35

INTERFERENCE

35.1. **IDENTIFY:** The sound will be maximally reinforced when the path difference is an integral multiple of wavelengths and cancelled when it is an odd number of half wavelengths.

SET UP: Constructive interference occurs for $r_2 - r_1 = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$. Destructive interference occurs for $r_2 - r_1 = (m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2 \ldots$. For this problem, $r_2 = 150$ cm and $r_1 = x$. The path taken by the person ensures that x is in the range $0 \le x \le 150$ cm.

EXECUTE: **(a)** $150\text{ cm} - x = m(34\text{ cm})$. $x = 150\text{ cm} - m(34\text{ cm})$. For $m = 0, 1, 2, 3, 4$ the values of x are 150 cm, 116 cm, 82 cm, 48 cm, 14 cm.

(b) $150\text{ cm} - x = (m + \frac{1}{2})(34\text{ cm})$. $x = 150\text{ cm} - (m + \frac{1}{2})(34\text{ cm})$. For $m = 0, 1, 2, 3$ the values of x are 133 cm, 99 cm, 65 cm, 31 cm.

EVALUATE: When $x = 116$ cm the path difference is $150\text{ cm} - 116\text{ cm} = 34$ cm, which is one wavelength. When $x = 133$ cm the path difference is 17 cm, which is one-half wavelength.

35.2. **IDENTIFY:** The sound will be maximally reinforced when the path difference is an integral multiple of wavelengths and cancelled when it is an odd number of half wavelengths.

SET UP: When she is at the midpoint between the two speakers the path difference $r_2 - r_1$ is zero. When she walks a distance d toward one speaker, r_2 increases by d and r_1 decreases by d, so the path difference changes by $2d$. Path difference $= m\lambda$ $(m = 0, \pm 1, \pm 2, \ldots)$ gives constructive interference and path difference $= (m + \frac{1}{2})\lambda$ $(m = 0, \pm 1, \pm 2, \ldots)$ gives destructive interference.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{340.0\text{ m/s}}{250.0\text{ Hz}} = 1.36\text{ m}.$

(a) The path difference is zero, so the interference is constructive.

(b) Destructive interference occurs, so the path difference equals $\lambda/2$. $2d = \dfrac{\lambda}{2}$ which gives

$$d = \frac{\lambda}{4} = \frac{1.36\text{ m}}{4} = 34.0\text{ cm}.$$

(c) Constructive interference occurs, so the path difference equals λ. $2d = \lambda$ which gives

$$d = \frac{\lambda}{2} = \frac{1.36\text{ m}}{2} = 68.0\text{ cm}.$$

EVALUATE: If she keeps walking, she will possibly find additional places where constructive and destructive interference occur.

35.3. **IDENTIFY:** The sound will be maximally reinforced when the path difference is an integral multiple of wavelengths and cancelled when it is an odd number of half wavelengths.

SET UP: $v = f\lambda$. Constructive interference occurs when the path difference $r_2 - r_1$ from the two sources is $r_2 - r_1 = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$. Destructive interference occurs when the path difference $r_2 - r_1$ is $r_2 - r_1 = (m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$.

EXECUTE: **(a)** The path difference from the two speakers is a half-integer number of wavelengths and the interference is destructive.

(b) The path difference changes by $\frac{\lambda}{2}$, so $\frac{\lambda}{2} = 0.398\text{ m}$ and $\lambda = 0.796\text{ m}$. $f = \frac{v}{\lambda} = \frac{340\text{ m/s}}{0.796\text{ m}} = 427\text{ Hz}$.

(c) The speaker must be moved a distance $\lambda = 0.796\text{ m}$, so the path difference will change by λ.

EVALUATE: In reality, sound interference effects are often difficult to hear clearly due to reflections off of surrounding surfaces, such as, wall, the ceiling and the floor.

35.4. **IDENTIFY:** For destructive interference the path difference is $(m+\frac{1}{2})\lambda, m = 0, \pm 1, \pm 2, \ldots$. The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda, m = 0, \pm 1, \pm 2, \ldots$ The longest wavelength is for $m = 1$.

SET UP: The path difference is 120 m.

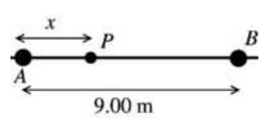
EXECUTE: **(a)** For destructive interference $\frac{\lambda}{2} = 120\text{ m} \Rightarrow \lambda = 240\text{ m}$.

(b) The longest wavelength for constructive interference is $\lambda = 120\text{ m}$.

EVALUATE: The path difference doesn't depend on the distance of point Q from B.

35.5. **IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from A to P and from B to P. For constructive interference this path difference is an integer multiple of the wavelength.

SET UP: Consider Figure 35.5.



The distance of point P from each coherent source is $r_A = x$ and $r_B = 9.00\text{ m} - x$.

Figure 35.5

EXECUTE: The path difference is $r_B - r_A = 9.00\text{ m} - 2x$.

$r_B - r_A = m\lambda, m = 0, \pm 1, \pm 2, \ldots$

$$\lambda = \frac{c}{f} = \frac{2.998\times 10^8\text{ m/s}}{120\times 10^6\text{ Hz}} = 2.50\text{ m}$$

Thus $9.00\text{ m} - 2x = m(2.50\text{ m})$ and $x = \frac{9.00\text{ m} - m(2.50\text{ m})}{2} = 4.50\text{ m} - (1.25\text{ m})m$. x must lie in the range 0 to 9.00 m since P is said to be between the two antennas.

$m = 0$ gives $x = 4.50\text{ m}$

$m = +1$ gives $x = 4.50\text{ m} - 1.25\text{ m} = 3.25\text{ m}$

$m = +2$ gives $x = 4.50\text{ m} - 2.50\text{ m} = 2.00\text{ m}$

$m = +3$ gives $x = 4.50\text{ m} - 3.75\text{ m} = 0.75\text{ m}$

$m = -1$ gives $x = 4.50\text{ m} + 1.25\text{ m} = 5.75\text{ m}$

$m = -2$ gives $x = 4.50\text{ m} + 2.50\text{ m} = 7.00\text{ m}$

$m = -3$ gives $x = 4.50\text{ m} + 3.75\text{ m} = 8.25\text{ m}$

All other values of m give values of x out of the allowed range. Constructive interference will occur for x = 0.75 m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

35.6. **IDENTIFY:** For constructive interference the path difference d is related to λ by $d = m\lambda$, $m = 0, 1, 2, \ldots$ For destructive interference $d = (m+\frac{1}{2})\lambda, m = 0, 1, 2, \ldots$

SET UP: $d = 2040\text{ nm}$

EXECUTE: **(a)** The brightest wavelengths are when constructive interference occurs:

$d = m\lambda_m \Rightarrow \lambda_m = \dfrac{d}{m} \Rightarrow \lambda_3 = \dfrac{2040\text{ nm}}{3} = 680\text{ nm},\ \lambda_4 = \dfrac{2040\text{ nm}}{4} = 510\text{ nm and } \lambda_5 = \dfrac{2040\text{ nm}}{5} = 408\text{ nm}.$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m+\frac{1}{2})\lambda_m$ so $\lambda_m = \dfrac{d}{m+\frac{1}{2}} = \dfrac{2040\text{ nm}}{m+\frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583$ nm and $\lambda_4 = 453$ nm.

EVALUATE: The wavelengths for constructive interference are between those for destructive interference.

35.7. **IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.

SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0$ m and $P_2 = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10.0$ m. The path distance is

$$\Delta P = P_2 - P_1 = 10.0\text{ m} - 8.0\text{ m} = 2.0\text{ m}$$

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0$ m and $P_2 = \sqrt{(6.0\text{ m})^2 + (17.0\text{ m})^2} = 18.0$ m. The path difference is $18.0\text{ m} - 17.0\text{ m} = 1.0$ m, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference $2\lambda, 3\lambda, 4\lambda$, etc., and destructive interference occurs if it is $\lambda/2, 3\lambda/2, 5\lambda/2$, etc.

35.8. **IDENTIFY:** At an antinode the interference is constructive and the path difference is an integer number of wavelengths; path difference $= m\lambda, m = 0, \pm 1, \pm 2, \ldots$ at an antinode.

SET UP: The maximum magnitude of the path difference is the separation d between the two sources.

