18

THERMAL PROPERTIES OF MATTER

18.1. **(a) IDENTIFY:** We are asked about a single state of the system.

SET UP: Use Eq. (18.2) to calculate the number of moles and then apply the ideal-gas equation.

EXECUTE: $n = \dfrac{m_{tot}}{M} = \dfrac{4.86 \times 10^{-4}\ \text{kg}}{4.00 \times 10^{-3}\ \text{kg/mol}} = 0.122\ \text{mol}.$

(b) $pV = nRT$ implies $p = nRT/V.$ T must be in kelvins, so $T = (18 + 273)\ \text{K} = 291\ \text{K}.$

$p = \dfrac{(0.122\ \text{mol})(8.3145\ \text{J/mol} \cdot \text{K})(291\ \text{K})}{20.0 \times 10^{-3}\ \text{m}^3} = 1.47 \times 10^4\ \text{Pa}.$

$p = (1.47 \times 10^4\ \text{Pa})(1.00\ \text{atm}/1.013 \times 10^5\ \text{Pa}) = 0.145\ \text{atm}.$

EVALUATE: The tank contains about 1/10 mole of He at around standard temperature, so a pressure around 1/10 atmosphere is reasonable.

18.2. **IDENTIFY:** $pV = nRT.$

SET UP: $T_1 = 41.0°\text{C} = 314\ \text{K}.$ $R = 0.08206\ \text{L} \cdot \text{atm/mol} \cdot \text{K}.$

EXECUTE: n and R are constant so $\dfrac{pV}{T} = nR$ is constant. $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}.$

$T_2 = T_1 \left(\dfrac{p_2}{p_1} \right)\left(\dfrac{V_2}{V_1} \right) = (314\ \text{K})(2)(2) = 1.256 \times 10^3\ \text{K} = 983°\text{C}.$

(b) $n = \dfrac{pV}{RT} = \dfrac{(0.180\ \text{atm})(2.60\ \text{L})}{(0.08206\ \text{L} \cdot \text{atm/mol} \cdot \text{K})(314\ \text{K})} = 0.01816\ \text{mol}.$

$m_{tot} = nM = (0.01816\ \text{mol})(4.00\ \text{g/mol}) = 0.0727\ \text{g}.$

EVALUATE: T is directly proportional to p and to $V,$ so when p and V are each doubled the Kelvin temperature increases by a factor of 4.

18.3. **IDENTIFY:** $pV = nRT.$

SET UP: T is constant.

EXECUTE: nRT is constant so $p_1 V_1 = p_2 V_2.$

$p_2 = p_1 \left(\dfrac{V_1}{V_2} \right) = (0.355\ \text{atm})\left(\dfrac{0.110\ \text{m}^3}{0.390\ \text{m}^3} \right) = 0.100\ \text{atm}.$

EVALUATE: For T constant, p decreases as V increases.

18.4. **IDENTIFY:** $pV = nRT.$

SET UP: $T_1 = 20.0°\text{C} = 293\ \text{K}.$

EXECUTE: **(a)** $n,$ R and V are constant. $\dfrac{p}{T} = \dfrac{nR}{V} = \text{constant}.$ $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}.$

$T_2 = T_1 \left(\dfrac{p_2}{p_1} \right) = (293\ \text{K})\left(\dfrac{1.00\ \text{atm}}{3.00\ \text{atm}} \right) = 97.7\ \text{K} = -175°\text{C}.$

(b) $p_2 = 1.00$ atm, $V_2 = 3.00$ L. $p_3 = 3.00$ atm. n, R and T are constant so $pV = nRT = $ constant.

$p_2 V_2 = p_3 V_3$.

$V_3 = V_2 \left(\dfrac{p_2}{p_3} \right) = (3.00 \text{ L}) \left(\dfrac{1.00 \text{ atm}}{3.00 \text{ atm}} \right) = 1.00$ L.

EVALUATE: The final volume is one-third the initial volume. The initial and final pressures are the same, but the final temperature is one-third the initial temperature.

18.5. **IDENTIFY:** We know the pressure and temperature and want to find the density of the gas. The ideal gas law applies.

SET UP: $M_{CO_2} = (12 + 2[16])$ g/mol $= 44$ g/mol. $M_{N_2} = 28$ g/mol. $\rho = \dfrac{pM}{RT}$.

$R = 8.315$ J/mol·K. T must be in kelvins. Express M in kg/mol and p in Pa. 1 atm $= 1.013 \times 10^5$ Pa.

EXECUTE: **(a)** *Mars*: $\rho = \dfrac{(650 \text{ Pa})(44 \times 10^{-3} \text{ kg/mol})}{(8.315 \text{ J/mol·K})(253 \text{ K})} = 0.0136$ kg/m³.

Venus: $\rho = \dfrac{(92 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(44 \times 10^{-3} \text{ kg/mol})}{(8.315 \text{ J/mol·K})(730 \text{ K})} = 67.6$ kg/m³.

Titan: $T = -178 + 273 = 95$ K.

$\rho = \dfrac{(1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(28 \times 10^{-3} \text{ kg/mol})}{(8.315 \text{ J/mol·K})(95 \text{ K})} = 5.39$ kg/m³.

EVALUATE: **(b)** Table 12.1 gives the density of air at 20°C and $p = 1$ atm to be 1.20 kg/m³. The density of the atmosphere of Mars is much less, the density for Venus is much greater and the density for Titan is somewhat greater.

18.6. **IDENTIFY:** $pV = nRT$ and the mass of the gas is $m_{tot} = nM$.

SET UP: The temperature is $T = 22.0°C = 295.15$ K. The average molar mass of air is

$M = 28.8 \times 10^{-3}$ kg/mol. For helium $M = 4.00 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $m_{tot} = nM = \dfrac{pV}{RT}M = \dfrac{(1.00 \text{ atm})(0.900 \text{ L})(28.8 \times 10^{-3} \text{ kg/mol})}{(0.08206 \text{ L·atm/mol·K})(295.15 \text{ K})} = 1.07 \times 10^{-3}$ kg.

(b) $m_{tot} = nM = \dfrac{pV}{RT}M = \dfrac{(1.00 \text{ atm})(0.900 \text{ L})(4.00 \times 10^{-3} \text{ kg/mol})}{(0.08206 \text{ L·atm/mol·K})(295.15 \text{ K})} = 1.49 \times 10^{-4}$ kg.

EVALUATE: $n = \dfrac{N}{N_A} = \dfrac{pV}{RT}$ says that in each case the balloon contains the same number of molecules.

The mass is greater for air since the mass of one molecule is greater than for helium.

18.7. **IDENTIFY:** We are asked to compare two states. Use the ideal gas law to obtain T_2 in terms of T_1 and ratios of pressures and volumes of the gas in the two states.

SET UP: $pV = nRT$ and n, R constant implies $pV/T = nR = $ constant and $p_1 V_1 / T_1 = p_2 V_2 / T_2$

EXECUTE: $T_1 = (27 + 273)$ K $= 300$ K

$p_1 = 1.01 \times 10^5$ Pa

$p_2 = 2.72 \times 10^6$ Pa $+ 1.01 \times 10^5$ Pa $= 2.82 \times 10^6$ Pa (in the ideal gas equation the pressures must be absolute, not gauge, pressures)

$T_2 = T_1 \left(\dfrac{p_2}{p_1} \right) \left(\dfrac{V_2}{V_1} \right) = 300 \text{ K} \left(\dfrac{2.82 \times 10^6 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right) \left(\dfrac{46.2 \text{ cm}^3}{499 \text{ cm}^3} \right) = 776$ K

$T_2 = (776 - 273)°C = 503°C$

EVALUATE: The units cancel in the V_2/V_1 volume ratio, so it was not necessary to convert the volumes in cm³ to m³. It was essential, however, to use T in kelvins.

18.8.   **IDENTIFY:** $pV = nRT$ and $m = nM$.

SET UP: We must use absolute pressure in $pV = nRT$. $p_1 = 4.01 \times 10^5$ Pa, $p_2 = 2.81 \times 10^5$ Pa.

$T_1 = 310$ K, $T_2 = 295$ K.

EXECUTE: **(a)** $n_1 = \dfrac{p_1 V_1}{RT_1} = \dfrac{(4.01 \times 10^5 \text{ Pa})(0.075 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(310 \text{ K})} = 11.7$ mol.

$m = nM = (11.7 \text{ mol})(32.0 \text{ g/mol}) = 374$ g.

(b) $n_2 = \dfrac{p_2 V_2}{RT_2} = \dfrac{(2.81 \times 10^5 \text{ Pa})(0.075 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(295 \text{ K})} = 8.59$ mol. $m = 275$ g.

The mass that has leaked out is $374 \text{ g} - 275 \text{ g} = 99$ g.

EVALUATE: In the ideal gas law we must use absolute pressure, expressed in Pa, and T must be in kelvins.

18.9.   **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 300$ K, $T_2 = 430$ K.

EXECUTE: **(a)** n, R are constant so $\dfrac{pV}{T} = nR = \text{constant}$. $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$.

$p_2 = p_1 \left(\dfrac{V_1}{V_2} \right) \left(\dfrac{T_2}{T_1} \right) = (7.50 \times 10^3 \text{ Pa}) \left(\dfrac{0.750 \text{ m}^3}{0.480 \text{ m}^3} \right) \left(\dfrac{430 \text{ K}}{300 \text{ K}} \right) = 1.68 \times 10^4$ Pa.

EVALUATE: Since the temperature increased while the volume decreased, the pressure must have increased. In $pV = nRT$, T must be in kelvins, even if we use a ratio of temperatures.

18.10.   **IDENTIFY:**   Use the ideal-gas equation to calculate the number of moles, n. The mass m_{total} of the gas is

$m_{\text{total}} = nM$.

SET UP: The volume of the cylinder is $V = \pi r^2 l$, where $r = 0.450$ m and $l = 1.50$ m.

$T = 22.0°\text{C} = 293.15$ K.   1 atm $= 1.013 \times 10^5$ Pa. $M = 32.0 \times 10^{-3}$ kg/mol. $R = 8.314$ J/mol · K.

EXECUTE: **(a)** $pV = nRT$ gives

$n = \dfrac{pV}{RT} = \dfrac{(21.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm}) \pi (0.450 \text{ m})^2 (1.50 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K})} = 827$ mol.

(b) $m_{\text{total}} = (827 \text{ mol})(32.0 \times 10^{-3} \text{ kg/mol}) = 26.5$ kg

EVALUATE: In the ideal-gas law, T must be in kelvins. Since we used R in units of J/mol · K we had to express p in units of Pa and V in units of m^3.

18.11.   **IDENTIFY:**   We are asked to compare two states. Use the ideal-gas law to obtain V_1 in terms of V_2 and the ratio of the temperatures in the two states.

SET UP: $pV = nRT$ and n, R, p are constant so $V/T = nR/p = \text{constant}$ and $V_1/T_1 = V_2/T_2$

EXECUTE: $T_1 = (19 + 273) \text{ K} = 292$ K   (T must be in kelvins)

$V_2 = V_1(T_2/T_1) = (0.600 \text{ L})(77.3 \text{ K}/292 \text{ K}) = 0.159$ L

EVALUATE: p is constant so the ideal-gas equation says that a decrease in T means a decrease in V.

