_2 = 2\lambda/d, \sin\theta_3 = 3\lambda/d, \sin\theta_4 = 4\lambda/d$

(b) At the interference bright fringes, $\cos^2\phi/2 = 1$ and $\beta/2 = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/3)\sin\theta}{\lambda}$.

At θ_1, $\sin\theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3$. The intensity is therefore

$$I_1 = I_0\left(\cos^2\frac{\phi}{2}\right)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0(1)\left(\frac{\sin\pi/3}{\pi/3}\right)^2 = 0.684\,I_0$$

At θ_2, $\sin\theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3$. Using the same procedure as for θ_1, we have

$I_2 = I_0(1)\left(\frac{\sin 2\pi/3}{2\pi/3}\right)^2 = 0.171\,I_0$.

At θ_3, we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin\ \pi = 0$.

At θ_4, $\sin\ \theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0\left(\frac{\sin 4\pi/3}{4\pi/3}\right)^2 = 0.0427\,I_0$

(c) Since $d = 3a$, every third interference maximum is missing.

(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

EVALUATE: The result in this problem is different from that in Figure 36.12c in the textbook because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the "envelope" of the intensity function decreases more rapidly here than in Figure 36.12c in the text because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

36.21. **(a) IDENTIFY** and **SET UP:** The interference fringes (maxima) are located by $d\sin\theta = m\lambda$, with $m = 0, \pm 1, \pm 2, \ldots$. The intensity *I* in the diffraction pattern is given by $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, with $\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta$. We want $m = \pm 3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d\sin\theta = m\lambda$ gives $\beta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{3\lambda}{d}\right) = 2\pi(3a/d)$.

$I = 0$ says $\frac{\sin\beta/2}{\beta/2} = 0$ so $\beta = 2\pi$ and then $2\pi = 2\pi(3a/d)$ and $(d/a) = 3$.

(b) IDENTIFY and **SET UP:** Fringes $m = 0, \pm 1, \pm 2$ are within the central diffraction maximum and the $m = \pm 3$ fringes coincide with the first diffraction minimum. Find the value of *m* for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = 2\pi(m/3)$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm 4$ and $m = +5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

36.22. **IDENTIFY** and **SET UP:** Use Figure 36.14b in the textbook. There is totally destructive interference between slits whose phasors are in opposite directions.

EXECUTE: By examining the diagram, we see that every fourth slit cancels each other.

EVALUATE: The total electric field is zero so the phasor diagram corresponds to a point of zero intensity. The first two maxima are at $\phi = 0$ and $\phi = \pi$, so this point is not midway between two maxima.

36.23. **(a) IDENTIFY** and **SET UP:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by $d\sin\theta = m\lambda$ so $\sin\theta = m\lambda/d$. Solve for θ.

EXECUTE: 1st order maximum: $m = 1$, so $\sin\theta = \dfrac{\lambda}{d} = \dfrac{580\times10^{-9}\text{ m}}{0.530\times10^{-3}\text{ m}} = 1.094\times10^{-3}$; $\theta = 0.0627°$

2nd order maximum: $m = 2$, so $\sin\theta = \dfrac{2\lambda}{d} = 2.188\times10^{-3}$; $\theta = 0.125°$

(b) IDENTIFY and **SET UP:** The intensity is given by Eq. (36.12): $I = I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$. Calculate ϕ and β at each θ from part (a).

EXECUTE: $\phi = \left(\dfrac{2\pi d}{\lambda}\right)\sin\theta = \left(\dfrac{2\pi d}{\lambda}\right)\left(\dfrac{m\lambda}{d}\right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1$

(Since the angular positions in part (a) correspond to interference maxima.)

$$\beta = \left(\frac{2\pi a}{\lambda}\right)\sin\theta = \left(\frac{2\pi a}{\lambda}\right)\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = m2\pi\left(\frac{0.320\text{ mm}}{0.530\text{ mm}}\right) = m(3.794\text{ rad})$$

1st order maximum: $m = 1$, so $I = I_0(1)\left(\dfrac{\sin(3.794/2)\text{rad}}{(3.794/2)\text{rad}}\right)^2 = 0.249I_0$

2nd order maximum: $m = 2$, so $I = I_0(1)\left(\dfrac{\sin 3.794\text{ rad}}{3.794\text{ rad}}\right)^2 = 0.0256I_0$

EVALUATE: The first diffraction minimum is at an angle θ given by $\sin\theta = \lambda/a$ so $\theta = 0.104°$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0.

36.24. **IDENTIFY:** The intensity at the screen is due to a combination of single-slit diffraction and double-slit interference.

SET UP: $I = I_0\left(\cos^2\dfrac{\phi}{2}\right)\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\phi = \dfrac{2\pi d}{\lambda}\sin\theta$ and $\beta = \dfrac{2\pi}{\lambda}a\sin\theta$.

EXECUTE: $\tan\theta = \dfrac{9.00\times10^{-4}\text{ m}}{0.750\text{ m}} = 1.200\times10^{-3}$. θ is small, so $\sin\theta \approx \tan\theta$.

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi(0.640\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3}) = 8.4956\text{ rad.}$$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi(0.434\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3}) = 5.7611\text{ rad.}$$

$$I = (5.00\times10^{-4}\text{ W/m}^2)(\cos 4.2478\text{ rad})^2\left[\frac{\sin 2.8805\text{ rad}}{2.8805}\right]^2 = 8.06\times10^{-7}\text{ W/m}^2.$$

EVALUATE: The intensity as decreased by a factor of almost a thousand, so it would be difficult to see the light at the screen.

36.25. **IDENTIFY** and **SET UP:** The phasor diagrams are similar to those in Figure 36.14. in the textbook. An interference minimum occurs when the phasors add to zero.
EXECUTE: **(a)** The phasor diagram is given in Figure 36.25a.

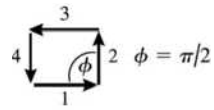


Figure 36.25a

There is destructive interference between the light through slits 1 and 3 and between 2 and 4.
(b) The phasor diagram is given in Figure 36.25b.

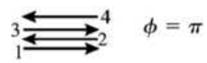


Figure 36.25b

There is destructive interference between the light through slits 1 and 2 and between 3 and 4.
(c) The phasor diagram is given in Figure 36.25c.

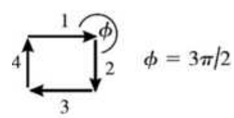


Figure 36.25c

There is destructive interference between light through slits 1 and 3 and between 2 and 4.
EVALUATE: Maxima occur when $\phi = 0, 2\pi, 4\pi$, etc. Our diagrams show that there are three minima between the maxima at $\phi = 0$ and $\phi = 2\pi$. This agrees with the general result that for N slits there are $N-1$ minima between each pair of principal maxima.

36.26. **IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.
SET UP: Single-slit diffraction dark fringes occur when $a \sin\theta = m\lambda$, and double-slit interference bright fringes occur when $d \sin\theta = m'\lambda$.
EXECUTE: **(a)** The angle at which the first bright fringe occurs is given by
$\tan\theta_1 = (1.53\text{ mm})/(2500\text{ mm}) \Rightarrow \theta_1 = 0.03507°$. $d\sin\theta_1 = \lambda$ and

$$d = \lambda/(\sin\theta_1) = (632.8\text{ nm})/\sin(0.03507°) = 0.00103\text{ m} = 1.03\text{ mm}$$

(b) The 7^{th} double-slit interference bright fringe is just cancelled by the 1^{st} diffraction dark fringe, so
$\sin\theta_{\text{diff}} = \lambda/a$ and $\sin\theta_{\text{interf}} = 7\lambda/d$
The angles are equal, so $\lambda/a = 7\lambda/d \to a = d/7 = (1.03\text{ mm})/7 = 0.148\text{ mm}$.