EXECUTE: **(a)** At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y-axis, so $m = +4$. At $S_2, r_2 - r_1 = -4\lambda$, and the path difference below this point, along the negative y-axis, stays the same, so $m = -4$.

(b) The wave pattern is sketched in Figure 35.8.

(c) The maximum and minimum m-values are determined by the largest integer less than or equal to $\dfrac{d}{\lambda}$.

(d) If $d = 7\frac{1}{2}\lambda \Rightarrow -7 \le m \le +7$, there will be a total of 15 antinodes between the sources.

EVALUATE: We are considering points close to the two sources and the antinodal curves are not straight lines.

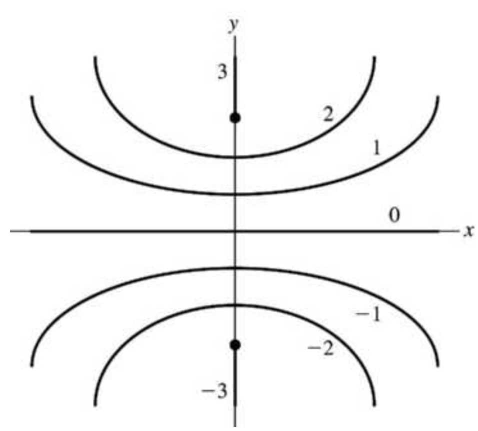


Figure 35.8

35.9. **IDENTIFY:** The value of y_{20} is much smaller than R and the approximate expression $y_m = R\dfrac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6\times10^{-3}$ m.

EXECUTE: $d = \dfrac{20R\lambda}{y_{20}} = \dfrac{(20)(1.20\text{ m})(502\times10^{-9}\text{ m})}{10.6\times10^{-3}\text{ m}} = 1.14\times10^{-3}\text{ m} = 1.14\text{ mm}$

EVALUATE: $\tan\theta_{20} = \dfrac{y_{20}}{R}$ so $\theta_{20} = 0.51°$ and the approximation $\sin\theta_{20} \approx \tan\theta_{20}$ is very accurate.

35.10. **IDENTIFY:** Since the dark fringes are eqully spaced, $R \gg y_m$, the angles are small and the dark bands are located by $y_{m+\frac{1}{2}} = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \dfrac{R\lambda}{d}$.

EXECUTE: $\Delta y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{\Delta y} = \dfrac{(1.80\text{ m})(4.50\times10^{-7}\text{ m})}{4.20\times10^{-3}\text{ m}} = 1.93\times10^{-4}\text{ m} = 0.193\text{ mm}.$

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.11. **IDENTIFY** and **SET UP:** The dark lines correspond to destructive interference and hence are located by Eq. (35.5):

$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \text{ so } \sin\theta = \frac{\left(m+\frac{1}{2}\right)\lambda}{d},\ m = 0, \pm1, \pm2, \ldots$$

Solve for θ that locates the second and third dark lines. Use $y = R\tan\theta$ to find the distance of each of the dark lines from the center of the screen.

EXECUTE: 1st dark line is for $m = 0$

2nd dark line is for $m = 1$ and $\sin\theta_1 = \dfrac{3\lambda}{2d} = \dfrac{3(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 1.667\times10^{-3}$ and $\theta_1 = 1.667\times10^{-3}$ rad

3rd dark line is for $m = 2$ and $\sin\theta_2 = \dfrac{5\lambda}{2d} = \dfrac{5(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 2.778\times10^{-3}$ and $\theta_2 = 2.778\times10^{-3}$ rad

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin\theta \approx \tan\theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R\tan\theta$, where $R = 0.850$ m is the distance to the screen.

$\tan\theta \approx \theta$ so $y_m = R\theta_m$

$y_1 = R\theta_1 = (0.750\text{ m})(1.667\times10^{-3}\text{ rad}) = 1.25\times10^{-3}$ m

$y_2 = R\theta_2 = (0.750\text{ m})(2.778\times10^{-3}\text{ rad}) = 2.08\times10^{-3}$ m

$\Delta y = y_2 - y_1 = 2.08\times10^{-3}\text{ m} - 1.25\times10^{-3}\text{ m} = 0.83\text{ mm}$

EVALUATE: Since θ_1 and θ_2 are very small we could have used Eq. (35.6), generalized to destructive interference: $y_m = R\left(m+\dfrac{1}{2}\right)\lambda/d$.

35.12. **IDENTIFY:** The water changes the wavelength of the light, but the rest of the analysis is the same as in Exercise 35.11.

SET UP: Water has $n = 1.333$. In water the wavelength is $\lambda = \dfrac{\lambda_0}{n}$. θ is very small for these dark lines and the approximate expression $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$ is accurate. Adjacent dark lines are separated by

$$\Delta y = y_{m+1} - y_m = \frac{R\lambda}{d}.$$

EXECUTE: $\Delta y = \dfrac{R\lambda_0}{dn} = \dfrac{(0.750\text{ m})(500\times 10^{-9}\text{ m})}{(0.450\times 10^{-3}\text{ m})(1.333)} = 6.25\times 10^{-4}\text{ m} = 0.625\text{ mm}.$

EVALUATE: λ is smaller in water and the dark lines are closer together when the apparatus is immersed in water.

35.13. **IDENTIFY:** Bright fringes are located at angles θ given by $d\sin\theta = m\lambda$.

SET UP: The largest value $\sin\theta$ can have is 1.00.

EXECUTE: **(a)** $m = \dfrac{d\sin\theta}{\lambda}$. For $\sin\theta = 1$, $m = \dfrac{d}{\lambda} = \dfrac{0.0116\times 10^{-3}\text{ m}}{5.85\times 10^{-7}\text{ m}} = 19.8$. Therefore, the largest *m* for fringes on the screen is $m = 19$. There are $2(19)+1 = 39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m = \pm 19$. $\sin\theta = m\dfrac{\lambda}{d} = \pm 19\left(\dfrac{5.85\times 10^{-7}\text{ m}}{0.0116\times 10^{-3}\text{ m}}\right) = \pm 0.958$ and $\theta = \pm 73.3°$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.14. **IDENTIFY:** The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200\times 10^{-3}$ m. $R = 4.00$ m.

EXECUTE: The distance between any two successive minima is

$y_{m+1} - y_m = R\dfrac{\lambda}{d} = (4.00\text{ m})\dfrac{(400\times 10^{-9}\text{m})}{(0.200\times 10^{-3}\text{ m})} = 8.00\text{ mm}$. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m \ll R$, the interference minima are equally spaced.

35.15. **IDENTIFY** and **SET UP:** The dark lines are located by $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$. The distance of each line from the center of the screen is given by $y = R\tan\theta$.

EXECUTE: First dark line is for $m = 0$ and $d\sin\theta_1 = \lambda/2$.

$\sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{550\times 10^{-9}\text{ m}}{2(1.80\times 10^{-6}\text{ m})} = 0.1528$ and $\theta_1 = 8.789°$. Second dark line is for $m = 1$ and $d\sin\theta_2 = 3\lambda/2$.

$\sin\theta_2 = \dfrac{3\lambda}{2d} = 3\left(\dfrac{550\times 10^{-9}\text{ m}}{2(1.80\times 10^{-6}\text{ m})}\right) = 0.4583$ and $\theta_2 = 27.28°$.

$y_1 = R\tan\theta_1 = (0.350\text{ m})\tan 8.789° = 0.0541\text{ m}$

$y_2 = R\tan\theta_2 = (0.350\text{ m})\tan 27.28° = 0.1805\text{ m}$

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805\text{ m} - 0.0541\text{ m} = 0.126\text{ m} = 12.6\text{ cm}$.

EVALUATE: $\sin\theta_1 = 0.1528$ and $\tan\theta_1 = 0.1546$. $\sin\theta_2 = 0.4583$ and $\tan\theta_2 = 0.5157$. As the angle increases, $\sin\theta \approx \tan\theta$ becomes a poorer approximation.