18.12.   **IDENTIFY:**   Apply $pV = nRT$ and the van der Waals equation (Eq. 18.7) to calculate p.

SET UP: 400 cm^3 $= 400 \times 10^{-6}$ m^3. $R = 8.314$ J/mol · K.

EXECUTE: **(a)** The ideal gas law gives $p = nRT/V = 7.28 \times 10^6$ Pa while Eq. (18.7) gives 5.87×10^6 Pa.

(b) The van der Waals equation, which accounts for the attraction between molecules, gives a pressure that is 20% lower.

(c) The ideal gas law gives $p = 7.28 \times 10^5$ Pa. Eq. (18.7) gives $p = 7.13 \times 10^5$ Pa, for a 2.1% difference.

EVALUATE: **(d)** As n/V decreases, the formulas and the numerical values for the two equations approach each other.

18.13.   **IDENTIFY:**   We know the volume of the gas at STP on the earth and want to find the volume it would occupy on Venus where the pressure and temperature are much greater.

SET UP: STP is $T = 273$ K and $p = 1$ atm. Set up a ratio using $pV = nRT$ with nR constant.

$T_V = 1003 + 273 = 1276$ K.

EXECUTE: $pV = nRT$ gives $\dfrac{pV}{T} = nR = $ constant, so $\dfrac{p_E V_E}{T_E} = \dfrac{p_V V_V}{T_V}$.

$V_V = V_E \left(\dfrac{p_E}{p_V} \right) \left(\dfrac{T_V}{T_E} \right) = V \left(\dfrac{1 \text{ atm}}{92 \text{ atm}} \right) \left(\dfrac{1276 \text{ K}}{273 \text{ K}} \right) = 0.0508V$.

EVALUATE: Even though the temperature on Venus is higher than it is on Earth, the pressure there is much greater than on Earth, so the volume of the gas on Venus is only about 5% what it is on Earth.

18.14.   **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 277$ K. $T_2 = 296$ K. Assume the number of moles of gas in the bubble remains constant.

EXECUTE: **(a)** n, R are constant so $\dfrac{pV}{T} = nR = $ constant. $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$ and

$\dfrac{V_2}{V_1} = \left(\dfrac{p_1}{p_2} \right) \left(\dfrac{T_2}{T_1} \right) = \left(\dfrac{3.50 \text{ atm}}{1.00 \text{ atm}} \right) \left(\dfrac{296 \text{ K}}{277 \text{ K}} \right) = 3.74$.

(b) This increase in volume of air in the lungs would be dangerous.

EVALUATE: The large decrease in pressure results in a large increase in volume.

18.15.   **IDENTIFY:** We are asked to compare two states. First use $pV = nRT$ to calculate p_1. Then use it to obtain T_2 in terms of T_1 and the ratio of pressures in the two states.

(a) SET UP: $pV = nRT$. Find the initial pressure p_1.

EXECUTE: $p_1 = \dfrac{nRT_1}{V} = \dfrac{(11.0 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(23.0 + 273.15)\text{K}}{3.10 \times 10^{-3} \text{ m}^3} = 8.737 \times 10^6$ Pa

SET UP: $p_2 = 100$ atm$(1.013 \times 10^5$ Pa/1 atm$) = 1.013 \times 10^7$ Pa

$p/T = nR/V = $ constant, so $p_1/T_1 = p_2/T_2$

EXECUTE: $T_2 = T_1 \left(\dfrac{p_2}{p_1} \right) = (296.15 \text{ K}) \left(\dfrac{1.013 \times 10^7 \text{ Pa}}{8.737 \times 10^6 \text{ Pa}} \right) = 343.4$ K $= 70.2°$C

(b) EVALUATE: The coefficient of volume expansion for a gas is much larger than for a solid, so the expansion of the tank is negligible.

18.16.   **IDENTIFY:** $F = pA$ and $pV = nRT$

SET UP: For a cube, $V/A = L$.

EXECUTE: **(a)** The force of any side of the cube is $F = pA = (nRT/V)A = (nRT)/L$, since the ratio of area to volume is $A/V = 1/L$. For $T = 20.0°$C $= 293.15$ K,

$$F = \dfrac{nRT}{L} = \dfrac{(3 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})}{0.200 \text{ m}} = 3.66 \times 10^4 \text{ N}.$$

(b) For $T = 100.00°$C $= 373.15$ K,

$$F = \dfrac{nRT}{L} = \dfrac{(3 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(373.15 \text{ K})}{0.200 \text{ m}} = 4.65 \times 10^4 \text{ N}.$$

EVALUATE: When the temperature increases while the volume is kept constant, the pressure increases and therefore the force increases. The force increases by the factor T_2/T_1.

18.17.   **IDENTIFY:** Example 18.4 assumes a temperature of $0°$C at all altitudes and neglects the variation of g with elevation. With these approximations, $p = p_0 e^{-Mgy/RT}$.

SET UP: $\ln(e^{-x}) = -x$. For air, $M = 28.8 \times 10^{-3}$ kg/mol.

EXECUTE: We want y for $p = 0.90 p_0$ so $0.90 = e^{-Mgy/RT}$ and $y = -\dfrac{RT}{Mg} \ln(0.90) = 850$ m.

EVALUATE: This is a commonly occurring elevation, so our calculation shows that 10% variations in atmospheric pressure occur at many locations.

18.18. **IDENTIFY:** From Example 18.4, the pressure at elevation y above sea level is $p = p_0 e^{-Mgy/RT}$.

SET UP: The average molar mass of air is $M = 28.8 \times 10^{-3}$ kg/mol.

EXECUTE: At an altitude of 100 m, $\dfrac{Mgy_1}{RT} = \dfrac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(100 \text{ m})}{(8.3145 \text{ J/mol} \cdot \text{K})(273.15 \text{ K})} = 0.01243$, and the

percent decrease in pressure is $1 - p/p_0 = 1 - e^{-0.01243} = 0.0124 = 1.24\%$. At an altitude of 1000 m,

$Mgy_2/RT = 0.1243$ and the percent decrease in pressure is $1 - e^{-0.1243} = 0.117 = 11.7\%$.

EVALUATE: These answers differ by a factor of $(11.7\%)/(1.24\%) = 9.44$, which is less than 10 because the variation of pressure with altitude is exponential rather than linear.

18.19. **IDENTIFY:** We know the volume, pressure and temperature of the gas and want to find its mass and density.

SET UP: $V = 3.00 \times 10^{-3}$ m³. $T = 295$ K. $p = 2.03 \times 10^{-8}$ Pa. The ideal gas law, $pV = nRT$, applies.

EXECUTE: **(a)** $pV = nRT$ gives

$n = \dfrac{pV}{RT} = \dfrac{(2.03 \times 10^{-8} \text{ Pa})(3.00 \times 10^{-3} \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(295 \text{ K})} = 2.48 \times 10^{-14}$ mol. The mass of this amount of gas is

$m = nM = (2.48 \times 10^{-14} \text{ mol})(28.0 \times 10^{-3} \text{ kg/mol}) = 6.95 \times 10^{-16}$ kg.

(b) $\rho = \dfrac{m}{V} = \dfrac{6.95 \times 10^{-16} \text{ kg}}{3.00 \times 10^{-3} \text{ m}^3} = 2.32 \times 10^{-13}$ kg/m³.

EVALUATE: The density at this level of vacuum is 13 orders of magnitude less than the density of air at STP, which is 1.20 kg/m³.

18.20. **IDENTIFY:** $p = p_0 e^{-Mgy/RT}$ from Example 18.4 gives the variation of air pressure with altitude. The

density ρ of the air is $\rho = \dfrac{pM}{RT}$, so ρ is proportional to the pressure p. Let ρ_0 be the density at the

surface, where the pressure is p_0.

SET UP: From Example 18.4, $\dfrac{Mg}{RT} = \dfrac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)}{(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K})} = 1.244 \times 10^{-4}$ m⁻¹.

EXECUTE: $p = p_0 e^{-(1.244 \times 10^{-4} \text{ m}^{-1})(1.00 \times 10^3 \text{ m})} = 0.883 p_0$. $\dfrac{\rho}{p} = \dfrac{M}{RT} = $ constant, so $\dfrac{\rho}{p} = \dfrac{\rho_0}{p_0}$ and

$\rho = \rho_0 \left(\dfrac{p}{p_0} \right) = 0.883 \rho_0$.

The density at an altitude of 1.00 km is 88.3% of its value at the surface.

EVALUATE: If the temperature is assumed to be constant, then the decrease in pressure with increase in altitude corresponds to a decrease in density.

18.21. **IDENTIFY:** Use Eq. (18.5) and solve for p.
SET UP: $\rho = pM/RT$ and $p = RT\rho/M$

$T = (-56.5 + 273.15) \text{ K} = 216.6 \text{ K}$

For air $M = 28.8 \times 10^{-3}$ kg/mol (Example 18.3)

EXECUTE: $p = \dfrac{(8.3145 \text{ J/mol} \cdot \text{K})(216.6 \text{ K})(0.364 \text{ kg/m}^3)}{28.8 \times 10^{-3} \text{ kg/mol}} = 2.28 \times 10^4$ Pa

EVALUATE: The pressure is about one-fifth the pressure at sea-level.

18.22. **IDENTIFY:** The molar mass is $M = N_A m$, where m is the mass of one molecule.

SET UP: $N_A = 6.02 \times 10^{23}$ molecules/mol.

EXECUTE: $M = N_A m = (6.02 \times 10^{23} \text{ molecules/mol})(1.41 \times 10^{-21} \text{ kg/molecule}) = 849$ kg/mol.

EVALUATE: For a carbon atom, $M = 12 \times 10^{-3}$ kg/mol. If this molecule is mostly carbon, so the average mass of its atoms is the mass of carbon, the molecule would contain $\dfrac{849 \text{ kg/mol}}{12 \times 10^{-3} \text{ kg/mol}} = 71{,}000$ atoms.

18.23. **IDENTIFY:** The mass m_{tot} is related to the number of moles n by $m_{\text{tot}} = nM$. Mass is related to volume by $\rho = m/V$.

SET UP: For gold, $M = 196.97$ g/mol and $\rho = 19.3 \times 10^3$ kg/m^3. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $m_{\text{tot}} = nM = (3.00 \text{ mol})(196.97 \text{ g/mol}) = 590.9$ g. The value of this mass of gold is $(590.9 \text{ g})(\$14.75/\text{g}) = \$8720$.

(b) $V = \dfrac{m}{\rho} = \dfrac{0.5909 \text{ kg}}{19.3 \times 10^3 \text{ kg/m}^3} = 3.06 \times 10^{-5}$ m^3. $V = \frac{4}{3}\pi r^3$ gives

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3[3.06 \times 10^{-5} \text{ m}^3]}{4\pi}\right)^{1/3} = 0.0194 \text{ m} = 1.94 \text{ cm. The diameter is } 2r = 3.88 \text{ cm.}$$

EVALUATE: The mass and volume are directly proportional to the number of moles.

18.24. **IDENTIFY:** Use $pV = nRT$ to calculate the number of moles and then the number of molecules would be $N = nN_A$.

SET UP: 1 atm $= 1.013 \times 10^5$ Pa. 1.00 cm$^3 = 1.00 \times 10^{-6}$ m^3. $N_A = 6.022 \times 10^{23}$ molecules/mol.