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.27. **IDENTIFY:** The diffraction minima are located by $\sin\theta = \dfrac{m_{\text{d}}\lambda}{a}$ and the two-slit interference maxima are located by $\sin\theta = \dfrac{m_{\text{i}}\lambda}{d}$. The third bright band is missing because the first order single-slit minimum occurs at the same angle as the third order double-slit maximum.
SET UP: The pattern is sketched in Figure 36.27. $\tan\theta = \dfrac{3\text{ cm}}{90\text{ cm}}$, so $\theta = 1.91°$.

EXECUTE: Single-slit dark spot: $a\sin\theta = \lambda$ and $a = \dfrac{\lambda}{\sin\theta} = \dfrac{500\text{ nm}}{\sin 1.91°} = 1.50\times10^4\text{ nm} = 15.0\ \mu\text{m}$ (width)

Double-slit bright fringe: $d\sin\theta = 3\lambda$ and $d = \dfrac{3\lambda}{\sin\theta} = \dfrac{3(500\text{ nm})}{\sin 1.91°} = 4.50\times10^4\text{ nm} = 45.0\ \mu\text{m}$ (separation).

EVALUATE: Note that $d/a = 3.0$.

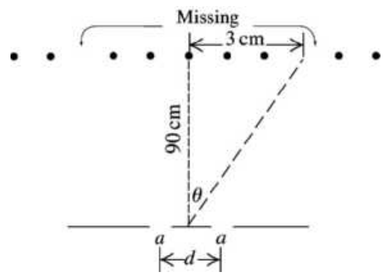


Figure 36.27

36.28. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.
SET UP: The order corresponds to the values of m.
EXECUTE: First-order: $d\sin\theta_1 = \lambda$. Fourth-order: $d\sin\theta_4 = 4\lambda$.

$\dfrac{d\sin\theta_4}{d\sin\theta_1} = \dfrac{4\lambda}{\lambda}$, $\sin\theta_4 = 4\sin\theta_1 = 4\sin 8.94°$ and $\theta_4 = 38.4°$.

EVALUATE: We did not have to solve for d.

36.29. **IDENTIFY** and **SET UP:** The bright bands are at angles θ given by $d\sin\theta = m\lambda$. Solve for d and then solve for θ for the specified order.

EXECUTE: **(a)** $\theta = 78.4°$ for $m = 3$ and $\lambda = 681$ nm, so $d = m\lambda/\sin\theta = 2.086\times10^{-4}$ cm

The number of slits per cm is $1/d = 4790$ slits/cm.

(b) 1st order: $m = 1$, so $\sin\theta = \lambda/d = (681\times10^{-9}\text{ m})/(2.086\times10^{-6}\text{ m})$ and $\theta = 19.1°$

2nd order: $m = 2$, so $\sin\theta = 2\lambda/d$ and $\theta = 40.8°$

(c) For $m = 4$, $\sin\theta = 4\lambda/d$ is greater than 1.00, so there is no 4th-order bright band.

EVALUATE: The angular position of the bright bands for a particular wavelength increases as the order increases.

36.30. **IDENTIFY:** The bright spots are located by $d\sin\theta = m\lambda$.
SET UP: Third-order means $m = 3$ and second-order means $m = 2$.

EXECUTE: $\dfrac{m\lambda}{\sin\theta} = d = \text{constant}$, so $\dfrac{m_r\lambda_r}{\sin\theta_r} = \dfrac{m_v\lambda_v}{\sin\theta_v}$.

$\sin\theta_v = \sin\theta_r\left(\dfrac{m_v}{m_r}\right)\left(\dfrac{\lambda_v}{\lambda_r}\right) = (\sin 65.0°)\left(\dfrac{2}{3}\right)\left(\dfrac{400\text{ nm}}{700\text{ nm}}\right) = 0.345$ and $\theta_v = 20.2°$.

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.31. **IDENTIFY** and **SET UP:** Calculate d for the grating. Use Eq. (36.13) to calculate θ for the longest wavelength in the visible spectrum and verify that θ is small. Then use Eq. (36.3) to relate the linear separation of lines on the screen to the difference in wavelength.

EXECUTE: **(a)** $d = \left(\dfrac{1}{900}\right)\text{cm} = 1.111\times10^{-5}$ m

For $\lambda = 700$ nm, $\lambda/d = 6.3\times10^{-2}$. The first-order lines are located at $\sin\theta = \lambda/d$; $\sin\theta$ is small enough for $\sin\theta \approx \theta$ to be an excellent approximation.

(b) $y = x\lambda/d$, where $x = 2.50$ m.

The distance on the screen between first-order bright bands for two different wavelengths is

$\Delta y = x(\Delta\lambda)/d$, so $\Delta\lambda = d(\Delta y)/x = (1.111\times10^{-5}\text{ m})(3.00\times10^{-3}\text{ m})/(2.50\text{ m}) = 13.3$ nm.

EVALUATE: The smaller d is (greater number of lines per cm) the smaller the $\Delta\lambda$ that can be measured.

36.32. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda.$

SET UP: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^5\ \text{m}^{-1}} = 2.86\times10^{-6}$ m

EXECUTE: **(a)** $m=1$: $\theta_{400} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{4.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 8.05°.$

$\theta_{700} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{7.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 14.18°.$ $\Delta\theta_1 = 14.18° - 8.05° = 6.13°.$

(b) $m=3$: $\theta_{400} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(4.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 24.8°.$

$\theta_{700} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(7.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 47.3°.$ $\Delta\theta_1 = 47.3° - 24.8° = 22.5°.$

EVALUATE: $\Delta\theta$ is larger in third order.

36.33. **IDENTIFY:** Knowing the wavelength of the light and the location of the first interference maxima, we can calculate the line density of the grating.

SET UP: The line density in lines/cm is $1/d$, with d in cm. The bright spots are located by $d\sin\theta = m\lambda,$ $m = 0, \pm1, \pm2, \ldots.$

EXECUTE: **(a)** $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{(1)(632.8\times10^{-9}\ \text{m})}{\sin 17.8°} = 2.07\times10^{-6}\ \text{m} = 2.07\times10^{-4}\ \text{cm}.$ $\dfrac{1}{d} = 4830$ lines/cm.

(b) $\sin\theta = \dfrac{m\lambda}{d} = m\left(\dfrac{632.8\times10^{-9}\ \text{m}}{2.07\times10^{-6}\ \text{m}}\right) = m(0.3057).$ For $m=\pm2,$ $\theta = \pm37.7°.$ For $m = \pm3,$ $\theta = \pm66.5°.$

EVALUATE: The angles are large, so they are not equally spaced; $37.7° \neq 2(17.8°)$ and $66.5° \neq 3(17.8°)$

36.34. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda.$

SET UP: 5000 slits/cm $\Rightarrow d = \dfrac{1}{5.00\times10^5\ \text{m}^{-1}} = 2.00\times10^{-6}$ m.

EXECUTE: **(a)** $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(2.00\times10^{-6}\ \text{m})\sin 13.5°}{1} = 4.67\times10^{-7}$ m.

(b) $m=2$: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{2(4.67\times10^{-7}\ \text{m})}{2.00\times10^{-6}\ \text{m}}\right) = 27.8°.$

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.35. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda.$

SET UP: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^5\ \text{m}^{-1}} = 2.86\times10^{-6}$ m

EXECUTE: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m(5.20\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = \arcsin((0.182)m).$

$m=1$: $\theta = 10.5°$; $m=2$: $\theta = 21.3°$; $m=3$: $\theta = 33.1°.$

EVALUATE: The angles are not precisely proportional to m, and deviate more from being proportional as the angles increase.