35.16. **IDENTIFY:** Using Eq. (35.6) for small angles: $y_m = R\dfrac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(5.00\text{ m})(1)}{(0.300\times 10^{-3}\text{ m})}(660-470)\times(10^{-9}\text{ m}) = 3.17\text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

35.17. **IDENTIFY** and **SET UP:** Use the information given about the bright fringe to find the distance d between the two slits. Then use Eq. (35.5) and $y = R\tan\theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

EXECUTE: $y_1 = \dfrac{R\lambda_1}{d}$, so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times10^{-9}\text{ m})}{4.84\times10^{-3}\text{ m}} = 3.72\times10^{-4}$ m. (R is much greater than d, so Eq. 35.6 is valid.) The dark fringes are located by $d\sin\theta = \left(m+\dfrac{1}{2}\right)\lambda,\ m = 0, \pm1, \pm2, \ldots$ The first-order dark fringe is located by $\sin\theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R\tan\theta \approx R\sin\theta = \frac{\lambda_2 R}{2d}$$

We want λ_2 such that $y = y_1$. This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200$ nm.

EVALUATE: For $\lambda = 600$ nm the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200$ nm.

35.18. **IDENTIFY:** Bright fringes are located at $y_m = R\dfrac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m+\frac{1}{2})\lambda$ and $y = R\tan\theta$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.32\times10^{14}\text{ Hz}} = 4.75\times10^{-7}$ m. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: **(a)** $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(4.75\times10^{-7}\text{ m})(0.850\text{ m})}{0.0311\text{ m}} = 3.89\times10^{-5}\text{ m} = 0.0389\text{ mm}$

(b) $\sin\theta = (2+\frac{1}{2})\dfrac{\lambda}{d} = (2.5)\left(\dfrac{4.75\times10^{-7}\text{ m}}{3.89\times10^{-5}\text{ m}}\right) = 0.0305$ and $\theta = 1.75°$.

$y = R\tan\theta = (85.0\text{ cm})\tan1.75° = 2.60$ cm.

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.19. **IDENTIFY:** Eq. (35.10): $I = I_0\cos^2(\phi/2)$. Eq. (35.11): $\phi = (2\pi/\lambda)(r_2 - r_1)$.

SET UP: ϕ is the phase difference and $(r_2 - r_1)$ is the path difference.

EXECUTE: **(a)** $I = I_0(\cos 30.0°)^2 = 0.750 I_0$

(b) $60.0° = (\pi/3)$ rad. $(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80$ nm.

EVALUATE: $\phi = 360°/6$ and $(r_2 - r_1) = \lambda/6$.

35.20. **IDENTIFY:** $\dfrac{\phi}{2\pi} = \dfrac{\text{path difference}}{\lambda}$ relates the path difference to the phase difference ϕ.

SET UP: The sources and point P are shown in Figure 35.20.

EXECUTE: $\phi = 2\pi\left(\dfrac{524\text{ cm} - 486\text{ cm}}{2\text{ cm}}\right) = 119$ radians

EVALUATE: The distances from B to P and A to P aren't important, only the difference in these distances.

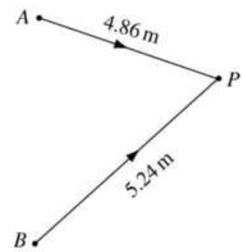


Figure 35.20

35.21. **IDENTIFY** and **SET UP:** The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta$ (Eq. 35.13.)

EXECUTE: $\phi = [2\pi(0.340\times10^{-3}\text{ m})/(500\times10^{-9}\text{ m})\sin 23.0° = 1670\text{ rad}$

EVALUATE: The *m*th bright fringe occurs when $\phi = 2\pi m$, so there are a large number of bright fringes within $23.0°$ from the centerline. Note that Eq. (35.13) gives ϕ in radians.

35.22. **(a) IDENTIFY** and **SET UP:** The minima are located at angles θ given by $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$. The first minimum corresponds to $m = 0$. Solve for θ. Then the distance on the screen is $y = R\tan\theta$.

EXECUTE: $\sin\theta = \dfrac{\lambda}{2d} = \dfrac{660\times10^{-9}\text{ m}}{2(0.260\times10^{-3}\text{ m})} = 1.27\times10^{-3}$ and $\theta = 1.27\times10^{-3}$ rad

$y = (0.700\text{ m})\tan(1.27\times10^{-3}\text{ rad}) = 0.889\text{ mm}.$

(b) IDENTIFY and **SET UP:** Eq. (35.15) given the intensity *I* as a function of the position *y* on the screen: $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$. Set $I = I_0/2$ and solve for *y*.

EXECUTE: $I = \dfrac{1}{2}I_0$ says $\cos^2\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{2}$

$\cos\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{\sqrt{2}}$ so $\dfrac{\pi dy}{\lambda R} = \dfrac{\pi}{4}$ rad

$y = \dfrac{\lambda R}{4d} = \dfrac{(660\times10^{-9}\text{ m})(0.700\text{ m})}{4(0.260\times10^{-3}\text{ m})} = 0.444\text{ mm}$

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0$.

35.23. **IDENTIFY:** The intensity decreases as we move away from the central maximum.

SET UP: The intensity is given by $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$.

EXECUTE: First find the wavelength: $\lambda = c/f = (3.00\times10^8\text{ m/s})/(12.5\text{ MHz}) = 24.00\text{ m}$

At the farthest the receiver can be placed, $I = I_0/4$, which gives

$$\frac{I_0}{4} = I_0\cos^2\left(\frac{\pi dy}{\lambda R}\right) \Rightarrow \cos^2\left(\frac{\pi dy}{\lambda R}\right) = \frac{1}{4} \Rightarrow \cos\left(\frac{\pi dy}{\lambda R}\right) = \pm\frac{1}{2}$$

The solutions are $\pi dy/\lambda R = \pi/3$ and $2\pi/3$. Using $\pi/3$, we get

$$y = \lambda R/3d = (24.00\text{ m})(500\text{ m})/[3(56.0\text{ m})] = 71.4\text{ m}$$

It must remain within 71.4 m of point *C*.

EVALUATE: Using $\pi dy/\lambda R = 2\pi/3$ gives $y = 142.8$ m. But to reach this point, the receiver would have to go beyond 71.4 m from *C*, where the signal would be too weak, so this second point is not possible.

35.24. **IDENTIFY:** The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \frac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$.

SET UP: $\lambda = \frac{c}{f} = \frac{3.00\times10^8 \text{ m/s}}{1.20\times10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = \frac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.25. **IDENTIFY:** Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.
SET UP: Consider Figure 35.25.

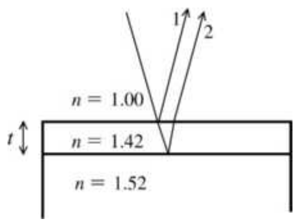


Figure 35.25

Both rays (1) and (2) undergo a $180°$ phase change on reflection, so there is no net phase difference introduced and the condition for destructive interference is $2t = \left(m + \frac{1}{2}\right)\lambda$.

EXECUTE: $t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2}$; thinnest film says $m = 0$ so $t = \frac{\lambda}{4}$.

$\lambda = \frac{\lambda_0}{1.42}$ and $t = \frac{\lambda_0}{4(1.42)} = \frac{650\times10^{-9} \text{ m}}{4(1.42)} = 1.14\times10^{-7} \text{ m} = 114 \text{ nm}$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

35.26. **IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.
SET UP: At the front surface of the film, light in air $(n = 1.00)$ reflects from the film $(n = 2.62)$ and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass $(n = 1.62)$ and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \frac{505 \text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\frac{505 \text{ nm}}{2[2.62]}\right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2, 3$ and 4: 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda_0/2n$. Compare this to Problem 35.25, where the minimum thickness for destructive interference is $t = \lambda_0/4n$.

35.27. **IDENTIFY:** The fringes are produced by interference between light reflected from the top and bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift, and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) is therefore $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.27. At a distance x from the point of contact of the two plates, the thickness of the air wedge is t.

EXECUTE: $\tan\theta = \dfrac{t}{x}$ so $t = x\tan\theta$. $t_m = (m+\tfrac{1}{2})\dfrac{\lambda}{2}$. $x_m = (m+\tfrac{1}{2})\dfrac{\lambda}{2\tan\theta}$ and $x_{m+1} = (m+\tfrac{3}{2})\dfrac{\lambda}{2\tan\theta}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \dfrac{\lambda}{2\tan\theta}$. $15.0 \text{ fringes/cm} = \dfrac{1.00}{\Delta x}$ and $\Delta x = \dfrac{1.00}{15.0 \text{ fringes/cm}} = 0.0667 \text{ cm}$. $\tan\theta = \dfrac{\lambda}{2\Delta x} = \dfrac{546\times10^{-9} \text{ m}}{2(0.0667\times10^{-2} \text{ m})} = 4.09\times10^{-4}$. The angle of the wedge is $4.09\times10^{-4} \text{ rad} = 0.0234°$.