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(9.00 \times 10^{-14} \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(1.00 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300.0 \text{ K})} = 3.655 \times 10^{-18}$ mol.

$N = nN_A = (3.655 \times 10^{-18} \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 2.20 \times 10^6$ molecules.

(b) $N = \dfrac{pVN_A}{RT}$ so $\dfrac{N}{p} = \dfrac{VN_A}{RT} = \text{constant}$ and $\dfrac{N_1}{p_1} = \dfrac{N_2}{p_2}$.

$N_2 = N_1\left(\dfrac{p_2}{p_1}\right) = (2.20 \times 10^6 \text{ molecules})\left(\dfrac{1.00 \text{ atm}}{9.00 \times 10^{-14} \text{ atm}}\right) = 2.44 \times 10^{19}$ molecules.

EVALUATE: The number of molecules in a given volume is directly proportional to the pressure. Even at the very low pressure in part (a) the number of molecules in 1.00 cm^3 is very large.

18.25. **IDENTIFY:** We are asked about a single state of the system.

SET UP: Use the ideal-gas law. Write n in terms of the number of molecules N.

(a) EXECUTE: $pV = nRT$, $n = N/N_A$ so $pV = (N/N_A)RT$

$$p = \left(\frac{N}{V}\right)\left(\frac{R}{N_A}\right)T$$

$$p = \left(\frac{80 \text{ molecules}}{1 \times 10^{-6} \text{ m}^3}\right)\left(\frac{8.3145 \text{ J/mol} \cdot \text{K}}{6.022 \times 10^{23} \text{ molecules/mol}}\right)(7500 \text{ K}) = 8.28 \times 10^{-12} \text{ Pa}$$

$p = 8.2 \times 10^{-17}$ atm. This is much lower than the laboratory pressure of 9×10^{-14} atm in Exercise 18.24.

(b) EVALUATE: The Lagoon Nebula is a very rarefied low pressure gas. The gas would exert *very* little force on an object passing through it.

18.26. **IDENTIFY:** $pV = nRT = NkT$

SET UP: At STP, $T = 273$ K, $p = 1.01 \times 10^5$ Pa. $N = 6 \times 10^9$ molecules.

EXECUTE: $V = \dfrac{NkT}{p} = \dfrac{(6 \times 10^9 \text{ molecules})(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(273 \text{ K})}{1.01 \times 10^5 \text{ Pa}} = 2.24 \times 10^{-16}$ m^3.

$L^3 = V$ so $L = V^{1/3} = 6.1 \times 10^{-6}$ m.

EVALUATE: This is a small cube.

18.27. **IDENTIFY:** $n = \dfrac{m}{M} = \dfrac{N}{N_A}$

SET UP: $N_A = 6.022 \times 10^{23}$ molecules/mol. For water, $M = 18 \times 10^{-3}$ kg/mol.

EXECUTE: $n = \dfrac{m}{M} = \dfrac{1.00 \text{ kg}}{18 \times 10^{-3} \text{ kg/mol}} = 55.6$ mol.

$N = n N_A = (55.6 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 3.35 \times 10^{25}$ molecules.

EVALUATE: Note that we converted M to kg/mol.

18.28. **IDENTIFY:** Use $pV = nRT$ and $n = \dfrac{N}{N_A}$ with $N = 1$ to calculate the volume V occupied by 1 molecule.

The length l of the side of the cube with volume V is given by $V = l^3$.

SET UP: $T = 27°C = 300$ K. $p = 1.00$ atm $= 1.013 \times 10^5$ Pa. $R = 8.314$ J/mol·K.

$N_A = 6.022 \times 10^{23}$ molecules/mol.

The diameter of a typical molecule is about 10^{-10} m. 0.3 nm $= 0.3 \times 10^{-9}$ m.

EXECUTE: **(a)** $pV = nRT$ and $n = \dfrac{N}{N_A}$ gives

$V = \dfrac{NRT}{N_A p} = \dfrac{(1.00)(8.314 \text{ J/mol·K})(300 \text{ K})}{(6.022 \times 10^{23} \text{ molecules/mol})(1.013 \times 10^5 \text{ Pa})} = 4.09 \times 10^{-26}$ m³. $l = V^{1/3} = 3.45 \times 10^{-9}$ m.

(b) The distance in part (a) is about 10 times the diameter of a typical molecule.

(c) The spacing is about 10 times the spacing of atoms in solids.

EVALUATE: There is space between molecules in a gas whereas in a solid the atoms are closely packed together.

18.29. **(a) IDENTIFY** and **SET UP:** Use the density and the mass of 5.00 mol to calculate the volume. $\rho = m/V$ implies $V = m/\rho$, where $m = m_{\text{tot}}$, the mass of 5.00 mol of water.

EXECUTE: $m_{\text{tot}} = nM = (5.00 \text{ mol})(18.0 \times 10^{-3} \text{ kg/mol}) = 0.0900$ kg

Then $V = \dfrac{m}{\rho} = \dfrac{0.0900 \text{ kg}}{1000 \text{ kg/m}^3} = 9.00 \times 10^{-5}$ m³

(b) One mole contains $N_A = 6.022 \times 10^{23}$ molecules, so the volume occupied by one molecule is

$\dfrac{9.00 \times 10^{-5} \text{ m}^3/\text{mol}}{(5.00 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol})} = 2.989 \times 10^{-29}$ m³/molecule

$V = a^3$, where a is the length of each side of the cube occupied by a molecule. $a^3 = 2.989 \times 10^{-29}$ m³, so $a = 3.1 \times 10^{-10}$ m.

(c) EVALUATE: Atoms and molecules are on the order of 10^{-10} m in diameter, in agreement with the above estimates.

18.30. **IDENTIFY:** $K_{\text{av}} = \tfrac{3}{2} kT$. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $M_{\text{Ne}} = 20.180$ g/mol, $M_{\text{Kr}} = 83.80$ g/mol and $M_{\text{Rn}} = 222$ g/mol.

EXECUTE: **(a)** $K_{\text{av}} = \tfrac{3}{2} kT$ depends only on the temperature so it is the same for each species of atom in the mixture.

(b) $\dfrac{v_{\text{rms,Ne}}}{v_{\text{rms,Kr}}} = \sqrt{\dfrac{M_{\text{Kr}}}{M_{\text{Ne}}}} = \sqrt{\dfrac{83.80 \text{ g/mol}}{20.18 \text{ g/mol}}} = 2.04.$ $\dfrac{v_{\text{rms,Ne}}}{v_{\text{rms,Rn}}} = \sqrt{\dfrac{M_{\text{Rn}}}{M_{\text{Ne}}}} = \sqrt{\dfrac{222 \text{ g/mol}}{20.18 \text{ g/mol}}} = 3.32.$

$\dfrac{v_{\text{rms,Kr}}}{v_{\text{rms,Rn}}} = \sqrt{\dfrac{M_{\text{Rn}}}{M_{\text{Kr}}}} = \sqrt{\dfrac{222 \text{ g/mol}}{83.80 \text{ g/mol}}} = 1.63.$

EVALUATE: The average kinetic energies are the same. The gas atoms with smaller mass have larger v_{rms}.

18.31.   **IDENTIFY** and **SET UP:** $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

EXECUTE: **(a)** v_{rms} is different for the two different isotopes, so the 235 isotope diffuses more rapidly.

(b) $\dfrac{v_{rms,235}}{v_{rms,238}} = \sqrt{\dfrac{M_{238}}{M_{235}}} = \sqrt{\dfrac{0.352 \text{ kg/mol}}{0.349 \text{ kg/mol}}} = 1.004$.

EVALUATE: The v_{rms} values each depend on T but their ratio is independent of T.

18.32.   **IDENTIFY** and **SET UP:**   With the multiplicity of each score denoted by n_i, the average score is

$\left(\dfrac{1}{150}\right)\Sigma n_i x_i$ and the rms score is $\left[\left(\dfrac{1}{150}\right)\Sigma n_i x_i^2\right]^{1/2}$.

EXECUTE: **(a)** 54.6

(b) 61.1

EVALUATE: The rms score is higher than the average score since the rms calculation gives more weight to the higher scores.

18.33.   **IDENTIFY:** $pV = nRT = \dfrac{N}{N_A}RT = \dfrac{m_{tot}}{M}RT$.

SET UP: We know that $V_A = V_B$ and that $T_A > T_B$.

EXECUTE: **(a)** $p = nRT/V$; we don't know n for each box, so either pressure could be higher.

(b) $pV = \left(\dfrac{N}{N_A}\right)RT$ so $N = \dfrac{pVN_A}{RT}$, where N_A is Avogadro's number. We don't know how the pressures

compare, so either N could be larger.

(c) $pV = (m_{tot}/M)RT$. We don't know the mass of the gas in each box, so they could contain the same gas or different gases.

(d) $\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$. $T_A > T_B$ and the average kinetic energy per molecule depends only on T, so the statement **must** be true.

(e) $v_{rms} = \sqrt{3kT/m}$. We don't know anything about the masses of the atoms of the gas in each box, so either set of molecules could have a larger v_{rms}.

EVALUATE: Only statement (d) must be true. We need more information in order to determine whether the other statements are true or false.

18.34.   **IDENTIFY:**   We can relate the temperature to the rms speed and the temperature to the pressure using the ideal gas law. The target variable is the pressure.

SET UP: $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ and $pV = nRT$, where $n = m/M$.

EXECUTE: Use v_{rms} to calculate T: $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ so

$T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{(28.014\times10^{-3} \text{ kg/mol})(182 \text{ m/s})^2}{3(8.314 \text{ J/mol}\cdot\text{K})} = 37.20$ K. The ideal gas law gives $p = \dfrac{nRT}{V}$.

$n = \dfrac{m}{M} = \dfrac{0.226\times10^{-3} \text{ kg}}{28.014\times10^{-3} \text{ kg/mol}} = 8.067\times10^{-3}$ mol. Solving for p gives

$p = \dfrac{(8.067\times10^{-3} \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(37.20 \text{ K})}{1.48\times10^{-3} \text{ m}^3} = 1.69\times10^3$ Pa.

EVALUATE: This pressure is around 1% of atmospheric pressure, which is not unreasonable since we have only around 1% of a mole of gas.

18.35.   **IDENTIFY:** $v_{rms} = \sqrt{\dfrac{3kT}{m}}$

SET UP: The mass of a deuteron is $m = m_p + m_n = 1.673 \times 10^{-27}$ kg $+ 1.675 \times 10^{-27}$ kg $= 3.35 \times 10^{-27}$ kg.

$c = 3.00 \times 10^8$ m/s. $k = 1.381 \times 10^{-23}$ J/molecule·K.

EXECUTE: **(a)** $v_{rms} = \sqrt{\dfrac{3(1.381 \times 10^{-23} \text{ J/molecule·K})(300 \times 10^6 \text{ K})}{3.35 \times 10^{-27} \text{ kg}}} = 1.93 \times 10^6$ m/s. $\dfrac{v_{rms}}{c} = 6.43 \times 10^{-3}$.

(b) $T = \left(\dfrac{m}{3k}\right)(v_{rms})^2 = \left(\dfrac{3.35 \times 10^{-27} \text{ kg}}{3(1.381 \times 10^{-23} \text{ J/molecule·K})}\right)(3.0 \times 10^7 \text{ m/s})^2 = 7.3 \times 10^{10}$ K.