36.36. **IDENTIFY:** The resolution is described by $R = \dfrac{\lambda}{\Delta\lambda} = Nm.$ Maxima are located by $d\sin\theta = m\lambda.$

SET UP: For 500 slits/mm, $d = (500\ \text{slits/mm})^{-1} = (500{,}000\ \text{slits/m})^{-1}.$

EXECUTE: **(a)** $N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{6.5645\times10^{-7}\ \text{m}}{2(6.5645\times10^{-7}\ \text{m} - 6.5627\times10^{-7}\ \text{m})} = 1820$ slits.

(b) $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) \Rightarrow \theta_1 = \sin^{-1}((2)(6.5645\times 10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0297°$ and

$\theta_2 = \sin^{-1}((2)(6.5627\times 10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0160°$. $\Delta\theta = 0.0137°$

EVALUATE: $d\cos\theta\, d\theta = \lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima and the first adjacent minimum is $\Delta\theta = \dfrac{\lambda}{Nd\cos\theta} = \dfrac{6.56\times 10^{-7}\text{ m}}{(1820)(2.0\times 10^{-6}\text{ m})\cos 41°} = 0.0137°$. This is the same as the angular separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.37. **IDENTIFY:** The resolving power depends on the line density and the width of the grating.

SET UP: The resolving power is given by $R = Nm = = \lambda/\Delta\lambda$.

EXECUTE: **(a)** $R = Nm = (5000\text{ lines/cm})(3.50\text{ cm})(1) = 17{,}500$

(b) The resolving power needed to resolve the sodium doublet is

$$R = \lambda/\Delta\lambda = (589\text{ nm})/(589.59\text{ nm} - 589.00\text{ nm}) = 998$$

so this grating can easily resolve the doublet.

(c) (i) $R = \lambda/\Delta\lambda$. Since $R = 17{,}500$ when $m = 1$, $R = 2\times 17{,}500 = 35{,}000$ for $m = 2$. Therefore

$$\Delta\lambda = \lambda/R = (587.8\text{ nm})/35{,}000 = 0.0168\text{ nm}$$

$$\lambda_{\text{min}} = \lambda + \Delta\lambda = 587.8002\text{ nm} + 0.0168\text{ nm} = 587.8170\text{ nm}$$

(ii) $\lambda_{\text{max}} = \lambda - \Delta\lambda = 587.8002\text{ nm} - 0.0168\text{ nm} = 587.7834\text{ nm}$

EVALUATE: (iii) Therefore the range of resolvable wavelengths is $587.7834\text{ nm} < \lambda < 587.8170\text{ nm}$.

36.38. **IDENTIFY** and **SET UP:** $\dfrac{\lambda}{\Delta\lambda} = Nm$

EXECUTE: $N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{587.8002\text{ nm}}{(587.9782\text{ nm} - 587.8002\text{ nm})} = \dfrac{587.8002}{0.178} = 3302\text{ slits}$.

$\dfrac{N}{1.20\text{ cm}} = \dfrac{3302}{1.20\text{ cm}} = 2752\dfrac{\text{slits}}{\text{cm}}$.

EVALUATE: A smaller number of slits would be needed to resolve these two lines in higher order.

36.39. **IDENTIFY** and **SET UP:** The maxima occur at angles θ given by Eq. (36.16), $2d\sin\theta = m\lambda$, where d is the spacing between adjacent atomic planes. Solve for d.

EXECUTE: Second order says $m = 2$.

$d = \dfrac{m\lambda}{2\sin\theta} = \dfrac{2(0.0850\times 10^{-9}\text{ m})}{2\sin 21.5°} = 2.32\times 10^{-10}\text{ m} = 0.232\text{ nm}$

EVALUATE: Our result is similar to d calculated in Example 36.5.

36.40. **IDENTIFY:** The maxima are given by $2d\sin\theta = m\lambda$, $m = 1, 2, \ldots$

SET UP: $d = 3.50\times 10^{-10}\text{ m}$.

EXECUTE: **(a)** $m = 1$ and $\lambda = \dfrac{2d\sin\theta}{m} = 2(3.50\times 10^{-10}\text{ m})\sin 15.0° = 1.81\times 10^{-10}\text{ m} = 0.181\text{ nm}$. This is an x ray.

(b) $\sin\theta = m\left(\dfrac{\lambda}{2d}\right) = m\left(\dfrac{1.81\times 10^{-10}\text{ m}}{2[3.50\times 10^{-10}\text{ m}]}\right) = m(0.2586)$. $m = 2$: $\theta = 31.1°$. $m = 3$: $\theta = 50.9°$. The equation doesn't have any solutions for $m > 3$.

EVALUATE: In this problem $\lambda/d = 0.52$.

36.41. **IDENTIFY:** The crystal behaves like a diffraction grating.

SET UP: The maxima are at angles θ given by $2d\sin\theta = m\lambda$, where $d = 0.440\text{ nm}$.

EXECUTE: $m = 1$. $\lambda = \dfrac{2d\sin\theta}{1} = 2(0.440\text{ nm})\sin 39.4° = 0.559\text{ nm}$.

EVALUATE: The result is a reasonable x ray wavelength.

36.42. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}.$

SET UP: $\theta = (1/60)°$

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = \dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(1/60)°} = 2.31\times10^{-3}\text{ m} = 2.3\text{ mm}$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.43. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}.$

SET UP: $\theta = \dfrac{W}{h},$ where $W = 28\text{ km}$ and $h = 1200\text{ km}.$ θ is small, so $\sin\theta \approx \theta.$

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = 1.22\lambda\dfrac{h}{W} = 1.22(0.036\text{ m})\dfrac{1.2\times10^{6}\text{ m}}{2.8\times10^{4}\text{ m}} = 1.88\text{ m}$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.44. **IDENTIFY:** The diameter D of the mirror determines the resolution.

SET UP: The resolving power is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}.$

EXECUTE: The same θ_{res} means that $\dfrac{\lambda_1}{D_1} = \dfrac{\lambda_2}{D_2}.$ $D_2 = D_1\dfrac{\lambda_2}{\lambda_1} = (8000\times10^{3}\text{ m})\left(\dfrac{550\times10^{-9}\text{ m}}{2.0\times10^{-2}\text{ m}}\right) = 220\text{ m}.$

EVALUATE: The Hubble telescope has an aperture of 2.4 m, so this would have to be an *enormous* optical telescope!

36.45. **IDENTIFY** and **SET UP:** The angular size of the first dark ring is given by $\sin\theta_1 = 1.22\lambda/D$ (Eq. 36.17). Calculate $\theta_1,$ and then the diameter of the ring on the screen is $2(4.5\text{ m})\tan\theta_1.$

EXECUTE: $\sin\theta_1 = 1.22\left(\dfrac{620\times10^{-9}\text{ m}}{7.4\times10^{-6}\text{ m}}\right) = 0.1022;\ \theta_1 = 0.1024\text{ rad}$

The radius of the Airy disk (central bright spot) is $r = (4.5\text{ m})\tan\theta_1 = 0.462\text{ m}.$ The diameter is $2r = 0.92\text{ m} = 92\text{ cm}.$

EVALUATE: $\lambda/D = 0.084.$ For this small D the central diffraction maximum is broad.