EVALUATE: The fringes are equally spaced; Δx is independent of m.

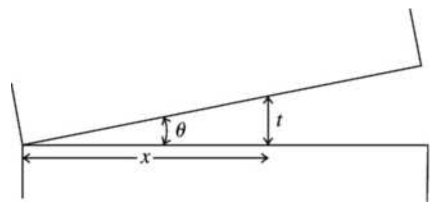


Figure 35.27

35.28. **IDENTIFY:** The fringes are produced by interference between light reflected from the top and from the bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) therefore is $2t = (m+\tfrac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.28.

EXECUTE: $\tan\theta = \dfrac{0.0800 \text{ mm}}{90.0 \text{ mm}} = 8.89\times10^{-4}$. $\tan\theta = \dfrac{t}{x}$ so $t = (8.89\times10^{-4})x$. $t_m = (m+\tfrac{1}{2})\dfrac{\lambda}{2}$.

$x_m = (m+\tfrac{1}{2})\dfrac{\lambda}{2(8.89\times10^{-4})}$ and $x_{m+1} = (m+\tfrac{3}{2})\dfrac{\lambda}{2(8.89\times10^{-4})}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \dfrac{\lambda}{2(8.89\times10^{-4})} = \dfrac{656\times10^{-9} \text{ m}}{2(8.89\times10^{-4})} = 3.69\times10^{-4} \text{ m} = 0.369 \text{ mm}$.

The number of fringes per cm is $\dfrac{1.00}{\Delta x} = \dfrac{1.00}{0.0369 \text{ cm}} = 27.1 \text{ fringes/cm}$.

EVALUATE: As $t \to 0$ the interference is destructive and there is a dark fringe at the line of contact between the two plates.

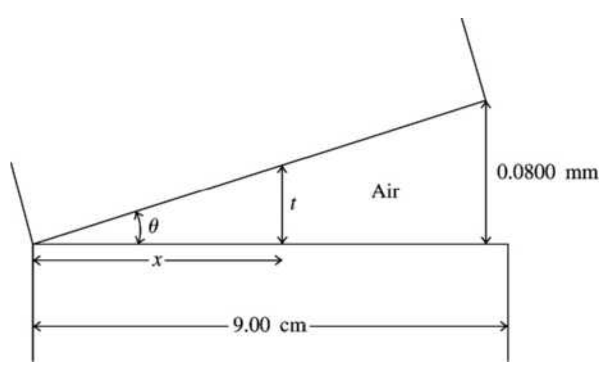


Figure 35.28

35.29. **IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: **(a)** Calling T the thickness of the film gives $2T = m\lambda_0/n$, which yields $T = m\lambda_0/(2n)$. Substituting the numbers gives

$$T = m\ (520.0\text{ nm})/[2(2.62)] = 99.237nm$$

T must be greater than 1036 nm, so $m = 11$, which gives $T = 1091.6$ nm, since we want to know the minimum thickness to add.

$$\Delta T = 1091.6\text{ nm} - 1036\text{ nm} = 55.6\text{ nm}.$$

(b) (i) Path difference $= 2T = 2(1092\text{ nm}) = 2184\text{ nm} = 2180\text{ nm}$.

(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0\text{ nm})/2.62 = 198.5\text{ nm}$.

$$\text{Path difference} = (2180\text{ nm})/[(198.5\text{ nm})/\text{wavelength}] = 11.0\text{ wavelengths}$$

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.30. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.

SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.

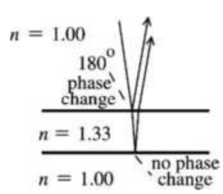
EXECUTE: **(a)** Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{550\text{ nm}}{4(1.85)} = 74.3\text{ nm}$.

(b) The next smallest thickness for constructive interference is with another half wavelength thickness added: $t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550\text{ nm})}{4(1.85)} = 223\text{ nm}$.

EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.31. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.

(a) SET UP: Consider Figure 35.31.



There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

Figure 35.31

EXECUTE: The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = \left(m + \dfrac{1}{2}\right)(\lambda_0/n)$, $m = 0, 1, 2, \ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(290\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{771.4\text{ nm}}{m+\frac{1}{2}}$$

for $m = 0, \lambda = 1543$ nm; for $m = 1, \lambda = 514$ nm; for $m = 2, \lambda = 308$ nm;… Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $\lambda_0 = \dfrac{2tn}{m+\frac{1}{2}} = \dfrac{2(340\text{ nm})(1.33)}{m+\frac{1}{2}} = \dfrac{904.4\text{ nm}}{m+\frac{1}{2}}$

for $m = 0, \lambda = 1809$ nm; for $m = 1, \lambda = 603$ nm; for $m = 2, \lambda = 362$ nm;… Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.32. **IDENTIFY:** The number of waves along the path is the path length divided by the wavelength. The path difference and the reflections determine the phase difference.

SET UP: The path length is $2t = 17.52\times10^{-6}$ m. The wavelength in the film is $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $\lambda = \dfrac{648\text{ nm}}{1.35} = 480$ nm. The number of waves is $\dfrac{2t}{\lambda} = \dfrac{17.52\times10^{-6}\text{ m}}{480\times10^{-9}\text{ m}} = 36.5.$

(b) The path difference introduces a $\lambda/2$, or $180°$, phase difference. The ray reflected at the top surface of the film undergoes a $180°$ phase shift upon reflection. The reflection at the lower surface introduces no phase shift. Both rays undergo a $180°$ phase shift, one due to reflection and one due to the path difference. The two effects cancel and the two rays are in phase as they leave the film.

EVALUATE: Note that we must use the wavelength in the film to determine the number of waves in the film.

35.33. **IDENTIFY:** Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is $2t = (m+\frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \dfrac{\lambda_0}{n}$. The minimum pit depth is for $m = 0$.

EXECUTE: $2t = \dfrac{\lambda}{2}$. $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{790\text{ nm}}{4(1.8)} = 110\text{ nm} = 0.11\ \mu\text{m}.$

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.34. **IDENTIFY:** Consider light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air $(n = 1.00)$ reflects from the film $(n = 1.33)$ and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film $(n = 1.33)$ reflects from air $(n = 1.00)$ and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{480\text{ nm}}{2.62}$.

EXECUTE: Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{480\text{ nm}}{2[1.33]}\right) = (180\text{ nm})m.$ The minimum thickness is 180 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 35.25, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.35. **IDENTIFY** and **SET UP:** Apply Eq. (35.19) and calculate y for $m = 1800$.

EXECUTE: Eq. (35.19): $y = m(\lambda/2) = 1800(633\times10^{-9}\text{ m})/2 = 5.70\times10^{-4}\text{ m} = 0.570\text{ mm}$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.36. **IDENTIFY:** Apply Eq. (35.19).

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

EXECUTE: **(a)** For Jan, the total shift was $y_1 = \dfrac{m\lambda_1}{2} = \dfrac{818(6.06\times10^{-7}\text{ m})}{2} = 2.48\times10^{-4}\text{ m}$. For Linda, the total shift was $y_2 = \dfrac{m\lambda_2}{2} = \dfrac{818(5.02\times10^{-7}\text{ m})}{2} = 2.05\times10^{-4}\text{ m}$.

(b) The net displacement of the mirror is the difference of the above values:

$$\Delta y = y_1 - y_2 = 0.248\text{ mm} - 0.205\text{ mm} = 0.043\text{ mm}.$$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.37. **IDENTIFY:** Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections,

$2t = \left(m + \dfrac{1}{2}\right)\lambda.\ t = R - \sqrt{R^2 - r^2}.$

EXECUTE: $\dfrac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2} \Rightarrow \sqrt{R^2 - r^2} = R - \dfrac{(2m+1)\lambda}{4}$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}},\ \text{for } R \gg \lambda.$$

The second bright ring is when $m = 1$:

$r \approx \sqrt{\dfrac{[2(1)+1](5.80\times10^{-7}\text{ m})(0.684\text{ m})}{2}} = 7.71\times10^{-4}\text{ m} = 0.771\text{ mm}$. So the diameter of the second bright ring is 1.54 mm.