EVALUATE: Even at very high temperatures and for this light nucleus, v_{rms} is a small fraction of the speed of light.

18.36. **IDENTIFY:** $v_{rms} = \sqrt{\dfrac{3RT}{M}}$, where T is in kelvins. $pV = nRT$ gives $\dfrac{n}{V} = \dfrac{p}{RT}$.

SET UP: $R = 8.314$ J/mol·K. $M = 44.0 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** For $T = 0.0°C = 273.15$ K, $v_{rms} = \sqrt{\dfrac{3(8.314 \text{ J/mol·K})(273.15 \text{ K})}{44.0 \times 10^{-3} \text{ kg/mol}}} = 393$ m/s. For

$T = -100.0°C = 173$ K, $v_{rms} = 313$ m/s. The range of speeds is 393 m/s to 313 m/s.

(b) For $T = 273.15$ K, $\dfrac{n}{V} = \dfrac{650 \text{ Pa}}{(8.314 \text{ J/mol·K})(273.15 \text{ K})} = 0.286$ mol/m^3. For $T = 173.15$ K,

$\dfrac{n}{V} = 0.452$ mol/m^3. The range of densities is 0.286 mol/m^3 to 0.452 mol/m^3.

EVALUATE: When the temperature decreases the rms speed decreases and the density increases.

18.37. **IDENTIFY** and **SET UP:** Apply the analysis of Section 18.3.

EXECUTE: **(a)** $\frac{1}{2} m(v^2)_{av} = \frac{3}{2} kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/molecule·K})(300 \text{ K}) = 6.21 \times 10^{-21}$ J

(b) We need the mass m of one molecule:

$m = \dfrac{M}{N_A} = \dfrac{32.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 5.314 \times 10^{-26}$ kg/molecule

Then $\frac{1}{2} m(v^2)_{av} = 6.21 \times 10^{-21}$ J (from part (a)) gives

$(v^2)_{av} = \dfrac{2(6.21 \times 10^{-21} \text{ J})}{m} = \dfrac{2(6.21 \times 10^{-21} \text{ J})}{5.314 \times 10^{-26} \text{ kg}} = 2.34 \times 10^5$ m^2/s^2

(c) $v_{rms} = \sqrt{(v^2)_{rms}} = \sqrt{2.34 \times 10^4 \text{ m}^2/\text{s}^2} = 484$ m/s

(d) $p = m v_{rms} = (5.314 \times 10^{-26} \text{ kg})(484 \text{ m/s}) = 2.57 \times 10^{-23}$ kg·m/s

(e) Time between collisions with one wall is $t = \dfrac{0.20 \text{ m}}{v_{rms}} = \dfrac{0.20 \text{ m}}{484 \text{ m/s}} = 4.13 \times 10^{-4}$ s

In a collision $\vec{v}$ changes direction, so $\Delta p = 2 m v_{rms} = 2(2.57 \times 10^{-23} \text{ kg·m/s}) = 5.14 \times 10^{-23}$ kg·m/s

$F = \dfrac{dp}{dt}$ so $F_{av} = \dfrac{\Delta p}{\Delta t} = \dfrac{5.14 \times 10^{-23} \text{ kg·m/s}}{4.13 \times 10^{-4} \text{ s}} = 1.24 \times 10^{-19}$ N

(f) pressure $= F/A = 1.24 \times 10^{-19}$ N/$(0.10 \text{ m})^2 = 1.24 \times 10^{-17}$ Pa (due to one molecule)

(g) pressure $= 1$ atm $= 1.013 \times 10^5$ Pa

Number of molecules needed is 1.013×10^5 Pa/$(1.24 \times 10^{-17}$ Pa/molecule$) = 8.17 \times 10^{21}$ molecules

(h) $pV = NkT$ (Eq. 18.18), so $N = \dfrac{pV}{kT} = \dfrac{(1.013 \times 10^5 \text{ Pa})(0.10 \text{ m})^3}{(1.381 \times 10^{-23} \text{ J/molecule·K})(300 \text{ K})} = 2.45 \times 10^{22}$ molecules

(i) From the factor of $\frac{1}{3}$ in $(v_x^2)_{av} = \frac{1}{3}(v^2)_{av}$.

EVALUATE: This exercise shows that the pressure exerted by a gas arises from collisions of the molecules of the gas with the walls.

18.38.   **IDENTIFY:** Apply Eq. (18.22) and calculate λ.

SET UP: 1 atm $=1.013\times10^5$ Pa, so $p=3.55\times10^{-8}$ Pa. $r=2.0\times10^{-10}$ m and $k=1.38\times10^{-23}$ J/K.

EXECUTE: $\lambda=\dfrac{kT}{4\pi\sqrt{2}r^2p}=\dfrac{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}{4\pi\sqrt{2}(2.0\times10^{-10}\text{ m})^2(3.55\times10^{-8}\text{ Pa})}=1.6\times10^5$ m

EVALUATE: At this very low pressure the mean free path is very large. If $v=484$ m/s, as in Example 18.8, then $t_{\text{mean}}=\dfrac{\lambda}{v}=330$ s. Collisions are infrequent.

18.39.   **IDENTIFY and SET UP:** Use equal v_{rms} to relate T and M for the two gases. $v_{\text{rms}}=\sqrt{3RT/M}$ (Eq. 18.19), so $v_{\text{rms}}^2/3R=T/M$, where T must be in kelvins. Same v_{rms} so same T/M for the two gases and $T_{\text{N}_2}/M_{\text{N}_2}=T_{\text{H}_2}/M_{\text{H}_2}$.

EXECUTE: $T_{\text{N}_2}=T_{\text{H}_2}\left(\dfrac{M_{\text{N}_2}}{M_{\text{H}_2}}\right)=((20+273)\text{K})\left(\dfrac{28.014\text{ g/mol}}{2.016\text{ g/mol}}\right)=4.071\times10^3$ K

$T_{\text{N}_2}=(4071-273)°\text{C}=3800°\text{C}$

EVALUATE: A N_2 molecule has more mass so N_2 gas must be at a higher temperature to have the same v_{rms}.

18.40.   **IDENTIFY:** $v_{\text{rms}}=\sqrt{\dfrac{3kT}{m}}$.

SET UP: $k=1.381\times10^{-23}$ J/molecule$\cdot$K.

EXECUTE: **(a)** $v_{\text{rms}}=\sqrt{\dfrac{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})(300\text{ K})}{3.00\times10^{-16}\text{ kg}}}=6.44\times10^{-3}$ m/s $=6.44$ mm/s

EVALUATE: **(b)** No. The rms speed depends on the average kinetic energy of the particles. At this T, H_2 molecules would have larger v_{rms} than the typical air molecules but would have the same average kinetic energy and the average kinetic energy of the smoke particles would be the same.

18.41.   **IDENTIFY:** Use Eq. (18.24), applied to a finite temperature change.

SET UP: $C_V=5R/2$ for a diatomic ideal gas and $C_V=3R/2$ for a monatomic ideal gas.

EXECUTE: **(a)** $Q=nC_V\Delta T=n\left(\tfrac{5}{2}R\right)\Delta T.$ $Q=(2.5\text{ mol})(\tfrac{5}{2})(8.3145\text{ J/mol}\cdot\text{K})(50.0\text{ K})=2600$ J.

(b) $Q=nC_V\Delta T=n(\tfrac{3}{2}R)\,\Delta T.$ $Q=(2.5\text{ mol})(\tfrac{3}{2})(8.3145\text{ J/mol}\cdot\text{K})(50.0\text{ K})=1560$ J.

EVALUATE: More heat is required for the diatomic gas; not all the heat that goes into the gas appears as translational kinetic energy, some goes into energy of the internal motion of the molecules (rotations).

18.42.   **IDENTIFY:** The heat Q added is related to the temperature increase ΔT by $Q=nC_V\Delta T$.

SET UP: For ideal H_2 (a diatomic gas), $C_{V,\text{H}_2}=5/2\,R$, and for ideal Ne (a monatomic gas), $C_{V,\text{Ne}}=3/2\,R$.

EXECUTE: $C_V\Delta T=\dfrac{Q}{n}=$ constant, so $C_{V,\text{H}_2}\Delta T_{\text{H}_2}=C_{V,\text{Ne}}\Delta T_{\text{Ne}}$.

$\Delta T_{\text{Ne}}=\left(\dfrac{C_{V,\text{H}_2}}{C_{V,\text{Ne}}}\right)\Delta T_{\text{H}_2}=\left(\dfrac{5/2\,R}{3/2\,R}\right)(2.50\text{ C}°)=4.17\text{ C}°=4.17\text{ K}.$

EVALUATE: The same amount of heat causes a smaller temperature increase for H_2 since some of the energy input goes into the internal degrees of freedom.

18.43.   **IDENTIFY:** $C=Mc$, where C is the molar heat capacity and c is the specific heat capacity.

$pV=nRT=\dfrac{m}{M}RT.$

SET UP: $M_{N_2} = 2(14.007 \text{ g/mol}) = 28.014 \times 10^{-3}$ kg/mol. For water, $c_w = 4190$ J/kg·K. For N_2,

$C_V = 20.76$ J/mol·K.

EXECUTE: **(a)** $c_{N_2} = \dfrac{C}{M} = \dfrac{20.76 \text{ J/mol} \cdot \text{K}}{28.014 \times 10^{-3} \text{ kg/mol}} = 741$ J/kg·K. $\dfrac{c_w}{c_{N_2}} = 5.65$; c_w is over five time larger.

(b) To warm the water, $Q = mc_w\Delta T = (1.00 \text{ kg})(4190 \text{ J/mol} \cdot \text{K})(10.0 \text{ K}) = 4.19 \times 10^4$ J. For air,

$m = \dfrac{Q}{c_{N_2}\Delta T} = \dfrac{4.19 \times 10^4 \text{ J}}{(741 \text{ J/kg} \cdot \text{K})(10.0 \text{ K})} = 5.65$ kg.

$V = \dfrac{mRT}{Mp} = \dfrac{(5.65 \text{ kg})(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{(28.014 \times 10^{-3} \text{ kg/mol})(1.013 \times 10^5 \text{ Pa})} = 4.85 \text{ m}^3$.

EVALUATE: c is smaller for N_2, so less heat is needed for 1.0 kg of N_2 than for 1.0 kg of water.

18.44. **(a) IDENTIFY** and **SET UP:** $\frac{1}{2}R$ contribution to C_V for each degree of freedom. The molar heat capacity C is related to the specific heat capacity c by $C = Mc$.

EXECUTE: $C_V = 6\left(\frac{1}{2}R\right) = 3R = 3(8.3145 \text{ J/mol} \cdot \text{K}) = 24.9$ J/mol·K. The specific heat capacity is

$c_V = C_V/M = (24.9 \text{ J/mol} \cdot \text{K})/(18.0 \times 10^{-3} \text{ kg/mol}) = 1380$ J/kg·K.

(b) For water vapor the specific heat capacity is $c = 2000$ J/kg·K. The molar heat capacity is

$C = Mc = (18.0 \times 10^{-3} \text{ kg/mol})(2000 \text{ J/kg} \cdot \text{K}) = 36.0$ J/mol·K.