36.46. **IDENTIFY:** Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin\ \theta \approx \theta = 1.22\lambda/D.$

EXECUTE: **(a)** Using Rayleigh's criterion

$$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(550\text{ nm})/(135/4\text{ mm}) = 1.99\times10^{-5}\text{ rad}$$

On the bear this angle subtends a distance $x.$ $\theta = x/R$ and

$$x = R\theta = (11.5\text{ m})(1.99\times10^{-5}\text{ rad}) = 2.29\times10^{-4}\text{m} = 0.23\text{ mm}$$

(b) At $f/22,$ D is $4/22$ times as large as at $f/4.$ Since θ is proportional to $1/D,$ and x is proportional to θ, x is $1/(4/22) = 22/4$ times as large as it was at $f/4.$ $x = (0.229\text{ mm})(22/4) = 1.3\text{ mm}$

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at $f/2$ and hence would have an even less resolving ability.

36.47. **IDENTIFY** and **SET UP:** Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D.$ The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \dfrac{250\times10^{3}\text{ m}}{5.93\times10^{11}\text{ m}},$ where $5.93\times10^{11}\text{ m}$ is the distance from earth to Jupiter. Thus $\theta = 4.216\times10^{-7}.$

Then $\theta = 1.22\dfrac{\lambda}{D}$ and $D = \dfrac{1.22\lambda}{\theta} = \dfrac{1.22(500\times10^{-9}\text{ m})}{4.216\times10^{-7}} = 1.45\text{ m}$

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

36.48. **IDENTIFY:** Rayleigh's criterion says $\theta_{\text{res}} = 1.22\frac{\lambda}{D}.$

SET UP: $D = 7.20$ cm. $\theta_{\text{res}} = \frac{y}{s},$ where *s* is the distance of the object from the lens and $y = 4.00$ mm.

EXECUTE: $\frac{y}{s} = 1.22\frac{\lambda}{D}.$ $s = \frac{yD}{1.22\lambda} = \frac{(4.00\times10^{-3}\text{ m})(7.20\times10^{-2}\text{ m})}{1.22(550\times10^{-9}\text{ m})} = 429\text{ m}.$

EVALUATE: The focal length of the lens doesn't enter into the calculation. In practice, it is difficult to achieve resolution that is at the diffraction limit.

36.49. **IDENTIFY** and **SET UP:** Let *y* be the separation between the two points being resolved and let *s* be their distance from the telescope. Then the limit of resolution corresponds to $1.22\ \frac{\lambda}{D} = \frac{y}{s}.$

EXECUTE: **(a)** Let the two points being resolved be the opposite edges of the crater, so *y* is the diameter of the crater. For the moon, $s = 3.8\times10^8$ m. $y = 1.22\lambda s/D.$

Hubble: $D = 2.4$ m and $\lambda = 400$ nm gives the maximum resolution, so $y = 77$ m

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1\times10^6$ m

(b) $s = \frac{yD}{1.22\lambda}.$ Let $y \approx 0.30$ m (the size of a license plate).

$s = (0.30\text{ m})(2.4\text{ m})/[(1.22)(400\times10^{-9}\text{ m})] = 1500\text{ km}.$

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

36.50. **IDENTIFY:** Apply $\sin\theta = 1.22\frac{\lambda}{D}.$

SET UP: θ is small, so $\sin\theta \approx \theta.$ Smallest resolving angle is for short-wavelength light (400 nm).

EXECUTE: $\theta \approx 1.22\frac{\lambda}{D} = (1.22)\frac{400\times10^{-9}\text{ m}}{5.08\text{ m}} = 9.61\times10^{-8}\text{ rad}.$ $\theta = \frac{10{,}000\text{ mi}}{R},$ where *R* is the distance to the star. $R = \frac{10{,}000\text{ mi}}{\theta} = \frac{16{,}000\text{ km}}{9.6\times10^{-8}\text{rad}} = 1.7\times10^{11}\text{ km}.$

EVALUATE: This is less than a light year, so there are no stars this close.

36.51. **IDENTIFY:** We can apply the equation for single-slit diffraction to the hair, with the thickness of the hair replacing the thickness of the slit.

SET UP: The dark fringes are located by $\sin\theta = m\frac{\lambda}{a}.$ The first dark fringes are for $m = \pm1.$ $y = R\tan\theta$ is the distance from the center of the screen. From the center to one minimum is 2.61 cm.

EXECUTE: $\tan\theta = \frac{y}{R} = \frac{2.61\text{ cm}}{125\text{ cm}} = 0.02088$ so $\theta = 1.20°.$ $a = \frac{\lambda}{\sin\theta} = \frac{632.8\times10^{-9}\text{ m}}{\sin 1.20°} = 30.2\ \mu\text{m}.$

EVALUATE: Although the thickness of human hairs can vary considerably, $30\ \mu$m is a reasonable thickness.

36.52. **IDENTIFY:** If the apparatus of Exercise 36.4 is placed in water, then all that changes is the wavelength $\lambda \to \lambda' = \frac{\lambda}{n}.$

SET UP: For $y \ll x,$ the distance between the two dark fringes on either side of the central maximum is $D' = 2y'.$ Let $D = 2y$ be the separation of 5.91×10^{-3} m found in Exercise 36.4.

EXECUTE: $2y'_1 = \frac{2x\lambda'}{a} = \frac{2x\lambda}{an} = \frac{D}{n} = \frac{5.91\times10^{-3}\text{ m}}{1.33} = 4.44\times10^{-3}\text{ m} = 4.44\text{ mm}.$

EVALUATE: The water shortens the wavelength and this decreases the width of the central maximum.

36.53. IDENTIFY: In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin\theta_m = m\lambda/a$, where $m = 1, 2, 3, \ldots,$ and the intensity is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** At the maximum possible angle, $\theta = 90°$, so

$$m_{max} = (a\sin 90°)/\lambda = (0.0250 \text{ mm})/(632.8 \text{ nm}) = 39.5$$

Since m must be an integer and $\sin\theta$ must be ≤ 1, $m_{max} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving

$$\sin\theta_{39} = (39)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{39} = \pm 80.8°$$

(c) The next closer dark fringe occurs at $\sin\theta_{38} = (38)(632.8 \text{ nm})/(0.0250 \text{ mm}) \Rightarrow \theta_{38} = 74.1°$.

The angle midway these two extreme fringes is $(80.8° + 74.1°)/2 = 77.45°$, and the intensity at this angle is

$I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.0250 \text{ mm})\sin(77.45°)}{632.8 \text{ nm}} = 121.15 \text{ rad}$, which gives

$$I = (8.50 \text{W/m}^2)\left[\frac{\sin(121.15 \text{ rad})}{121.15 \text{ rad}}\right]^2 = 5.55\times 10^{-4} \text{ W/m}^2.$$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

36.54. IDENTIFY: The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin\theta = \lambda/2$.

EXECUTE: **(a)** First find the wavelength of the sound that strikes the openings in the wall. $\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344 \text{ m/s} - 80.0 \text{ m/s})/(1250 \text{ Hz}) = 0.211 \text{ m}$. Destructive interference first occurs where $d\sin\theta = \lambda/2$, which gives $d = \lambda/(2\sin\theta) = (0.211 \text{ m})/(2\sin 11.4°) = 0.534 \text{ m}$.

(b) $\lambda = v/f = (344 \text{ m/s})/(1250 \text{ Hz}) = 0.275 \text{ m}$. $\sin\theta = \lambda/2d = (0.275 \text{ m})/[2(0.480 \text{ m})] \rightarrow \theta = \pm 16.7°$.