EVALUATE: The diameter of the m^{th} ring is proportional to $\sqrt{2m+1}$, so the rings get closer together as m increases. This agrees with Figure 35.16b in the textbook.

35.38. **IDENTIFY:** As found in Problem 35.37, the radius of the m^{th} bright ring is $r \approx \sqrt{\dfrac{(2m+1)\lambda R}{2}}$, for $R \gg \lambda$.

SET UP: Introducing a liquid between the lens and the plate just changes the wavelength from λ to $\dfrac{\lambda}{n}$, where n is the refractive index of the liquid.

EXECUTE: $r(n) \approx \sqrt{\dfrac{(2m+1)\lambda R}{2n}} = \dfrac{r}{\sqrt{n}} = \dfrac{0.720\text{ mm}}{\sqrt{1.33}} = 0.624\text{ mm}.$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.39. **IDENTIFY** and **SET UP:** Consider the interference of the rays reflected from each side of the film. At the front of the film light in air reflects off the film $(n = 1.432)$ and there is a 180° phase shift. At the back of the film light in the film $(n = 1.432)$ reflects off the glass $(n = 1.62)$ and there is a 180° phase shift. Therefore, the reflections introduce no net phase shift. The path difference is 2*t*, where *t* is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{\lambda_{\text{air}}}{n}$.

EXECUTE: **(a)** Since there is no net phase difference produced by the reflections, the condition for destructive interference is $2t = (m + \tfrac{1}{2})\lambda.\ t = (m + \tfrac{1}{2})\dfrac{\lambda}{2}$ and the minimum thickness is $t = \dfrac{\lambda}{4} = \dfrac{\lambda_{\text{air}}}{4n} = \dfrac{550\text{ nm}}{4(1.432)} = 96.0\text{ nm}$.

(b) For destructive interference, $2t = (m + \tfrac{1}{2})\dfrac{\lambda_{\text{air}}}{n}$ and $\lambda_{\text{air}} = \dfrac{2tn}{m + \frac{1}{2}} = \dfrac{275\text{ nm}}{m + \frac{1}{2}}$. $m = 0$: $\lambda_{\text{air}} = 550\text{ nm}$.

$m=1$: $\lambda_{\text{air}}=183$ nm. All other λ_{air} values are shorter. For constructive interference, $2t=m\dfrac{\lambda_{\text{air}}}{n}$ and $\lambda_{\text{air}}=\dfrac{2tn}{m}=\dfrac{275\text{ nm}}{m}$. For $m=1$, $\lambda_{\text{air}}=275$ nm and all other λ_{air} values are shorter.

EVALUATE: The only visible wavelength in air for which there is destructive interference is 550 nm. There are no visible wavelengths in air for which there is constructive interference.

35.40. **IDENTIFY** and **SET UP:** Consider reflection from either side of the film. **(a)** At the front of the film, light in air $(n=1.00)$ reflects off the film $(n=1.45)$ and there is a 180° phase shift. At the back of the film, light in the film $(n=1.45)$ reflects off the cornea $(n=1.38)$ and there is no phase shift. The reflections produce a net 180° phase difference so the condition for constructive interference is $2t=(m+\frac{1}{2})\lambda$, where $\lambda=\dfrac{\lambda_{\text{air}}}{n}$. $t=(m+\frac{1}{2})\dfrac{\lambda_{\text{air}}}{2n}$.

EXECUTE: The minimum thickness is for $m=0$, and is given by $t=\dfrac{\lambda_{\text{air}}}{4n}=\dfrac{600\text{ nm}}{4(1.45)}=103\text{ nm}$ (103.4 nm with less rounding).

(b) $\lambda_{\text{air}}=\dfrac{2nt}{m+\frac{1}{2}}=\dfrac{2(1.45)(103.4\text{ nm})}{m+\frac{1}{2}}=\dfrac{300\text{ nm}}{m+\frac{1}{2}}$. For $m=0$, $\lambda_{\text{air}}=600$ nm. For $m=1$, $\lambda_{\text{air}}=200$ nm and all other values are smaller. No other visible wavelengths are reinforced. The condition for destructive interference is $2t=m\dfrac{\lambda_{\text{air}}}{n}$. $\lambda=\dfrac{2tn}{m}=\dfrac{300\text{ nm}}{m}$. For $m=1$, $\lambda_{\text{air}}=300$ nm and all other values are shorter. There are no visible wavelengths for which there is destructive interference.

(c) Now both rays have a 180° phase change on reflection and the reflections don't introduce any net phase shift. The expression for constructive interference in parts (a) and (b) now gives destructive interference and the expression in (a) and (b) for destructive interference now gives constructive interference. The only visible wavelength for which there will be destructive interference is 600 nm and there are no visible wavelengths for which there will be constructive interference.

EVALUATE: Changing the net phase shift due to the reflections can convert the interference for a particular thickness from constructive to destructive, and vice versa.

35.41. **IDENTIFY:** The insertion of the metal foil produces a wedge of air, which is an air film of varying thickness. This film causes a path difference between light reflected off the top and bottom of this film.

SET UP: The two sheets of glass are sketched in Figure 35.41. The thickness of the air wedge at a distance x from the line of contact is $t=x\tan\theta$. Consider rays 1 and 2 that are reflected from the top and bottom surfaces, respectively, of the air film. Ray 1 has no phase change when it reflects and ray 2 has a 180° phase change when it reflects, so the reflections introduce a net 180° phase difference. The path difference is $2t$ and the wavelength in the film is $\lambda=\lambda_{\text{air}}$.

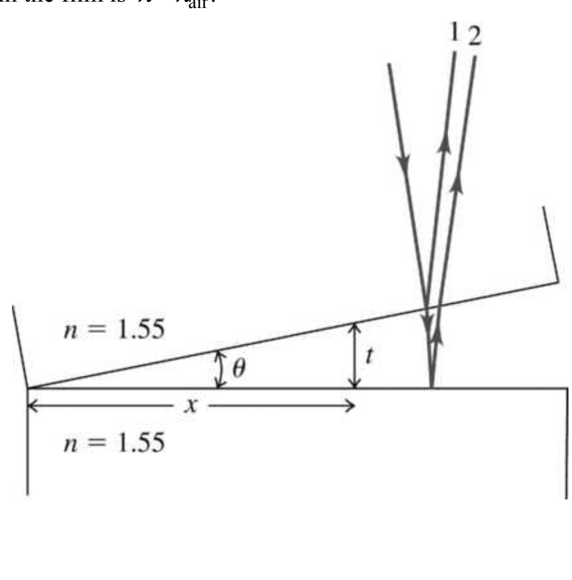


Figure 35.41

EXECUTE: **(a)** Since there is a 180° phase difference from the reflections, the condition for constructive interference is $2t = (m+\frac{1}{2})\lambda$. The positions of first enhancement correspond to $m = 0$ and $2t = \dfrac{\lambda}{2}$.

$x\tan\theta = \dfrac{\lambda}{4}$. θ is a constant, so $\dfrac{x_1}{\lambda_1} = \dfrac{x_2}{\lambda_2}$. $x_1 = 1.15$ mm, $\lambda_1 = 400.0$ nm. $x_2 = x_1\left(\dfrac{\lambda_2}{\lambda_1}\right)$. For

$\lambda_2 = 550$ nm (green), $x_2 = (1.15 \text{ mm})\left(\dfrac{550 \text{ nm}}{400 \text{ nm}}\right) = 1.58$ mm. For $\lambda_2 = 600$ nm (orange),

$x_2 = (1.15 \text{ mm})\left(\dfrac{600 \text{ nm}}{400 \text{ nm}}\right) = 1.72$ mm.

(b) The positions of next enhancement correspond to $m = 1$ and $2t = \dfrac{3\lambda}{2}$. $x\tan\theta = \dfrac{3\lambda}{4}$. The values of x are 3 times what they are in part (a). Violet: 3.45 mm; green: 4.74 mm; orange: 5.16 mm.