EVALUATE: The difference is $36.0 \text{ J/mol} \cdot \text{K} - 24.9 \text{ J/mol} \cdot \text{K} = 11.1$ J/mol·K, which is about $2.7\left(\frac{1}{2}R\right)$;

the vibrational degrees of freedom make a significant contribution.

18.45. **IDENTIFY:** $C_V = 3R$ gives C_V in units of J/mol·K. The atomic mass M gives the mass of one mole.

SET UP: For aluminum, $M = 26.982 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $C_V = 3R = 24.9$ J/mol·K. $c_V = \dfrac{24.9 \text{ J/mol} \cdot \text{K}}{26.982 \times 10^{-3} \text{ kg/mol}} = 923$ J/kg·K.

(b) Table 17.3 gives 910 J/kg·K. The value from Eq. (18.28) is too large by about 1.4%.

EVALUATE: As shown in Figure 18.21 in the textbook, C_V approaches the value $3R$ as the temperature increases. The values in Table 17.3 are at room temperature and therefore are somewhat smaller than $3R$.

18.46. **IDENTIFY:** Table 18.2 gives the value of v/v_{rms} for which 94.7% of the molecules have a smaller value of

v/v_{rms}. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: For N_2, $M = 28.0 \times 10^{-3}$ kg/mol. $v/v_{rms} = 1.60$.

EXECUTE: $v_{rms} = \dfrac{v}{1.60} = \sqrt{\dfrac{3RT}{M}}$, so the temperature is

$T = \dfrac{Mv^2}{3(1.60)^2 R} = \dfrac{(28.0 \times 10^{-3} \text{ kg/mol})}{3(1.60)^2(8.3145 \text{ J/mol} \cdot \text{K})}v^2 = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)v^2$.

(a) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(1500 \text{ m/s})^2 = 987$ K

(b) $T = (4.385 \times 10^{-4} \text{K} \cdot \text{s}^2/\text{m}^2)(1000 \text{ m/s})^2 = 438$ K

(c) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(500 \text{ m/s})^2 = 110$ K

EVALUATE: As T decreases the distribution of molecular speeds shifts to lower values.

18.47. **IDENTIFY:** Apply Eqs. (18.34), (18.35) and (18.36).

SET UP: Note that $\dfrac{k}{m} = \dfrac{R/N_A}{M/N_A} = \dfrac{R}{M}$. $M = 44.0 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $v_{mp} = \sqrt{2(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(44.0 \times 10^{-3} \text{ kg/mol})} = 3.37 \times 10^2$ m/s.

(b) $v_{av} = \sqrt{8(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(\pi(44.0 \times 10^{-3} \text{ kg/mol}))} = 3.80 \times 10^2 \text{ m/s}.$

(c) $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})/(44.0 \times 10^{-3} \text{ kg/mol})} = 4.12 \times 10^2 \text{ m/s}.$

EVALUATE: The average speed is greater than the most probable speed and the rms speed is greater than the average speed.

18.48.   IDENTIFY and SET UP: Eq. (18.33): $f(v) = \dfrac{8\pi}{m}\left(\dfrac{m}{2\pi kT}\right)^{3/2} \epsilon e^{-\epsilon/kT}$

At the maximum of $f(\epsilon)$, $\dfrac{df}{d\epsilon} = 0.$

EXECUTE: $\dfrac{df}{d\epsilon} = \dfrac{8\pi}{m}\left(\dfrac{m}{2\pi kT}\right)^{3/2} \dfrac{d}{d\epsilon}(\epsilon e^{-\epsilon/kT}) = 0$

This requires that $\dfrac{d}{d\epsilon}(\epsilon e^{-\epsilon/kT}) = 0.$

$e^{-\epsilon/kT} - (\epsilon/kT)e^{-\epsilon/kT} = 0$

$(1 - \epsilon/kT)e^{-\epsilon/kT} = 0$

This requires that $1 - \epsilon/kT = 0$ so $\epsilon = kT$, as was to be shown. And then since $\epsilon = \frac{1}{2}mv^2$, this gives

$\frac{1}{2}mv_{mp}^2 = kT$ and $v_{mp} = \sqrt{2kT/m}$, which is Eq. (18.34).

EVALUATE: $v_{rms} = \sqrt{\frac{3}{2}}v_{mp}$. The average of v^2 gives more weight to larger v.

18.49.   IDENTIFY: Refer to the phase diagram in Figure 18.24 in the textbook.

SET UP: For water the triple-point pressure is 610 Pa and the critical-point pressure is 2.212×10^7 Pa.

EXECUTE: (a) To observe a solid to liquid (melting) phase transition the pressure must be greater than the triple-point pressure, so $p_1 = 610$ Pa. For $p < p_1$ the solid to vapor (sublimation) phase transition is observed.

(b) No liquid to vapor (boiling) phase transition is observed if the pressure is greater than the critical-point pressure. $p_2 = 2.212 \times 10^7$ Pa. For $p_1 < p < p_2$ the sequence of phase transitions are solid to liquid and then liquid to vapor.

EVALUATE: Normal atmospheric pressure is approximately 1.0×10^5 Pa, so the solid to liquid to vapor sequence of phase transitions is normally observed when the material is water.

18.50.   IDENTIFY and SET UP: If the temperature at altitude y is below the freezing point only cirrus clouds can form. Use $T = T_0 - \alpha y$ to find the y that gives $T = 0.0°C$.

EXECUTE: $y = \dfrac{T_0 - T}{\alpha} = \dfrac{15.0°C - 0.0°C}{6.0 \text{ C}°/\text{km}} = 2.5 \text{ km}$

EVALUATE: The solid-liquid phase transition occurs at $0°C$ only for $p = 1.01 \times 10^5$ Pa. Use the results of Example 18.4 to estimate the pressure at an altitude of 2.5 km.

$p_2 = p_1 e^{Mg(y_2 - y_1)/RT}$

$Mg(y_2 - y_1)/RT = 1.10(2500 \text{ m}/8863 \text{ m}) = 0.310$ (using the calculation in Example 18.4)

Then $p_2 = (1.01 \times 10^5 \text{ Pa})e^{-0.31} = 0.74 \times 10^5$ Pa.

This pressure is well above the triple point pressure for water. Figure 18.24 in the textbook shows that the fusion curve has large slope and it takes a large change in pressure to change the phase transition temperature very much. Using $0.0°C$ introduces little error.

18.51.   IDENTIFY: Figure 18.24 in the textbook shows that there is no liquid phase below the triple point pressure.

SET UP: Table 18.3 gives the triple point pressure to be 610 Pa for water and 5.17×10^5 Pa for CO_2.

EXECUTE: The atmospheric pressure is below the triple point pressure of water, and there can be no liquid water on Mars. The same holds true for CO_2.

EVALUATE: On earth $p_{atm} = 1 \times 10^5$ Pa, so on the surface of the earth there can be liquid water but not liquid CO_2.

18.52. **IDENTIFY:** The ideal gas law will tell us the number of moles of gas in the room, which we can use to find the number of molecules.

SET UP: $pV = nRT$, $N = nN_A$, and $m = nM$.

EXECUTE: **(a)** $T = 27.0°C + 273 = 300$ K. $p = 1.013 \times 10^5$ Pa.

$$n = \frac{pV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(216 \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 8773 \text{ mol}.$$

$N = nN_A = (8773 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 5.28 \times 10^{27}$ molecules.

(b) $V = (216 \text{ m}^3)(1 \text{ cm}^3/10^{-6} \text{ m}^3) = 2.16 \times 10^8 \text{ cm}^3$. The particle density is

$$\frac{5.28 \times 10^{27} \text{ molecules}}{2.16 \times 10^8 \text{ cm}^3} = 2.45 \times 10^{19} \text{ molecules/cm}^3.$$

(c) $m = nM = (8773 \text{ mol})(28.014 \times 10^{-3} \text{ kg/mol}) = 246$ kg.

EVALUATE: A cubic centimeter of air (about the size of a sugar cube) contains around 10^{19} molecules, and the air in the room weighs about 500 lb!

18.53. **IDENTIFY:** We can model the atmosphere as a fluid of constant density, so the pressure depends on the depth in the fluid, as we saw in Section 12.2.

SET UP: The pressure difference between two points in a fluid is $\Delta p = \rho gh$, where h is the difference in height of two points.

EXECUTE: **(a)** $\Delta p = \rho gh = (1.2 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m}) = 1.18 \times 10^4$ Pa.

(b) At the bottom of the mountain, $p = 1.013 \times 10^5$ Pa. At the top, $p = 8.95 \times 10^4$ Pa.

$$pV = nRT = \text{constant} \quad \text{so} \quad p_b V_b = p_t V_t \quad \text{and} \quad V_t = V_b \left(\frac{p_b}{p_t} \right) = (0.50 \text{ L}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{8.95 \times 10^4 \text{ Pa}} \right) = 0.566 \text{ L}.$$

EVALUATE: The pressure variation with altitude is affected by changes in air density and temperature and we have neglected those effects. The pressure decreases with altitude and the volume increases. You may have noticed this effect: bags of potato chips "puff up" when taken to the top of a mountain.

18.54. **IDENTIFY:** As the pressure on the bubble changes, its volume will change. As we saw in Section 12.2, the pressure in a fluid depends on the depth.

SET UP: The pressure at depth h in a fluid is $p = p_0 + \rho gh$, where p_0 is the pressure at the surface. $p_0 = p_{air} = 1.013 \times 10^5$ Pa. The density of water is $\rho = 1000 \text{ kg/m}^3$.

EXECUTE: $p_1 = p_0 + \rho gh = 1.013 \times 10^5$ Pa $+ (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(25 \text{ m}) = 3.463 \times 10^5$ Pa.

$p_2 = p_{air} = 1.013 \times 10^5$ Pa. $V_1 = 1.0 \text{ mm}^3$. n, R and T are constant so $pV = nRT = \text{constant}$. $p_1 V_1 = p_2 V_2$

and $V_2 = V_1 \left(\dfrac{p_1}{p_2} \right) = (1.0 \text{ mm}^3) \left(\dfrac{3.463 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) = 3.4 \text{ mm}^3$.

EVALUATE: This is a large change and would have serious effects.

18.55. **IDENTIFY:** The buoyant force on the balloon must be equal to the weight of the load plus the weight of the gas.

SET UP: The buoyant force is $F_B = \rho_{air} Vg$. A lift of 290 kg means $\dfrac{F_B}{g} - m_{hot} = 290$ kg, where m_{hot} is the mass of hot air in the balloon. $m = \rho V$.

EXECUTE: $m_{hot} = \rho_{hot} V$. $\dfrac{F_B}{g} - m_{hot} = 290$ kg gives $(\rho_{air} - \rho_{hot})V = 290$ kg.

Solving for ρ_{hot} gives $\rho_{\text{hot}} = \rho_{\text{air}} - \dfrac{290 \text{ kg}}{V} = 1.23 \text{ kg/m}^3 - \dfrac{290 \text{ kg}}{500.0 \text{ m}^3} = 0.65 \text{ kg/m}^3$. $\rho_{\text{hot}} = \dfrac{pM}{RT_{\text{hot}}}$.

$\rho_{\text{air}} = \dfrac{pM}{RT_{\text{air}}}$. $\rho_{\text{hot}} T_{\text{hot}} = \rho_{\text{air}} T_{\text{air}}$ so

$T_{\text{hot}} = T_{\text{air}} \left(\dfrac{\rho_{\text{air}}}{\rho_{\text{hot}}} \right) = (288 \text{ K}) \left(\dfrac{1.23 \text{ kg/m}^3}{0.65 \text{ kg/m}^3} \right) = 545 \text{ K} = 272 °\text{C}$.