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

36.55. IDENTIFY and SET UP: $\sin\theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin\theta_{air} = \dfrac{\lambda_{air}}{a}$; $\sin\theta_{liquid} = \dfrac{\lambda_{liquid}}{a}$. $\lambda_{liquid} = \lambda_{air}\left(\dfrac{\sin\theta_{liquid}}{\sin\theta_{air}}\right) = \lambda_{air}\dfrac{\sin 21.6°}{\sin 38.2°} = 0.5953\lambda_{air}$.

$\lambda_{liquid} = \lambda_{air}/n$ (Eq. 33.5), so $n = \lambda_{air}/\lambda_{liquid} = \dfrac{\lambda_{air}}{0.5953\lambda_{air}} = 1.68$.

EVALUATE: Light travels faster in air and n must be > 1.00. The smaller λ in the liquid reduces θ that located the first dark band.

36.56. IDENTIFY: $d = \dfrac{1}{N}$, so the bright fringes are located by $\dfrac{1}{N}\sin\theta = \lambda$.

SET UP: Red: $\dfrac{1}{N}\sin\lambda_R = 700 \text{ nm}$. Violet: $\dfrac{1}{N}\sin\lambda_V = 400 \text{ nm}$.

EXECUTE: **(a)** $\dfrac{\sin\theta_R}{\sin\theta_V} = \dfrac{7}{4}$. $\theta_R - \theta_V = 21.0° \rightarrow \theta_R = \theta_V + 21.0°$. $\dfrac{\sin(\theta_V + 21.0°)}{\sin\theta_V} = \dfrac{7}{4}$. Using a trig identity from Appendix B gives $\dfrac{\sin\theta_V\cos 21.0° + \cos\theta_V\sin 21.0°}{\sin\theta_V} = 7/4$. $\cos 21.0° + \cot\theta_V\sin 21.0° = 7/4$.

$\tan\theta_V = 0.4390 \Rightarrow \theta_V = 23.7°$ and $\theta_R = \theta_V + 21.0° = 23.7° + 21.0° = 44.7°$. Then $\dfrac{1}{N}\sin\theta_R = 700 \text{ nm}$

gives $N = \dfrac{\sin\theta_R}{700\text{ nm}} = \dfrac{\sin 44.7°}{700\times10^{-9}\text{m}} = 1.00\times10^6$ lines/m $= 1.00\times10^4$ lines/cm.

(b) The spectrum begins at 23.7° and ends at 44.7°.

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.57. **(a) IDENTIFY** and **SET UP:** The angular position of the first minimum is given by $a\sin\theta = m\lambda$ (Eq. 36.2), with $m=1$. The distance of the minimum from the center of the pattern is given by $y = x\tan\theta$.

$$\sin\theta = \frac{\lambda}{a} = \frac{540\times10^{-9}\text{ m}}{0.360\times10^{-3}\text{ m}} = 1.50\times10^{-3};\ \theta = 1.50\times10^{-3}\text{ rad}$$

$y_1 = x\tan\theta = (1.20\text{ m})\tan(1.50\times10^{-3}\text{ rad}) = 1.80\times10^{-3}\text{ m} = 1.80\text{ mm}.$

(Note that θ is small enough for $\theta \approx \sin\theta \approx \tan\theta,$ and Eq. (36.3) applies.)

(b) IDENTIFY and **SET UP:** Find the phase angle β where $I = I_0/2$. Then use Eq. (36.6) to solve for θ and $y = x\tan\theta$ to find the distance.

EXECUTE: Eq. (36.5) gives that $I = \frac{1}{2}I_0$ when $\beta = 2.78$ rad.

$\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$ (Eq. (36.6)), so $\sin\theta = \dfrac{\beta\lambda}{2\pi a}.$

$$y = x\tan\theta \approx x\sin\theta \approx \frac{\beta\lambda x}{2\pi a} = \frac{(2.78\text{ rad})(540\times10^{-9}\text{ m})(1.20\text{ m})}{2\pi(0.360\times10^{-3}\text{ m})} = 7.96\times10^{-4}\text{ m} = 0.796\text{ mm}$$

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum; see Exercise 36.15.

36.58. **IDENTIFY:** $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\dfrac{d\sin\gamma}{d\gamma} = \cos\gamma.\ \dfrac{d}{d\gamma}\left(\dfrac{1}{\gamma}\right) = -\dfrac{1}{\gamma^2}.$

EXECUTE: $\dfrac{dI}{d\gamma} = I_0\dfrac{d}{d\gamma}\left(\dfrac{\sin\gamma}{\gamma}\right)^2 = 2\left(\dfrac{\sin\gamma}{\gamma}\right)\left(\dfrac{\cos\gamma}{\gamma} - \dfrac{\sin\gamma}{\gamma^2}\right) = 0.\ \dfrac{\cos\gamma}{\gamma} - \dfrac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma \Rightarrow \gamma = \tan\gamma.$

(b) The graph in Figure 36.58 is a plot of $f(\gamma) = \gamma - \tan\gamma$. When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the three smallest γ-values are $\gamma = 4.49$ rad 7.73 rad, and 10.9 rad.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first three secondary maxima. The first four minima are at $\gamma = 3.14$ rad, 6.28 rad, 9.42 rad, and 12.6 rad. The maxima are between adjacent minima, but not precisely midway between them.

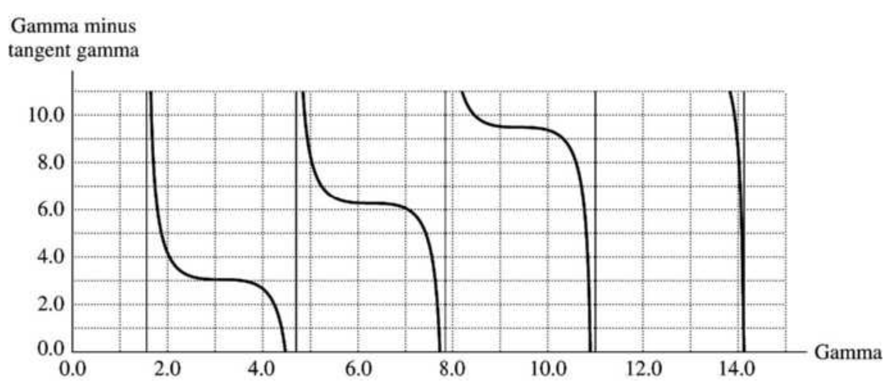


Figure 36.58

36.59. **IDENTIFY** and **SET UP:** Relate the phase difference between adjacent slits to the sum of the phasors for all slits. The phase difference between adjacent slits is $\phi = \dfrac{2\pi d}{\lambda}\sin\theta \approx \dfrac{2\pi d\theta}{\lambda}$ when θ is small and $\sin\theta \approx \theta$. Thus $\theta = \dfrac{\lambda\phi}{2\pi d}$.

EXECUTE: A principal maximum occurs when $\phi = \phi_{\text{max}} = m2\pi$, where m is an integer, since then all the phasors add. The first minima on either side of the m^{th} principal maximum occur when $\phi = \phi^{\pm}_{\text{min}} = m2\pi \pm (2\pi/N)$ and the phasor diagram for N slits forms a closed loop and the resultant phasor is zero. The angular position of a principal maximum is $\theta = \left(\dfrac{\lambda}{2\pi d}\right)\phi_{\text{max}}$. The angular position of the adjacent minimum is $\theta^{\pm}_{\text{min}} = \left(\dfrac{\lambda}{2\pi d}\right)\phi^{\pm}_{\text{min}}$.

$$\theta^{+}_{\text{min}} = \left(\frac{\lambda}{2\pi d}\right)\left(\phi_{\text{max}} + \frac{2\pi}{N}\right) = \theta + \left(\frac{0}{2\pi d}\right)\left(\frac{2\pi}{N}\right) = \theta + \frac{\lambda}{Nd}$$

$$\theta^{-}_{\text{min}} = \left(\frac{\lambda}{2\pi d}\right)\left(\phi_{\text{max}} - \frac{2\pi}{N}\right) = \theta - \frac{\lambda}{Nd}$$

The angular width of the principal maximum is $\theta^{+}_{\text{min}} - \theta^{-}_{\text{min}} = \dfrac{2\lambda}{Nd}$, as was to be shown.