(c) $\tan\theta = \dfrac{\lambda}{4x} = \dfrac{400.0\times10^{-9} \text{ m}}{4(1.15\times10^{-3} \text{ m})} = 8.70\times10^{-5}$. $\tan\theta = \dfrac{t_{\text{foil}}}{11.0 \text{ cm}}$, so $t_{\text{foil}} = 9.57\times10^{-4}$ cm $= 9.57$ μm.

EVALUATE: The thickness of the foil must be very small to cause these observable interference effects. If it is too thick, the film is no longer a "thin film."

35.42. **IDENTIFY** and **SET UP:** Figure 35.41 for Problem 35.41 also applies in this case, but now the wedge is jelly instead of air and $\lambda = \dfrac{\lambda_{\text{air}}}{n}$. Ray 1 has a 180° phase shift upon reflection and ray 2 has no phase change. As in Problem 35.41, the reflections introduce a net 180° phase difference. Since the reflections introduce a net 180° phase difference, the condition for destructive interference is $2t = m\dfrac{\lambda_{\text{air}}}{n}$.

EXECUTE: $2t = m\dfrac{\lambda_{\text{air}}}{n}$. $t = x\tan\theta$ so $x = m\dfrac{\lambda_{\text{air}}}{2n\tan\theta}$. The separation Δx between adjacent dark fringes is

$\Delta x = \dfrac{\lambda_{\text{air}}}{2n\tan\theta}$ and $n = \dfrac{\lambda_{\text{air}}}{2(\Delta x)\tan\theta}$. $\Delta x = \dfrac{6.33 \text{ mm}}{10} = 0.633$ mm. $\tan\theta = \dfrac{0.015\times10^{-3} \text{ m}}{8.00\times10^{-2} \text{ m}} = 1.875\times10^{-4}$.

$$n = \frac{525\times10^{-9} \text{ m}}{2(0.633\times10^{-3} \text{ m})(1.875\times10^{-4})} = 2.21.$$

EVALUATE: $n > 1$, as it must be, and $n = 2.21$ is not unreasonable for jelly.

35.43. **IDENTIFY:** The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d\sin\theta = (m+\frac{1}{2})\lambda$. For a liquid with refractive index n,

$\lambda_{\text{liq}} = \dfrac{\lambda_{\text{air}}}{n}$.

EXECUTE: $\dfrac{\sin\theta}{\lambda} = \dfrac{(m+\frac{1}{2})}{d} =$ constant, so $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and

$$n = \frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \frac{\sin 35.20°}{\sin 19.46°} = 1.730.$$

EVALUATE: In the liquid the wavelength is shorter and $\sin\theta = (m+\frac{1}{2})\dfrac{\lambda}{d}$ gives a smaller θ than in air, for the same m.

35.44. **IDENTIFY:** As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d\sin\theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0\, dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at $d\sin\theta = \lambda/2 \Rightarrow \sin\theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin\theta = \lambda/2w$

Taking differentials gives $d(\sin\theta) = d(\lambda/2w)$ and $\cos\theta\, d\theta = -\lambda/2\; dw/w^2$. For thermal expansion, $dw = \alpha w_0 dT$, which gives $\cos\theta\, d\theta = -\dfrac{\lambda}{2}\dfrac{\alpha w_0 dT}{w_0^2} = -\dfrac{\lambda\alpha dT}{2w_0}$. Solving for $d\theta$ gives $d\theta = -\dfrac{\lambda\alpha dT}{2w_0\cos\theta_0}$.

Get λ: $w_0 \sin\theta_0 = \lambda/2 \to \lambda = 2w_0\sin\theta_0$. Substituting this quantity into the equation for $d\theta$ gives

$$d\theta = -\frac{2w_0\sin\theta_0\,\alpha dT}{2w_0\cos\theta_0} = -\tan\theta_0\,\alpha dT.$$

$$d\theta = -\tan(32.5°)(2.0\times10^{-5}\ \text{K}^{-1})(115\ \text{K}) = -0.001465\ \text{rad} = -0.084°$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin\theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.45. **IDENTIFY:** *Both* frequencies will interfere constructively when the path difference from both of them is an integral number of wavelengths.

SET UP: Constructive interference occurs when $\sin\theta = m\lambda/d$.

EXECUTE: First find the two wavelengths.

$$\lambda_1 = v/f_1 = (344\ \text{m/s})/(900\ \text{Hz}) = 0.3822\ \text{m}$$

$$\lambda_2 = v/f_2 = (344\ \text{m/s})/(1200\ \text{Hz}) = 0.2867\ \text{m}$$

To interfere constructively at the same angle, the angles must be the same, and hence the sines of the angles must be equal. Each sine is of the form $\sin\,\theta = m\lambda/d$, so we can equate the sines to get

$$m_1\lambda_1/d = m_2\lambda_2/d$$
$$m_1(0.3822\ \text{m}) = m_2(0.2867\ \text{m})$$
$$m_2 = 4/3\ m_1$$

Since both m_1 and m_2 must be integers, the allowed pairs of values of m_1 and m_2 are

$$m_1 = m_2 = 0$$
$$m_1 = 3,\ m_2 = 4$$
$$m_1 = 6,\ m_2 = 8$$
$$m_1 = 9,\ m_2 = 12$$
etc.

For $m_1 = m_2 = 0$, we have $\theta = 0$,

For $m_1 = 3$, $m_2 = 4$, we have $\sin\theta_1 = (3)(0.3822\ \text{m})/(2.50\ \text{m})$, giving $\theta_1 = 27.3°$.

For $m_1 = 6$, $m_2 = 8$, we have $\sin\theta_1 = (6)(0.3822\ \text{m})/(2.50\ \text{m})$, giving $\theta_1 = 66.5°$.

For $m_1 = 9$, $m_2 = 12$, we have $\sin\theta_1 = (9)(0.3822\ \text{m})/(2.50\ \text{m}) = 1.38 > 1$, so no angle is possible.

EVALUATE: At certain other angles, one frequency will interfere constructively, but the other will not.

35.46. **IDENTIFY:** For destructive interference, $d = r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda.$

SET UP: $r_2 - r_1 = \sqrt{(200\ \text{m})^2 + x^2} - x$

EXECUTE: $(200\ \text{m})^2 + x^2 = x^2 + \left[\left(m + \frac{1}{2}\right)\lambda\right]^2 + 2x\left(m + \frac{1}{2}\right)\lambda.$

$x = \dfrac{20{,}000\ \text{m}^2}{\left(m + \frac{1}{2}\right)\lambda} - \dfrac{1}{2}\left(m + \dfrac{1}{2}\right)\lambda.$ The wavelength is calculated by $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\ \text{m/s}}{5.80\times10^6\ \text{Hz}} = 51.7\ \text{m}.$

$m = 0: x = 761\ \text{m}$; $m = 1: x = 219\ \text{m}$; $m = 2: x = 90.1\ \text{m}$; $m = 3; x = 20.0\ \text{m}.$

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181\ \text{m}.$ The maximum possible path difference is the separation of 200 m between the sources.

35.47. **IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However, the light is traveling through a medium (plastic) that is different from air.

SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d\ \sin\theta = ½\ \lambda/n,$ where n is the refractive index of the plastic.

EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan\theta = [(5.82\text{ mm})/2]/(3250\text{ mm})$, giving $\theta = \pm 0.0513°$.

For destructive interference, we have $d\ \sin\theta = ½\ \lambda/n$ and

$$n = \lambda/(2d\sin\theta) = (632.8\text{ nm})/[2(0.000225\text{ m})(\sin\ 0.0513°)] = 1.57$$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

35.48. **IDENTIFY:** Interference occurs due to the path difference of light in the thin film.

SET UP: Originally the path difference was an odd number of half-wavelengths for cancellation to occur. If the path difference decreases by ½ wavelength, it will be a multiple of the wavelength, so constructive interference will occur.

EXECUTE: Calling ΔT the thickness that must be removed, we have

$$\text{path difference} = 2\Delta T = ½\ \lambda/n \text{ and } \Delta T = \lambda/4n = (525\text{ nm})/[4(1.40)] = 93.75\text{ nm}$$

At 4.20 nm/yr, we have $(4.20\text{ nm/yr})t = 93.75$ nm and $t = 22.3$ yr.

EVALUATE: If you were giving a warranty on this film, you certainly could not give it a "lifetime guarantee"!