EVALUATE: This temperature is well above normal air temperatures, so the air in the balloon would need considerable heating.

18.56. **IDENTIFY:** $\Delta V = \beta V_0 \Delta T - V_0 k \Delta p$

SET UP: For steel, $\beta = 3.6 \times 10^{-5} \text{ K}^{-1}$ and $k = 6.25 \times 10^{-12} \text{ Pa}^{-1}$.

EXECUTE: $\beta V_0 \Delta T = (3.6 \times 10^{-5} \text{ K}^{-1})(11.0 \text{ L})(21 \text{ C}°) = 0.0083 \text{ L}$.

$-k V_0 \Delta p = -(6.25 \times 10^{-12}/\text{Pa})(11 \text{ L})(2.1 \times 10^7 \text{ Pa}) = -0.0014 \text{ L}$. The total change in volume is

$\Delta V = 0.0083 \text{ L} - 0.0014 \text{ L} = 0.0069 \text{ L}$.

(b) Yes; ΔV is much less than the original volume of 11.0 L.

EVALUATE: Even for a large pressure increase and a modest temperature increase, the magnitude of the volume change due to the temperature increase is much larger than that due to the pressure increase.

18.57. **IDENTIFY:** We are asked to compare two states. Use the ideal-gas law to obtain m_2 in terms of m_1 and the ratio of pressures in the two states. Apply Eq. (18.4) to the initial state to calculate m_1.

SET UP: $pV = nRT$ can be written $pV = (m/M)RT$

T, V, M, R are all constant, so $p/m = RT/MV = $ constant.

So $p_1/m_1 = p_2/m_2$, where m is the mass of the gas in the tank.

EXECUTE: $p_1 = 1.30 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 1.40 \times 10^6 \text{ Pa}$

$p_2 = 2.50 \times 10^5 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 3.51 \times 10^5 \text{ Pa}$

$m_1 = p_1 V M / RT$; $V = hA = h\pi r^2 = (1.00 \text{ m})\pi (0.060 \text{ m})^2 = 0.01131 \text{ m}^3$

$m_1 = \dfrac{(1.40 \times 10^6 \text{ Pa})(0.01131 \text{ m}^3)(44.1 \times 10^{-3} \text{ kg/mol})}{(8.3145 \text{ J/mol} \cdot \text{K})((22.0 + 273.15)\text{K})} = 0.2845 \text{ kg}$

Then $m_2 = m_1 \left(\dfrac{p_2}{p_1} \right) = (0.2845 \text{ kg}) \left(\dfrac{3.51 \times 10^5 \text{ Pa}}{1.40 \times 10^6 \text{ Pa}} \right) = 0.0713 \text{ kg}$.

m_2 is the mass that remains in the tank. The mass that has been used is

$m_1 - m_2 = 0.2845 \text{ kg} - 0.0713 \text{ kg} = 0.213 \text{ kg}$.

EVALUATE: Note that we have to use absolute pressures. The absolute pressure decreases by a factor of four and the mass of gas in the tank decreases by a factor of four.

18.58. **IDENTIFY:** Apply $pV = nRT$ to the air inside the diving bell. The pressure p at depth y below the surface of the water is $p = p_{\text{atm}} + \rho g y$.

SET UP: $p = 1.013 \times 10^5 \text{ Pa}$. $T = 300.15 \text{ K}$ at the surface and $T' = 280.15 \text{ K}$ at the depth of 13.0 m.

EXECUTE: **(a)** The height h' of the air column in the diving bell at this depth will be proportional to the volume, and hence inversely proportional to the pressure and proportional to the Kelvin temperature:

$h' = h \dfrac{p}{p'} \dfrac{T'}{T} = h \dfrac{p_{\text{atm}}}{p_{\text{atm}} + \rho g y} \dfrac{T'}{T}$.

$h' = (2.30 \text{ m}) \dfrac{(1.013 \times 10^5 \text{ Pa})}{(1.013 \times 10^5 \text{ Pa}) + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(73.0 \text{ m})} \left(\dfrac{280.15 \text{ K}}{300.15 \text{ K}} \right) = 0.26 \text{ m}$.

The height of the water inside the diving bell is $h - h' = 2.04 \text{ m}$.

(b) The necessary gauge pressure is the term $\rho g y$ from the above calculation, $p_{\text{gauge}} = 7.37 \times 10^5 \text{ Pa}$.

EVALUATE: The gauge pressure required in part (b) is about 7 atm.

18.59.   **IDENTIFY:** $pV = NkT$ gives $\dfrac{N}{V} = \dfrac{p}{kT}$.

SET UP: 1 atm $= 1.013 \times 10^5$ Pa. $T_K = T_C + 273.15$. $k = 1.381 \times 10^{-23}$ J/molecule $\cdot$ K.

EXECUTE: **(a)** $T_C = T_K - 273.15 = 94$ K $- 273.15 = -179°$C

(b) $\dfrac{N}{V} = \dfrac{p}{kT} = \dfrac{(1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(94 \text{ K})} = 1.2 \times 10^{26}$ molecules/m^3

(c) For the earth, $p = 1.0$ atm $= 1.013 \times 10^5$ Pa and $T = 22°$C $= 295$ K.

$\dfrac{N}{V} = \dfrac{(1.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(295 \text{ K})} = 2.5 \times 10^{25}$ molecules/m^3. The atmosphere of Titan is about

five times denser than earth's atmosphere.

EVALUATE: Though it is smaller than Earth and has weaker gravity at its surface, Titan can maintain a dense atmosphere because of the very low temperature of that atmosphere.

18.60.   **IDENTIFY:**   For constant temperature, the variation of pressure with altitude is calculated in Example 18.4

to be $p = p_0 e^{-Mgy/RT}$. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $g_{Earth} = 9.80$ m/s^2. $T = 460°$C $= 733$ K. $M = 44.0$ g/mol $= 44.0 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $\dfrac{Mgy}{RT} = \dfrac{(44.0 \times 10^{-3} \text{ kg/mol})(0.894)(9.80 \text{ m/s}^2)(1.00 \times 10^3 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(733 \text{ K})} = 0.06326.$

$p = p_0 e^{-Mgy/RT} = (92 \text{ atm})e^{-0.06326} = 86$ atm. The pressure is 86 earth-atmospheres, or 0.94 Venus-atmospheres.

(b) $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3(8.314 \text{ J/mol} \cdot \text{K})(733 \text{ K})}{44.0 \times 10^{-3} \text{ kg/mol}}} = 645$ m/s. v_{rms} has this value both at the surface and at an

altitude of 1.00 km.

EVALUATE: v_{rms} depends only on T and the molar mass of the gas. For Venus compared to earth, the surface temperature, in kelvins, is nearly a factor of three larger and the molecular mass of the gas in the atmosphere is only about 50% larger, so v_{rms} for the Venus atmosphere is larger than it is for the earth's atmosphere.

18.61.   **IDENTIFY:** $pV = nRT$

SET UP: In $pV = nRT$ we must use the absolute pressure. $T_1 = 278$ K. $p_1 = 2.72$ atm. $T_2 = 318$ K.

EXECUTE: n, R constant, so $\dfrac{pV}{T} = nR = $ constant. $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$ and

$p_2 = p_1 \left(\dfrac{V_1}{V_2} \right) \left(\dfrac{T_2}{T_1} \right) = (2.72 \text{ atm}) \left(\dfrac{0.0150 \text{ m}^3}{0.0159 \text{ m}^3} \right) \left(\dfrac{318 \text{ K}}{278 \text{ K}} \right) = 2.94$ atm. The final gauge pressure is

2.94 atm $- 1.02$ atm $= 1.92$ atm.

EVALUATE: Since a ratio is used, pressure can be expressed in atm. But absolute pressures must be used. The ratio of gauge pressures is not equal to the ratio of absolute pressures.

18.62.   **IDENTIFY:**   In part (a), apply $pV = nRT$ to the ethane in the flask. The volume is constant once the

stopcock is in place. In part (b) apply $pV = \dfrac{m_{tot}}{M} RT$ to the ethane at its final temperature and pressure.

SET UP: 1.50 L $= 1.50 \times 10^{-3}$ m^3. $M = 30.1 \times 10^{-3}$ kg/mol. Neglect the thermal expansion of the flask.

EXECUTE: **(a)** $p_2 = p_1 (T_2/T_1) = (1.013 \times 10^5 \text{ Pa})(300 \text{ K}/490 \text{ K}) = 6.20 \times 10^4$ Pa.

(b) $m_{tot} = \left(\dfrac{p_2 V}{RT_2} \right) M = \left(\dfrac{(6.20 \times 10^4 \text{ Pa})(1.50 \times 10^{-3} \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} \right)(30.1 \times 10^{-3} \text{ Kg/mol}) = 1.12$ g.

EVALUATE: We could also calculate m_{tot} with $p = 1.013 \times 10^5$ Pa and $T = 490$ K, and we would obtain the same result. Originally, before the system was warmed, the mass of ethane in the flask was

$$m = (1.12 \text{ g}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{6.20 \times 10^4 \text{ Pa}} \right) = 1.83 \text{ g}.$$

18.63. **(a) IDENTIFY:** Consider the gas in one cylinder. Calculate the volume to which this volume of gas expands when the pressure is decreased from $(1.20 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa}) = 1.30 \times 10^6$ Pa to 1.01×10^5 Pa. Apply the ideal-gas law to the two states of the system to obtain an expression for V_2 in terms of V_1 and the ratio of the pressures in the two states.

SET UP: $pV = nRT$

n, R, T constant implies $pV = nRT = $ constant, so $p_1 V_1 = p_2 V_2$.

EXECUTE: $V_2 = V_1 (p_1/p_2) = (1.90 \text{ m}^3) \left(\frac{1.30 \times 10^6 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right) = 24.46 \text{ m}^3$

The number of cylinders required to fill a 750 m³ balloon is $750 \text{ m}^3 / 24.46 \text{ m}^3 = 30.7$ cylinders.

EVALUATE: The ratio of the volume of the balloon to the volume of a cylinder is about 400. Fewer cylinders than this are required because of the large factor by which the gas is compressed in the cylinders.

(b) IDENTIFY: The upward force on the balloon is given by Archimedes's principle (Chapter 12): $B = $ weight of air displaced by balloon $= \rho_{air} V g$. Apply Newton's second law to the balloon and solve for the weight of the load that can be supported. Use the ideal-gas equation to find the mass of the gas in the balloon.

SET UP: The free-body diagram for the balloon is given in Figure 18.63.
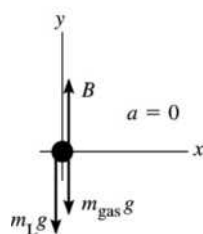
m_{gas} is the mass of the gas that is inside the balloon; m_L is the mass of the load that is supported by the balloon.

EXECUTE: $\Sigma F_y = ma_y$

$B - m_L g - m_{gas} g = 0$

Figure 18.63

$\rho_{air} V g - m_L g - m_{gas} g = 0$

$m_L = \rho_{air} V - m_{gas}$

Calculate m_{gas}, the mass of hydrogen that occupies 750 m³ at 15°C and $p = 1.01 \times 10^5$ Pa.