EVALUATE: The angular width of the principal maximum decreases like $1/N$ as N increases.

36.60. **IDENTIFY:** Heating the plate causes it to expand, which widens the slit. The increased slit width changes the angles at which destructive interference occurs.

SET UP: First minimum is at angle θ given by $\tan\theta = \dfrac{(2.75\times10^{-3}/2)}{0.620}$. Therefore, θ is small and the equation $y_m = x\dfrac{m\lambda}{a}$ is accurate. The width of the central maximum is $w = \dfrac{2x\lambda}{a}$. The change in slit width is $\Delta a = a\alpha\Delta T$.

EXECUTE: $dw = 2x\lambda\left(-\dfrac{da}{a^2}\right) = -\dfrac{2x\lambda}{a^2}da = -\dfrac{w}{a}da$. Therefore, $\Delta w = -\dfrac{w}{a}\Delta a$. The equation for thermal expansion says $\Delta a = a\alpha\Delta T$, so $\Delta w = -w\alpha\Delta T = -(2.75\text{ mm})(2.4\times10^{-5}\text{ K}^{-1})(500\text{ K}) = -0.033\text{ mm}$. When the temperature of the plate increases, the width of the slit increases and the width of the central maximum decreases.

EVALUATE: The fractional change in the width of the slit is $(0.033\text{ mm})/(2.75\text{ mm}) = 1.2\%$. This is small, but observable.

36.61. **IDENTIFY** and **SET UP:** Draw the specified phasor diagrams. There is totally destructive interference between two slits when their phasors are in opposite directions.

EXECUTE: **(a)** For eight slits, the phasor diagrams must have eight vectors. The diagrams for each specified value of ϕ are sketched in Figure 36.61a. In each case the phasors all sum to zero.

(b) The additional phasor diagrams for $\phi = 3\pi/2$ and $3\pi/4$ are sketched in Figure 36.61b.

For $\phi = \dfrac{3\pi}{4}, \phi = \dfrac{5\pi}{4}$, and $\phi = \dfrac{7\pi}{4}$, totally destructive interference occurs between slits four apart. For $\phi = \dfrac{3\pi}{2}$, totally destructive interference occurs with every second slit.

EVALUATE: At a minimum the phasors for all slits sum to zero.

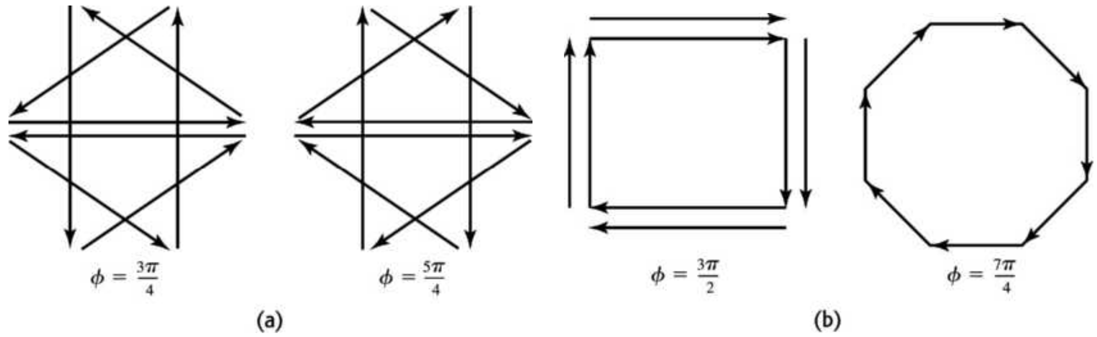


Figure 36.61

36.62. **IDENTIFY:** The wavelength of the helium spectral line from the receding galaxy will be different from the spectral line on earth due to the Doppler shift in the light from the galaxy.

SET UP: $d\sin\theta = m\lambda$. $\sin\theta_{\text{lab}} = \dfrac{2\lambda_{\text{lab}}}{d}$. $\sin\theta_{\text{galaxy}} = \dfrac{2\lambda_{\text{galaxy}}}{d}$. $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\left(\dfrac{\lambda_{\text{lab}}}{\lambda_{\text{galaxy}}}\right)$. The Doppler formula says $f_{\text{R}} = \sqrt{\dfrac{c-v}{c+v}}f_{\text{S}}$. Using $f = \dfrac{c}{\lambda}$, we have $\dfrac{1}{\lambda_{\text{R}}} = \dfrac{1}{\lambda_{\text{S}}}\sqrt{\dfrac{c-v}{c+v}}$. Since the lab is the receiver R and the galaxy is the source S, this becomes $\lambda_{\text{lab}} = \lambda_{\text{galaxy}}\sqrt{\dfrac{c+v}{c-v}}$.

EXECUTE: $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\sqrt{\dfrac{c+v}{c-v}} = \sin(18.9^\circ)\sqrt{\dfrac{2.998\times10^8\text{ m/s} + 2.65\times10^7\text{ m/s}}{2.998\times10^8\text{ m/s} - 2.65\times10^7\text{ m/s}}}$ which gives $\theta_{\text{galaxy}} = 20.7^\circ$.

EVALUATE: The galaxy is moving away, so the wavelength of its light will be lengthened, which means that the angle should be increased compared to the angle from light on earth, as we have found.

36.63. **IDENTIFY** and **SET UP:** The condition for an intensity maximum is $d\sin\theta = m\lambda$, $m = 0, \pm1, \pm2,\ldots$ Third order means $m = 3$. The longest observable wavelength is the one that gives $\theta = 90^\circ$ and hence $\theta = 1$.

EXECUTE: 9200 lines/cm so 9.2×10^5 lines/m and $d = \dfrac{1}{9.2\times10^5}\text{ m} = 1.087\times10^{-6}\text{ m}$.

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.087\times10^{-6}\text{ m})(1)}{3} = 3.6\times10^{-7}\text{ m} = 360\text{ nm}.$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

36.64. **IDENTIFY** and **SET UP:** As the rays first reach the slits there is already a phase difference between adjacent slits of $\dfrac{2\pi d\sin\theta'}{\lambda}$. This, added to the usual phase difference introduced after passing through the slits, yields the condition for an intensity maximum. For a maximum the total phase difference must equal $2\pi m$.

EXECUTE: $\dfrac{2\pi d\sin\theta}{\lambda} + \dfrac{2\pi d\sin\theta'}{\lambda} = 2\pi m \Rightarrow d(\sin\theta + \sin\theta') = m\lambda$

(b) 600 slits/mm $\Rightarrow d = \dfrac{1}{6.00\times10^5\text{ m}^{-1}} = 1.67\times10^{-6}\text{ m}$.

For $\theta' = \theta°$,

$m = 0$: $\theta = \arcsin(0) = 0$.

$m = 1$: $\theta = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{6.50\times10^{-7}\text{m}}{1.67\times10^{-6}\text{m}}\right) = 22.9^\circ$.

$m = -1$: $\theta = \arcsin\left(-\dfrac{\lambda}{d}\right) = \arcsin\left(-\dfrac{6.50\times10^{-7}\text{m}}{1.67\times10^{-6}\text{m}}\right) = -22.9^\circ$.