35.49. **IDENTIFY:** For destructive interference the net phase difference must be $180°$, which is one-half a period, or $\lambda/2$. Part of this phase difference is due to the fact that the speakers are ¼ of a period out of phase, and the rest is due to the path difference between the sound from the two speakers.

SET UP: The phase of A is 90° or, $\lambda/4$, ahead of B. At points above the centerline, points are closer to A than to B and the signal from A gains phase relative to B because of the path difference. Destructive interference will occur when $d\sin\theta = (m+\frac{1}{4})\lambda,\ m = 0, 1, 2, \ldots$. At points at an angle θ below the centerline, the signal from B gains phase relative to A because of the phase difference. Destructive interference will occur when $d\sin\theta = (m+\frac{3}{4})\lambda, m = 0, 1, 2, \ldots$. $\lambda = \dfrac{v}{f}$.

EXECUTE: $\lambda = \dfrac{340\text{ m/s}}{444\text{ Hz}} = 0.766$ m.

Points above the centerline: $\sin\theta = (m+\frac{1}{4})\dfrac{\lambda}{d} = (m+\frac{1}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{1}{4})$. $m = 0$: $\theta = 3.14°$; $m = 1$: $\theta = 15.9°$; $m = 2$: $\theta = 29.5°$; $m = 3$: $\theta = 45.4°$; $m = 4$: $\theta = 68.6°$.

Points below the centerline: $\sin\theta = (m+\frac{3}{4})\dfrac{\lambda}{d} = (m+\frac{3}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{3}{4})$. $m = 0$: $\theta = 9.45°$; $m = 1$: $\theta = 22.5°$; $m = 2$: $\theta = 37.0°$; $m = 3$: $\theta = 55.2°$.

EVALUATE: It is *not* always true that the path difference for destructive interference must be $(m+\frac{1}{2})\lambda$, but it *is* always true that the phase difference must be 180° (or odd multiples of 180°).

35.50. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: Use $\cos(\omega t+\phi/2) = \cos(\omega t)\cos(\phi/2) - \sin(\omega t)\sin(\phi/2)$. Then $2\cos(\phi/2)\cos(\omega t+\phi/2) = 2\cos(\omega t)\cos^2(\phi/2) - 2\sin(\omega t)\sin(\phi/2)\cos(\phi/2)$. Then use $\cos^2(\phi/2) = \dfrac{1+\cos(\phi)}{2}$ and $2\sin(\phi/2)\cos(\phi/2) = \sin\phi$. This gives $\cos(\omega t) + (\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)) = \cos(\omega t) + \cos(\omega t+\phi)$, using again the trig identity for the cosine of the sum of two angles.

EXECUTE: **(a)** The electric field is the sum of the two fields and can be written as $E_P(t) = E_2(t) + E_1(t) = E\cos(\omega t) + E\cos(\omega t+\phi)$. $E_P(t) = 2E\cos(\phi/2)\cos(\omega t+\phi/2)$.

(b) $E_p(t) = A\cos(\omega t + \phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A = 2E|\cos(\phi/2)|$.

(c) To have an interference maximum, $\dfrac{\phi}{2} = 2\pi m$. So, for example, using $m = 1$, the relative phases are

E_2: 0; E_1: $\phi = 4\pi$; E_p: $\dfrac{\phi}{2} = 2\pi$, and all waves are in phase.

(d) To have an interference minimum, $\dfrac{\phi}{2} = \pi\left(m + \dfrac{1}{2}\right)$. So, for example using $m = 0$, relative phases are

E_2: 0; E_1: $\phi = \pi$; E_p: $\phi/2 = \pi/2$, and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.

(e) The instantaneous magnitude of the Poynting vector is

$$|\vec{S}| = \varepsilon_0 c E_p^2(t) = \varepsilon_0 c(4E^2 \cos^2(\phi/2)\cos^2(\omega t + \phi/2)).$$

For a time average, $\cos^2(\omega t + \phi/2) = \dfrac{1}{2}$, so $|S_{\text{av}}| = 2\varepsilon_0 c E^2 \cos^2(\phi/2)$.

EVALUATE: The result of part (e) shows that the intensity at a point depends on the phase difference ϕ at that point for the waves from each source.

35.51. **IDENTIFY** and **SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.

EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.

At $20.0°\text{C}$, $t_0 = (582.4\text{ nm})/[(2)(1.750)] = 166.4\text{ nm}$.

At $170°\text{C}$, $t = (588.5\text{ nm})/[(2)(1.750)] = 168.1\text{ nm}$.

$t = t_0(1 + \alpha\Delta T)$ so

$\alpha = (t - t_0)/(t_0\Delta T) = (1.7\text{ nm})/[(166.4\text{ nm})(150\text{C}°)] = 6.8\times10^{-5}(\text{C}°)^{-1}$

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$.The value we calculated for α is the same order of magnitude as those given in Table 17.1.

35.52. **IDENTIFY:** The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.

SET UP: For constructive interference, $\sin\theta = m\lambda/d$.

EXECUTE: **(a)** First find the wavelength of the UHF waves:

$$\lambda = c/f = (3.00\times10^8\text{ m/s})/(1575.42\text{ MHz}) = 0.1904\text{ m}$$

For maximum intensity $(\pi d\ \sin\theta)/\lambda = m\pi$, so

$$\sin\theta = m\lambda/d = m[(0.1904\text{ m})/(5.18\text{ m})] = 0.03676m$$

The maximum possible m would be for $\theta = 90°$, or $\sin\theta = 1$, so

$$m_{\text{max}} = d/\lambda = (5.18\text{ m})/(0.1904\text{ m}) = 27.2$$

which must be ±27 since m is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of $27 + 27 + 1 = 55$ bright fringes.

(b) Using $\sin\ \theta = m\lambda/d$, where $m = 0, \pm1, \pm2, \text{and} \pm3$, we have

$$\sin\theta = m\lambda/d = m[(0.1904\text{ m})/(5.18\text{ m})] = 0.03676m$$

$$m = \theta:\ \ \sin\theta = 0, \text{ which gives } \theta = 0°$$

$$m = \pm1: \sin\theta = \pm(0.03676)(1), \text{ which gives } \theta = \pm2.11°$$

$$m = \pm2: \sin\theta = \pm(0.03676)(2), \text{ which gives } \theta = \pm4.22°$$

$$m = \pm3: \sin\theta = \pm(0.03676)(3), \text{ which gives } \theta = \pm6.33°$$

(c) $I = I_0 \cos^2\left(\dfrac{\pi d \sin\theta}{\lambda}\right) = (2.00\,\text{W/m}^2)\cos^2\left[\dfrac{\pi(5.18\text{ m})\sin(4.65°)}{0.1904\text{ m}}\right] = 1.28\text{ W/m}^2.$

EVALUATE: Notice that $\sin\theta$ increases in integer steps, but θ only increases in integer steps for small θ.

35.53. **IDENTIFY:** Apply $I = I_0 \cos\left(\dfrac{\pi d}{\lambda}\sin\theta\right)$.

SET UP: $I = I_0/2$ when $\dfrac{\pi d}{\lambda}\sin\theta$ is $\dfrac{\pi}{4}$ rad, $\dfrac{3\pi}{4}$ rad,….

EXECUTE: First we need to find the angles at which the intensity drops by one-half from the value of the m^{th} bright fringe. $I = I_0\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right) = \dfrac{I_0}{2} \Rightarrow \dfrac{\pi d}{\lambda}\sin\theta \approx \dfrac{\pi d\theta_m}{\lambda} = (m+1/2)\dfrac{\pi}{2}.$

$m = 0 : \theta = \theta_m^- = \dfrac{\lambda}{4d}$; $m = 1 : \theta = \theta_m^+ = \dfrac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \dfrac{\lambda}{2d}.$

EVALUATE: There is no dependence on the *m*-value of the fringe, so all fringes at small angles have the same half-width.

35.54. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.

SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference $2t = (m_1 + \tfrac{1}{2})\lambda_1 = (m_2 + \tfrac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m-value, $m_2 = m_1 - 1$.