$pV = nRT = (m_{gas}/M) RT$ gives

$$m_{gas} = pVM/RT = \frac{(1.01 \times 10^5 \text{ Pa})(750 \text{ m}^3)(2.02 \times 10^{-3} \text{ kg/mol})}{(8.3145 \text{ J/mol} \cdot \text{K})(288 \text{ K})} = 63.9 \text{ kg}$$

Then $m_L = (1.23 \text{ kg/m}^3)(750 \text{ m}^3) - 63.9 \text{ kg} = 859$ kg, and the weight that can be supported is

$w_L = m_L g = (859 \text{ kg})(9.80 \text{ m/s}^2) = 8420$ N.

(c) $m_L = \rho_{air} V - m_{gas}$

$m_{gas} = pVM/RT = (63.9 \text{ kg})((4.00 \text{ g/mol})/(2.02 \text{ g/mol})) = 126.5$ kg (using the results of part (b)).

Then $m_L = (1.23 \text{ kg/m}^3)(750 \text{ m}^3) - 126.5 \text{ kg} = 796$ kg.

$w_L = m_L g = (796 \text{ kg})(9.80 \text{ m/s}^2) = 7800$ N.

EVALUATE: A greater weight can be supported when hydrogen is used because its density is less.

18.64.   **IDENTIFY:**   The upward force exerted by the gas on the piston must equal the piston's weight. Use
$pV = nRT$ to calculate the volume of the gas, and from this the height of the column of gas in the cylinder.

SET UP: $F = pA = p\pi r^2$, with $r = 0.100$ m and $p = 0.500$ atm $= 5.065 \times 10^4$ Pa. For the cylinder,
$V = \pi r^2 h.$

EXECUTE: (a) $p\pi r^2 = mg$ and $m = \dfrac{p\pi r^2}{g} = \dfrac{(5.065 \times 10^4 \text{ Pa})\pi(0.100 \text{ m})^2}{9.80 \text{ m/s}^2} = 162$ kg.

(b) $V = \pi r^2 h$ and $V = nRT/p$. Combining these equations gives $h = nRT/\pi r^2 p$, which gives

$h = \dfrac{(1.80 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})}{\pi(0.100 \text{ m})^2 (5.065 \times 10^4 \text{ Pa})} = 276$ m.

EVALUATE: The calculation assumes a vacuum $(p = 0)$ in the tank above the piston.

18.65.   **IDENTIFY:**   Apply Bernoulli's equation to relate the efflux speed of water out the hose to the height of
water in the tank and the pressure of the air above the water in the tank. Use the ideal-gas equation to relate
the volume of the air in the tank to the pressure of the air.

(a) **SET UP:**   Points 1 and 2 are shown in Figure 18.65.
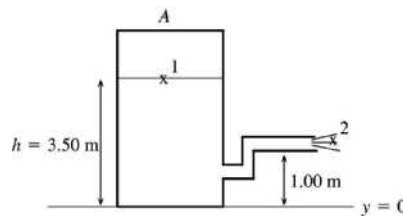
$p_1 = 4.20 \times 10^5$ Pa

$p_2 = p_{air} = 1.00 \times 10^5$ Pa

large tank implies $v_1 \approx 0$

Figure 18.65

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\frac{1}{2}\rho v_2^2 = p_1 - p_2 + \rho g(y_1 - y_2)$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 26.2$ m/s

(b) $\underline{h = 3.00 \text{ m}}$

The volume of the air in the tank increases so its pressure decreases. $pV = nRT = $ constant, so $pV = p_0 V_0$

(p_0 is the pressure for $h_0 = 3.50$ m and p is the pressure for $h = 3.00$ m)

$p(4.00 \text{ m} - h)A = p_0(4.00 \text{ m} - h_0)A$

$p = p_0 \left(\dfrac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa})\left(\dfrac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 3.00 \text{ m}} \right) = 2.10 \times 10^5$ Pa

Repeat the calculation of part (a), but now $p_1 = 2.10 \times 10^5$ Pa and $y_1 = 3.00$ m.

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 16.1$ m/s

$\underline{h = 2.00 \text{ m}}$

$p = p_0 \left(\dfrac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa})\left(\dfrac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 2.00 \text{ m}} \right) = 1.05 \times 10^5$ Pa

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 5.44$ m/s

(c) $v_2 = 0$ means $(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2) = 0$

$p_1 - p_2 = -\rho g(y_1 - y_2)$

$$y_1 - y_2 = h - 1.00 \text{ m}$$

$$p = p_0 \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa}) \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right). \text{ This is } p_1, \text{ so}$$

$$(4.20 \times 10^5 \text{ Pa}) \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right) - 1.00 \times 10^5 \text{ Pa} = (9.80 \text{ m/s}^2)(1000 \text{ kg/m}^3)(1.00 \text{ m} - h)$$

$$(210/(4.00 - h)) - 100 = 9.80 - 9.80h, \text{ with } h \text{ in meters.}$$

$$210 = (4.00 - h)(109.8 - 9.80h)$$

$$9.80h^2 - 149h + 229.2 = 0 \text{ and } h^2 - 15.20h + 23.39 = 0$$

quadratic formula: $h = \frac{1}{2} \left(15.20 \pm \sqrt{(15.20)^2 - 4(23.39)} \right) = (7.60 \pm 5.86) \text{ m}$

h must be less than 4.00 m, so the only acceptable value is $h = 7.60 \text{ m} - 5.86 \text{ m} = 1.74 \text{ m}$

EVALUATE: The flow stops when $p + \rho g(y_1 - y_2)$ equals air pressure. For $h = 1.74$ m, $p = 9.3 \times 10^4$ Pa and $\rho g(y_1 - y_2) = 0.7 \times 10^4$ Pa, so $p + \rho g(y_1 - y_2) = 1.0 \times 10^5$ Pa, which is air pressure.

18.66. **IDENTIFY:** Use the ideal gas law to find the number of moles of air taken in with each breath and from this calculate the number of oxygen molecules taken in. Then find the pressure at an elevation of 2000 m and repeat the calculation.

SET UP: The number of molecules in a mole is $N_A = 6.022 \times 10^{23}$ molecules/mol.

$R = 0.08206$ L·atm/mol·K. Example 18.4 shows that the pressure variation with altitude y, when constant temperature is assumed, is $p = p_0 e^{-Mgy/RT}$. For air, $M = 28.8 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** $pV = nRT$ gives $n = \dfrac{pV}{RT} = \dfrac{(1.00 \text{ atm})(0.50 \text{ L})}{(0.08206 \text{ L·atm/mol·K})(293.15 \text{ K})} = 0.0208$ mol.

$N = (0.210)nN_A = (0.210)(0.0208 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 2.63 \times 10^{21}$ molecules.

(b) $\dfrac{Mgy}{RT} = \dfrac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(2000 \text{ m})}{(8.314 \text{ J/mol·K})(293.15 \text{ K})} = 0.2316.$

$p = p_0 e^{-Mgy/RT} = (1.00 \text{ atm})e^{-0.2316} = 0.793$ atm.

N is proportional to n, which is in turn proportional to p, so

$N = \left(\dfrac{0.793 \text{ atm}}{1.00 \text{ atm}} \right)(2.63 \times 10^{21} \text{ molecules}) = 2.09 \times 10^{21}$ molecules.

(c) Less O_2 is taken in with each breath at the higher altitude, so the person must take more breaths per minute.

EVALUATE: A given volume of gas contains fewer molecules when the pressure is lowered and the temperature is kept constant.

18.67. **IDENTIFY and SET UP:** Apply Eq.(18.2) to find n and then use Avogadro's number to find the number of molecules.

EXECUTE: Calculate the number of water molecules N.

Number of moles: $n = \dfrac{m_{tot}}{M} = \dfrac{50 \text{ kg}}{18.0 \times 10^{-3} \text{ kg/mol}} = 2.778 \times 10^3$ mol

$N = nN_A = (2.778 \times 10^3 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 1.7 \times 10^{27}$ molecules

Each water molecule has three atoms, so the number of atoms is $3(1.7 \times 10^{27}) = 5.1 \times 10^{27}$ atoms

EVALUATE: We could also use the masses in Example 18.5 to find the mass m of one H_2O molecule:

$m = 2.99 \times 10^{-26}$ kg. Then $N = m_{tot}/m = 1.7 \times 10^{27}$ molecules, which checks.

18.68. **IDENTIFY:** $pV = nRT = \dfrac{N}{N_A} RT$. Deviations will be noticeable when the volume V of a molecule is on the order of 1% of the volume of gas that contains one molecule.

SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

EXECUTE: The volume of gas per molecule is $\frac{RT}{N_A p}$, and the volume of a molecule is about

$V_0 = \frac{4}{3}\pi(2.0 \times 10^{-10}\,\text{m})^3 = 3.4 \times 10^{-29}\,\text{m}^3$. Denoting the ratio of these volumes as f,

$$p = f\frac{RT}{N_A V_0} = f\frac{(8.3145\,\text{J/mol} \cdot \text{K})(300\,\text{K})}{(6.022 \times 10^{23}\,\text{molecules/mol})(3.4 \times 10^{-29}\,\text{m}^3)} = (1.2 \times 10^8\,\text{Pa})f.$$

"Noticeable deviations" is a subjective term, but f on the order of 1.0% gives a pressure of 10^6 Pa.

EVALUATE: The forces between molecules also cause deviations from ideal-gas behavior.

18.69. **IDENTIFY:** Eq. (18.16) says that the average translational kinetic energy of each molecule is equal to $\frac{3}{2}kT$.

$v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$.

SET UP: $k = 1.381 \times 10^{-23}$ J/molecule $\cdot$ K.

EXECUTE: **(a)** $\frac{1}{2}m(v^2)_{\text{av}}$ depends only on T and both gases have the same T, so both molecules have the

same average translational kinetic energy. v_{rms} is proportional to $m^{-1/2}$, so the lighter molecules, A, have

the greater v_{rms}.

(b) The temperature of gas B would need to be raised.

(c) $\sqrt{\dfrac{T}{m}} = \dfrac{v_{\text{rms}}}{\sqrt{3k}} = \text{constant}$, so $\dfrac{T_A}{m_A} = \dfrac{T_B}{m_B}$.

$T_B = \left(\dfrac{m_B}{m_A}\right)T_A = \left(\dfrac{5.34 \times 10^{-26}\,\text{kg}}{3.34 \times 10^{-27}\,\text{kg}}\right)(283.15\,\text{K}) = 4.53 \times 10^3\,\text{K} = 4250°\text{C}.$

(d) $T_B > T_A$ so the B molecules have greater translational kinetic energy per molecule.

EVALUATE: In $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$ and $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$ the temperature T must be in kelvins.

18.70. **IDENTIFY:** The equations derived in the subsection Collisions Between Molecules in Section 18.3 can be applied to the bees. The average distance a bee travels between collisions is the mean free path, λ. The average

time between collisions is the mean free time, t_{mean}. The number of collisions per second is $\dfrac{dN}{dt} = \dfrac{1}{t_{\text{mean}}}$.

SET UP: $V = (1.25\,\text{m})^3 = 1.95\,\text{m}^3$. $r = 0.750 \times 10^{-2}$ m. $v = 1.10$ m/s. $N = 2500$.