For $\theta' = 20.0°$,

$m = 0$: $\theta = \arcsin(-\sin 20.0°) = -20.0°$.

$m = 1$: $\theta = \arcsin\left(\dfrac{6.50\times10^{-7}\text{ m}}{1.67\times10^{-6}\text{ m}}\sin 20.0°\right) = 2.71°$.

$m = -1$: $\theta = \arcsin\left(-\dfrac{6.50\times10^{-7}\text{m}}{1.67\times10^{-6}\text{m}}\sin 20.0°\right) = -47.0°$.

EVALUATE: When $\theta' > 0$, the maxima are shifted downward on the screen, toward more negative angles.

36.65. **IDENTIFY:** The maxima are given by $d\sin\theta = m\lambda$. We need $\sin\theta = \dfrac{m\lambda}{d} \le 1$ in order for all the visible wavelengths to be seen.

SET UP: For 650 slits/mm $\Rightarrow d = \dfrac{1}{6.50\times10^{5}\text{ m}^{-1}} = 1.53\times10^{-6}$ m.

EXECUTE: $\lambda_1 = 4.00\times10^{-7}\text{m}$: $m=1$: $\dfrac{\lambda_1}{d} = 0.26$; $m=2$: $\dfrac{2\lambda_1}{d} = 0.52$; $m=3$: $\dfrac{3\lambda_1}{d} = 0.78$.

$\lambda_2 = 7.00\times10^{-7}\text{ m}$: $m=1$: $\dfrac{\lambda_2}{d} = 0.46$; $m=2$: $\dfrac{2\lambda_2}{d} = 0.92$; $m=3$: $\dfrac{3\lambda_2}{d} = 1.37$. So, the third order does not contain the violet end of the spectrum, and therefore only the first- and second-order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.66. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \dfrac{\Delta x}{R}$, where Δx is the size of the detail and $R = 7.2\times10^{8}$ ly.

$1\text{ ly} = 9.41\times10^{12}$ km. $\lambda = c/f$

EXECUTE: $\sin\theta = 1.22\dfrac{\lambda}{D} \approx \dfrac{\Delta x}{R} \Rightarrow \Delta x = \dfrac{1.22\lambda R}{D} = \dfrac{(1.22)cR}{Df} = \dfrac{(1.22)(3.00\times10^{5}\text{ km/s})(7.2\times10^{8}\text{ ly})}{(77.000\times10^{3}\text{ km})(1.665\times10^{9}\text{ Hz})} = 2.06$ ly.

$(9.41\times10^{12}\text{ km/ly})(2.06\text{ ly}) = 1.94\times10^{13}$ km.

EVALUATE: $\lambda = 18$ cm. λ/D is very small, so $\dfrac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.67. **IDENTIFY** and **SET UP:** Add the phases between adjacent sources.

EXECUTE: **(a)** $d\sin\theta = m\lambda$. Place 1st maximum at ∞ or $\theta = 90°$. $d = \lambda$. If $d < \lambda$, this puts the first maximum "beyond ∞." Thus, if $d < \lambda$ there is only a single principal maximum.

(b) At a principal maximum when $\delta = 0$, the phase difference due to the path difference between adjacent slits is $\Phi_{\text{path}} = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$. This just scales 2π radians by the fraction the wavelength is of the path difference between adjacent sources. If we add a relative phase δ between sources, we still must maintain a total phase difference of zero to keep our principal maximum.

$$\Phi_{\text{path}} \pm \delta = 0 \Rightarrow \frac{2\pi d\sin\theta}{\lambda} = \pm\delta \text{ or } \theta = \sin^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)$$

(c) $d = \dfrac{0.280\text{ m}}{14} = 0.0200$ m (count the number of spaces between 15 points). Let $\theta = 45°$. Also recall $f\lambda = c$, so

$$\delta_{\max} = \pm\frac{2\pi(0.0200\text{ m})(8.800\times10^{9}\text{ Hz})\sin 45°}{(3.00\times10^{8}\text{ m/s})} = \pm2.61\text{ radians.}$$

EVALUATE: δ must vary over a wider range in order to sweep the beam through a greater angle.

36.68. **IDENTIFY:** The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n$, and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D$.

EXECUTE: **(a)** In air the wavelength is $\lambda_0 = c/f = (3.00\times10^8\ \text{m/s})/(6.00\times10^{14}\ \text{Hz}) = 5.00\times10^{-7}\ \text{m}$. In water the wavelength is $\lambda = \lambda_0/n = (5.00\times10^{-7}\ \text{m})/1.33 = 3.76\times10^{-7}\ \text{m}$. With the lens open all the way, we have $D = f/2.8 = (35.0\ \text{mm})/2.80 = (0.0350\ \text{m})/2.80$. In the water, we have

$$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(3.76\times10^{-7}\ \text{m})[(0.0350\ \text{m})/2.80] = 3.67\times10^{-5}\ \text{rad}.$$

Calling w the width of the resolvable detail, we have

$$\theta = w/x \rightarrow w = x\theta = (2750\ \text{mm})(3.67\times10^{-5}\ \text{rad}) = 0.101\ \text{mm}$$

(b) $\theta = 1.22\lambda/D = (1.22)(5.00\times10^{-7}\ \text{m})/[(0.0350\ \text{m})/2.80] = 4.88\times10^{-5}\ \text{rad}$

$$w = x\theta = (2750\ \text{mm})(4.88\times10^{-5}\ \text{rad}) = 0.134\ \text{mm}$$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.69. **IDENTIFY:** The diameter D of the aperture limits the resolution due to diffraction, by Rayleigh's criterion.

SET UP: Rayleigh's criterion says that $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$. $D = 4.00$ mm. $\theta_{\text{res}} = \dfrac{y}{s}$, where s is the altitude and $y = 65.0$ m.

EXECUTE: Combining two equations above gives $\dfrac{y}{s} = 1.22\dfrac{\lambda}{D}$.

$$s = \frac{yD}{1.22\lambda} = \frac{(65.0\ \text{m})(4.00\times10^{-3}\ \text{m})}{1.22(550\times10^{-9}\ \text{m})} = 3.87\times10^5\ \text{m} = 387\ \text{km}.$$

EVALUATE: This is comparable to the altitude of the Hubble telescope.

36.70. **IDENTIFY:** The resolution of the eye is limited because light diffracts as it passes through the pupil. The size of the pupil determines the resolution.

SET UP: The smallest angular separation that can be resolved is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$. The angular size of the object is its height divided by its distance from the eye.

EXECUTE: **(a)** The angular size of the object is $\theta = \dfrac{50\times10^{-6}\ \text{m}}{25\times10^{-2}\ \text{m}} = 2.0\times10^{-4}\ \text{rad}$.

$\theta_{\text{res}} = 1.22\dfrac{\lambda}{D} = 1.22\left(\dfrac{550\times10^{-9}\ \text{m}}{2.0\times10^{-3}\ \text{m}}\right) = 3.4\times10^{-4}\ \text{rad}$. $\theta < \theta_{\text{res}}$ so the object cannot be resolved.

(b) $\theta_{\text{res}} = \dfrac{y}{s}$ and $y = s\theta_{\text{res}} = (25\ \text{cm})(3.4\times10^{-4}\ \text{rad}) = 8.5\times10^{-3}\ \text{cm} = 85\ \mu\text{m}$.

(c) $\theta = \theta_{\text{res}} = 3.4\times10^{-4}\ \text{rad} = 0.019° = 1.1\ \text{min}$. This is very close to the experimental value of 1 min.