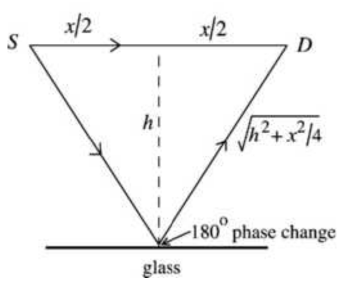
EXECUTE: $\left(m_1 + \dfrac{1}{2}\right)\lambda_1 = \left(m_1 - \dfrac{1}{2}\right)\lambda_2 \Rightarrow m_1(\lambda_2 - \lambda_1) = \dfrac{\lambda_1 + \lambda_2}{2} \Rightarrow m_1 = \dfrac{\lambda_1 + \lambda_2}{2(\lambda_2 - \lambda_1)}.$

$m_1 = \dfrac{477.0\text{ nm} + 540.6\text{ nm}}{2(540.6\text{ nm} - 477.0\text{ nm})} = 8.$ $2t = \left(8 + \dfrac{1}{2}\right)\dfrac{\lambda_{01}}{n} \Rightarrow t = \dfrac{17(477.0\text{ nm})}{4(1.52)} = 1334\text{ nm}.$

EVALUATE: Now that we have *t* we can calculate all the other wavelengths for which there is constructive interference.

35.55. **IDENTIFY:** Consider the phase difference due to the path difference and due to the reflection of one ray from the glass surface.

(a) SET UP: Consider Figure 35.55.



path difference $=$

$2\sqrt{h^2 + x^2/4} - x = \sqrt{4h^2 + x^2} - x$

Figure 35.55

Since there is a $180°$ phase change for the reflected ray, the condition for constructive interference is path difference $= \left(m + \dfrac{1}{2}\right)\lambda$ and the condition for destructive interference is path difference $= m\lambda$.

(b) EXECUTE: Constructive interference: $\left(m + \dfrac{1}{2}\right)\lambda = \sqrt{4h^2 + x^2} - x$ and $\lambda = \dfrac{\sqrt{4h^2 + x^2} - x}{m + \dfrac{1}{2}}$. Longest λ is for $m = 0$ and then $\lambda = 2\left(\sqrt{4h^2 + x^2} - x\right) = 2\left(\sqrt{4(0.24\text{ m})^2 + (0.14\text{ m})^2} - 0.14\text{ m}\right) = 0.72\text{ m}$

EVALUATE: For $\lambda = 0.72$ m the path difference is $\lambda/2$.

35.56. **IDENTIFY:** Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_{\text{g}} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{\text{g}}} \Rightarrow \lambda = \frac{2t_{\text{g}}n_{\text{g}}}{(m+\frac{1}{2})} = \frac{2(74 \text{ nm})(1.80)}{(m+\frac{1}{2})} = \frac{266 \text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533 \text{ nm } (m=0).$$

For the cytoplasm layers:

$$2t_{\text{c}} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{\text{c}}} \Rightarrow \lambda = \frac{2t_{\text{c}}n_{\text{c}}}{(m+\frac{1}{2})} = \frac{2(100 \text{ nm})(1.333)}{(m+\frac{1}{2})} = \frac{267 \text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533 \text{ nm } (m=0).$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\frac{100}{74} = \frac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.57. **IDENTIFY:** The slits will produce an interference pattern, but in the liquid, the wavelength of the light will be less than it was in air.

SET UP: The first bright fringe occurs when $d \sin\theta = \lambda/n$.

EXECUTE: In air: $d\sin 18.0° = \lambda$. In the liquid: $d\sin 12.6° = \lambda/n$. Dividing the equations gives

$$n = (\sin 18.0°)/(\sin 12.6°) = 1.42$$

EVALUATE: It was not necessary to know the spacing of the slits, since it was the same in both air and the liquid.

35.58. **IDENTIFY** and **SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.

EXECUTE: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700 \text{ nm}) = 2100 \text{ nm}$. For destructive interference: $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \frac{d\sin\theta}{m+\frac{1}{2}} = \frac{2100 \text{ nm}}{m+\frac{1}{2}}$. So the possible wavelengths are

$\lambda_2 = 600$ nm, for $m = 3$, and $\lambda_2 = 467$ nm, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.59. **(a) IDENTIFY:** The wavelength in the glass is decreased by a factor of $1/n$, so for light through the upper slit a shorter path is needed to produce the same phase at the screen. Therefore, the interference pattern is shifted downward on the screen.

(b) SET UP: Consider the total phase difference produced by the path length difference and also by the different wavelength in the glass.

EXECUTE: At a point on the screen located by the angle θ the difference in path length is $d\sin\theta$. This introduces a phase difference of $\phi = \left(\frac{2\pi}{\lambda_0}\right)(d\sin\theta)$, where λ_0 is the wavelength of the light in air or vacuum. In the thickness L of glass the number of wavelengths is $\frac{L}{\lambda} = \frac{nL}{\lambda_0}$. A corresponding length L of the path of the ray through the lower slit, in air, contains L/λ_0 wavelengths. The phase difference this introduces is $\phi = 2\pi\left(\frac{nL}{\lambda_0} - \frac{L}{\lambda_0}\right)$ and $\phi = 2\pi(n-1)(L/\lambda_0)$. The total phase difference is the sum of these two, $\left(\dfrac{2\pi}{\lambda_0}\right)(d\sin\theta) + 2\pi(n-1)(L/\lambda_0) = (2\pi/\lambda_0)(d\sin\theta + L(n-1))$. Eq. (35.10) then gives

$$I = I_0\cos^2\left[\left(\frac{\pi}{\lambda_0}\right)(d\sin\theta + L(n-1))\right].$$

(c) Maxima means $\cos\phi/2 = \pm 1$ and $\phi/2 = m\pi, m = 0, \pm 1, \pm 2, \ldots$ $(\pi/\lambda_0)(d\sin\theta + L(n-1)) = m\pi$

$$d\sin\theta + L(n-1) = m\lambda_0$$

$$\sin\theta = \frac{m\lambda_0 - L(n-1)}{d}$$

EVALUATE: When $L \to 0$ or $n \to 1$ the effect of the plate goes away and the maxima are located by Eq. (35.4).

35.60. **IDENTIFY:** Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d\sin\theta = \lambda/2$. The intensity at any angle θ is given by

$$I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right).$$

EXECUTE: **(a)** At the first dark fringe, we have $d\sin\theta = \lambda/2$. $d/\lambda = 1/(2\sin 19.0°) = 1.54$.

(b) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{I_0}{10} \Rightarrow \cos\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{1}{\sqrt{10}}$. $\dfrac{\pi d\sin\theta}{\lambda} = \arccos\left(\dfrac{1}{\sqrt{10}}\right) = 71.57° = 1.249$ rad.

Using the result from part (a), that $d/\lambda = 1.54$, we have

$\pi(1.54)\sin\theta = 1.249$. $\sin\theta = 0.2589$, so $\theta = \pm 15.0°$.

EVALUATE: Since the first dark fringes occur at $\pm 19.0°$, it is reasonable that at 15° the intensity is reduced to only 1/10 of its maximum central value.

35.61. **IDENTIFY:** There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda = \dfrac{\lambda_0}{n}$ and $\Delta n = n_0(2.50\times 10^{-5}\ (\text{C}°)^{-1})\Delta T$. $\Delta L = L_0(5.00\times 10^{-6}\ (\text{C}°)^{-1})\Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by: $\Delta N_1 = \dfrac{2n_{\text{glass}}\Delta L}{\lambda_0} - \dfrac{2n_{\text{air}}\Delta L}{\lambda_0} = \dfrac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$ and

$$\Delta N_1 = \frac{2(1.48-1)(0.030\text{ m})(5.00\times 10^{-6}/\text{C}°)(5.00\text{ C}°)}{5.89\times 10^{-7}\text{ m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}}L_0}{\lambda_0} = \frac{2(2.50\times 10^{-5}/\text{C}°)(5.00\text{ C}°/\text{min})(1.00\text{ min})(0.0300\text{ m})}{5.89\times 10^{-7}\text{m}} = 12.73.$$

So, the total change in the number of wavelengths as the rod expands is

$\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.62. **IDENTIFY:** Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent sources so the fringe spacing is $\Delta y = \dfrac{R\lambda}{d}$, where d is the distance between S_1 and S_2.

SET UP: For small angles, $\sin\theta \approx \theta$, with θ expressed in radians.

EXECUTE: **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source S_0, we can calculate the height of one image above the source:

$$\frac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1).$$

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m} + 0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.