(d) Diffraction is more important.

EVALUATE: We could not see any clearer if our retinal cells were much smaller than they are now because diffraction is more important in limiting the resolution of our vision.

36.71. **IDENTIFY:** The liquid reduces the wavlength of the light (compared to its value in air), and the scratch causes light passing through it to undergo single-slit diffraction.

SET UP: $\sin\theta = \dfrac{\lambda}{a}$, where λ is the wavelength in the liquid. $n = \dfrac{\lambda_{\text{air}}}{\lambda}$.

EXECUTE: $\tan\theta = \dfrac{(22.4/2)\ \text{cm}}{30.0\ \text{cm}}$ and $\theta = 20.47°$.

$\lambda = a\sin\theta = (1.25\times10^{-6}\ \text{m})\sin 20.47° = 4.372\times10^{-7}\ \text{m} = 437.2\ \text{nm}$. $n = \dfrac{\lambda_{\text{air}}}{\lambda} = \dfrac{612\ \text{nm}}{437.2\ \text{nm}} = 1.40$.

EVALUATE: $n > 1$, as it must be, and $n = 1.40$ is reasonable for many transparent films.

36.72. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}.$

SET UP: θ is small, so $\sin\theta \approx \dfrac{\Delta x}{R},$ where Δx is the size of the details and R is the distance to the earth. $1\text{ ly} = 9.41\times10^{15}\text{ m}.$

EXECUTE: **(a)** $R = \dfrac{D\Delta x}{1.22\lambda} = \dfrac{(6.00\times10^{6}\text{ m})(2.50\times10^{5}\text{ m})}{(1.22)(1.0\times10^{-5}\text{ m})} = 1.23\times10^{17}\text{ m} = 13.1\text{ ly}$

(b) $\Delta x = \dfrac{1.22\lambda R}{D} = \dfrac{(1.22)(1.0\times10^{-5}\text{ m})(4.22\text{ ly})(9.41\times10^{15}\text{ m/ly})}{1.0\text{ m}} = 4.84\times10^{8}\text{ km}.$ This is about 10,000 times the diameter of the earth! Not enough resolution to see an earth-like planet! Δx is about 3 times the distance from the earth to the sun.

(c) $\Delta x = \dfrac{(1.22)(1.0\times10^{-5}\text{ m})(59\text{ ly})(9.41\times10^{15}\text{ m/ly})}{6.00\times10^{6}\text{ m}} = 1.13\times10^{6}\text{ m} = 1130\text{ km}.$

$\dfrac{\Delta x}{D_{\text{planet}}} = \dfrac{1130\text{ km}}{1.38\times10^{5}\text{ km}} = 8.19\times10^{-3};$ Δx is small compared to the size of the planet.

EVALUATE: The very large diameter of *Planet Imager* allows it to resolve planet-sized detail at great distances.

36.73. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** From the segment dy', the fraction of the amplitude of E_0 that gets through is

$E_0\left(\dfrac{dy'}{a}\right) \Rightarrow dE = E_0\left(\dfrac{dy'}{a}\right)\sin(kx-\omega t).$

(b) The path difference between each little piece is

$y'\sin\theta \Rightarrow kx = k(D - y'\sin\theta) \Rightarrow dE = \dfrac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t).$ This can be rewritten as

$dE = \dfrac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\cos(kD-\omega t)).$

(c) So the total amplitude is given by the integral over the slit of the above.

$\Rightarrow E = \int_{-a/2}^{a/2} dE = \dfrac{E_0}{a}\int_{-a/2}^{a/2} dy'\,(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\cos(kD-\omega t)).$

But the second term integrates to zero, so we have:

$$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'(\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta)}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$$

$$\Rightarrow E = E_0\sin(kD-\omega x)\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega x)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$$

At $\theta = 0, \dfrac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega x).$

(d) Since $I \propto E^2 \Rightarrow I = I_0\left(\dfrac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used $I_0 = E_0^2\sin^2(kx-\omega t).$

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.74. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d\sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR-\omega t+n\phi)} = \cos(kR-\omega t+n\phi) + i\sin(kR-\omega t+n\phi)$. The real part is just $\cos(kR-\omega t+n\phi)$. So,

$\mathrm{Re}\left[\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega x+n\phi)}\right] = \sum_{n=0}^{N-1} E_0\cos(kR-\omega x+n\phi)$. (Note: Re means "the real part of…."). But this is just

$E_0\cos(kR-\omega t) + E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+2\phi)+\cdots+E_0\cos(kR-\omega t+(N-1)\phi)$

(c) $\sum_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0\sum_{n=0}^{N-1} e^{-i\omega t}e^{+ikR}e^{in\phi} = E_0 e^{i(kR-\omega t)}\sum_{n=0}^{N-1} e^{in\phi}$. $\sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1}(e^{i\phi})^n$. But recall

$\sum_{n=0}^{N-1} x^n = \dfrac{x^N-1}{x-1}$. Putting everything together:

$$\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\phi/2)}\frac{(e^{iN\phi/2}-e^{-iN\phi/2})}{(e^{i\phi/2}-e^{-i\phi/2})}$$

$$= E_0[\cos(kR-\omega t+(N-1)\phi/2)+i\sin(kR-\omega t+(N-1)\phi/2)]\left[\frac{\cos N\phi/2+\sin N\phi/2-\cos N\phi/2+i\sin N\phi/2}{\cos\phi/2+i\sin\phi/2-\cos\phi/2+i\sin\phi/2}\right]$$

Taking only the real part gives $\Rightarrow E_0\cos(kR-\omega t+(N-1)\phi/2)\dfrac{\sin(N\phi/2)}{\sin\phi/2} = E.$

(d) $I = |E|^2_{\text{av}} = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.) $I_0 \propto \dfrac{E_0^2}{2}$.

EVALUATE: **(e)** $N=2$. $I = I_0\dfrac{\sin^2(2\phi/2)}{\sin^2\phi/2} = \dfrac{I_0(2\sin\phi/2\cos\phi/2)^2}{\sin^2\phi/2} = 4I_0\cos^2\dfrac{\phi}{2}$. Looking at Eq. (35.9),

$I'_0 \propto 2E_0^2$ but for us $I_0 \propto \dfrac{E_0^2}{2} = \dfrac{I'_0}{4}$.

36.75. **IDENTIFY and SET UP:** From Problem 36.74, $I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2}$. Use this result to obtain each result specified in the problem.

EXECUTE: **(a)** $\lim_{\phi\to 0} I \to \dfrac{0}{0}$. Use 1'Hôpital's rule: $\lim_{\phi\to 0}\dfrac{\sin(N\phi/2)}{\sin\phi/2} = \lim_{\phi\to 0}\left(\dfrac{N/2}{1/2}\right)\dfrac{\cos(N\phi/2)}{\cos(\phi/2)} = N$. So

$\lim_{\phi\to 0} I = N^2 I_0$.

(b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\min} = \pi$ or $\phi_{\min} = \dfrac{2\pi}{N}$.

The width of the central maximum goes like $2\phi_{\min}$, so it is proportional to $\dfrac{1}{N}$.

(c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity.

That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\frac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\frac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n=0$ will be a maximum, but $n=1, 2\ldots, N-1$ will be minima with another maximum at $n=N$.

(e) Between maxima $\frac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e.,}\frac{\pi}{2},\frac{3\pi}{2}\text{etc.}\right)$ and if N is odd then $\frac{\sin^2(N\phi/2)}{\sin^2\phi/2}\rightarrow 1$, so $I\rightarrow I_0$.

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.