EXECUTE: **(a)** $\lambda = \dfrac{V}{4\pi\sqrt{2}r^2 N} = \dfrac{1.95\,\text{m}^3}{4\pi\sqrt{2}(0.750 \times 10^{-2}\,\text{m})^2(2500)} = 0.780\,\text{m} = 78.0\,\text{cm}$

(b) $\lambda = vt_{\text{mean}}$, so $t_{\text{mean}} = \dfrac{\lambda}{v} = \dfrac{0.780\,\text{m}}{1.10\,\text{m/s}} = 0.709\,\text{s}$.

(c) $\dfrac{dN}{dt} = \dfrac{1}{t_{\text{mean}}} = \dfrac{1}{0.709\,\text{s}} = 1.41$ collisions/s

EVALUATE: The calculation is valid only if the motion of each bee is random.

18.71. **IDENTIFY:** The mass of one molecule is the molar mass, M, divided by the number of molecules in a mole, N_A. The average translational kinetic energy of a single molecule is $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$. Use

$pV = NkT$ to calculate N, the number of molecules.

SET UP: $k = 1.381 \times 10^{-23}$ J/molecule $\cdot$ K. $M = 28.0 \times 10^{-3}$ kg/mol. $T = 295.15$ K. The volume of the

balloon is $V = \frac{4}{3}\pi(0.250\,\text{m})^3 = 0.0654\,\text{m}^3$. $p = 1.25$ atm $= 1.27 \times 10^5$ Pa.

EXECUTE: **(a)** $m = \dfrac{M}{N_A} = \dfrac{28.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 4.65 \times 10^{-26}$ kg

(b) $\frac{1}{2} m(v^2)_{av} = \frac{3}{2} kT = \frac{3}{2}(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(295.15 \text{ K}) = 6.11 \times 10^{-21}$ J

(c) $N = \dfrac{pV}{kT} = \dfrac{(1.27 \times 10^5 \text{ Pa})(0.0654 \text{ m}^3)}{(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(295.15 \text{ K})} = 2.04 \times 10^{24}$ molecules

(d) The total average translational kinetic energy is

$N\left(\frac{1}{2} m(v^2)_{av}\right) = (2.04 \times 10^{24} \text{ molecules})(6.11 \times 10^{-21} \text{ J/molecule}) = 1.25 \times 10^4$ J.

EVALUATE: The number of moles is $n = \dfrac{N}{N_A} = \dfrac{2.04 \times 10^{24} \text{ molecules}}{6.022 \times 10^{23} \text{ molecules/mol}} = 3.39$ mol.

$K_{tr} = \frac{3}{2} nRT = \frac{3}{2}(3.39 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K}) = 1.25 \times 10^4$ J, which agrees with our results in part (d).

18.72. **IDENTIFY:** $U = mgy$. The mass of one molecule is $m = M/N_A$. $K_{av} = \frac{3}{2} kT$.

SET UP: Let $y = 0$ at the surface of the earth and $h = 400$ m. $N_A = 6.022 \times 10^{23}$ molecules/mol and

$k = 1.38 \times 10^{-23}$ J/K. $15.0°\text{C} = 288$ K.

EXECUTE: **(a)** $U = mgh = \dfrac{M}{N_A} gh = \left(\dfrac{28.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}}\right)(9.80 \text{ m/s}^2)(400 \text{ m}) = 1.82 \times 10^{-22}$ J.

(b) Setting $U = \dfrac{3}{2} kT$, $T = \dfrac{2}{3}\left(\dfrac{1.82 \times 10^{-22} \text{ J}}{1.38 \times 10^{-23} \text{ J/K}}\right) = 8.80$ K.

EVALUATE: **(c)** The average kinetic energy at 15.0°C is much larger than the increase in gravitational potential energy, so it is energetically possible for a molecule to rise to this height. But Example 18.8 shows that the mean free path will be very much less than this and a molecule will undergo many collisions as it rises. These numerous collisions transfer kinetic energy between molecules and make it highly unlikely that a given molecule can have very much of its translational kinetic energy converted to gravitational potential energy.

18.73. **IDENTIFY** and **SET UP:** At equilibrium $F(r) = 0$. The work done to increase the separation from r_2 to ∞

is $U(\infty) - U(r_2)$.

(a) EXECUTE: $U(r) = U_0[(R_0/r)^{12} - 2(R_0/r)^6]$

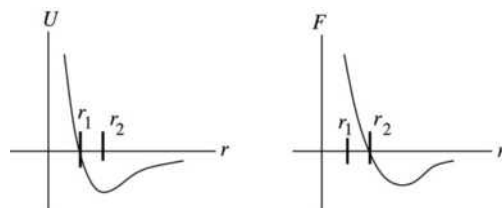Eq. (14.26): $F(r) = 12(U_0/R_0)[(R_0/r)^{13} - (R_0/r)^7]$. The graphs are given in Figure 18.73.

Figure 18.73

(b) equilibrium requires $F = 0$; occurs at point r_2. r_2 is where U is a minimum (stable equilibrium).

(c) $U = 0$ implies $[(R_0/r)^{12} 2(R_0/r)^6] = 0$

$(r_1/R_0)^6 = 1/2$ and $r_1 = R_0/(2)^{1/6}$

$F = 0$ implies $[(R_0/r)^{13} - (R_0/r)^7] = 0$

$(r_2/R_0)^6 = 1$ and $r_2 = R_0$

Then $r_1/r_2 = (R_0/2^{1/6})/R_0 = 2^{-1/6}$

(d) $W_{\text{other}} = \Delta U$

At $r \to \infty$, $U = 0$, so $W = -U(R_0) = -U_0[(R_0/R_0)^{12} - 2(R_0/R_0)^6] = +U_0$

EVALUATE: The answer to part (d), U_0, is the depth of the potential well shown in the graph of $U(r)$.

18.74. **IDENTIFY:** Use $pV = nRT$ to calculate the number of moles, n. Then $K_{\text{tr}} = \frac{3}{2}nRT$. The mass of the gas, m_{tot}, is given by $m_{\text{tot}} = nM$.

SET UP: $5.00 \text{ L} = 5.00 \times 10^{-3} \text{ m}^3$

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(1.01 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.2025$ moles

$K_{\text{tr}} = \frac{3}{2}(0.2025 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K}) = 758 \text{ J}.$

(b) $m_{\text{tot}} = nM = (0.2025 \text{ mol})(2.016 \times 10^{-3} \text{ kg/mol}) = 4.08 \times 10^{-4} \text{ kg}.$ The kinetic energy due to the speed of the jet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(4.08 \times 10^{-4} \text{ kg})(300.0 \text{ m/s})^2 = 18.4 \text{ J}.$ The total kinetic energy is

$K_{\text{tot}} = K + K_{\text{tr}} = 18.4 \text{ J} + 758 \text{ J} = 776 \text{ J}.$ The percentage increase is $\dfrac{K}{K_{\text{tot}}} \times 100\% = \dfrac{18.4 \text{ J}}{776 \text{ J}} \times 100\% = 2.37\%.$

(c) No. The temperature is associated with the random translational motion, and that hasn't changed.

EVALUATE: Eq. (18.13) gives $K_{\text{tr}} = \frac{3}{2}pV = \frac{3}{2}(1.01 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3) = 758 \text{ J},$ which agrees with our result in part (a). $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = 1.93 \times 10^3 \text{ m/s}.$ v_{rms} is a lot larger than the speed of the jet, so the percentage increase in the total kinetic energy, calculated in part (b), is small.

18.75. **IDENTIFY and SET UP:** Apply Eq. (18.19) for v_{rms}. The equation preceeding Eq. (18.12) relates v_{rms} and $(v_x)_{\text{rms}}$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{3RT/M}$

$v_{\text{rms}} = \sqrt{\dfrac{3(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{28.0 \times 10^{-3} \text{ kg/mol}}} = 517 \text{ m/s}$

(b) $(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$ so $\sqrt{(v_x^2)_{\text{av}}} = (1/\sqrt{3})\sqrt{(v^2)_{\text{av}}} = (1/\sqrt{3})v_{\text{rms}} = (1/\sqrt{3})(517 \text{ m/s}) = 298 \text{ m/s}$

EVALUATE: The speed of sound is approximately equal to $(v_x)_{\text{rms}}$ since it is the motion along the direction of propagation of the wave that transmits the wave.

18.76. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$

SET UP: $M = 1.99 \times 10^{30} \text{ kg}$, $R = 6.96 \times 10^8 \text{ m}$ and $G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3(1.38 \times 10^{-23} \text{ J/K})(5800 \text{ K})}{(1.67 \times 10^{-27} \text{ kg})}} = 1.20 \times 10^4 \text{ m/s}.$

(b) $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(6.96 \times 10^8 \text{ m})}} = 6.18 \times 10^5 \text{ m/s}.$

EVALUATE: **(c)** The escape speed is about 50 times the rms speed, and any of Figure 18.23 in the textbook, Eq. (18.32) or Table (18.2) will indicate that there is a negligibly small fraction of molecules with the escape speed.

18.77. **(a) IDENTIFY and SET UP:** Apply conservation of energy $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $U = -Gmm_p/r$. Let point 1 be at the surface of the planet, where the projectile is launched, and let point 2 be far from the earth. Just barely escapes says $v_2 = 0$.

EXECUTE: Only gravity does work says $W_{\text{other}} = 0$.

$U_1 = -Gmm_p/R_p$; $r_2 \to \infty$ so $U_2 = 0$; $v_2 = 0$ so $K_2 = 0$.

The conservation of energy equation becomes $K_1 - Gmm_p/R_p = 0$ and $K_1 = Gmm_p/R_p$.

But $g = Gm_p/R_p^2$ so $Gm_p/R_p = R_p g$ and $K_1 = mgR_p$, as was to be shown.

EVALUATE: The greater gR_p is, the more initial kinetic energy is required for escape.

(b) IDENTIFY and SET UP: Set K_1 from part (a) equal to the average kinetic energy of a molecule as given by Eq. (18.16). $\frac{1}{2}m(v^2)_{av} = mgR_p$ (from part (a)). But also, $\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$, so $mgR_p = \frac{3}{2}kT$

EXECUTE: $T = \dfrac{2mgR_p}{3k}$

nitrogen

$m_{N_2} = (28.0 \times 10^{-3} \text{ kg/mol})/(6.022 \times 10^{23} \text{ molecules/mol}) = 4.65 \times 10^{-26} \text{ kg/molecule}$

$T = \dfrac{2mgR_p}{3k} = \dfrac{2(4.65 \times 10^{-26} \text{ kg/molecule})(9.80 \text{ m/s}^2)(6.38 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 1.40 \times 10^5 \text{ K}$

hydrogen

$m_{H_2} = (2.02 \times 10^{-3} \text{ kg/mol})/(6.022 \times 10^{23} \text{ molecules/mol}) = 3.354 \times 10^{-27} \text{ kg/molecule}$

$T = \dfrac{2mgR_p}{3k} = \dfrac{2(3.354 \times 10^{-27} \text{ kg/molecule})(9.80 \text{ m/s}^2)(6.38 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 1.01 \times 10^4 \text{ K}$

(c) $T = \dfrac{2mgR_p}{3k}$

nitrogen

$T = \dfrac{2(4.65 \times 10^{-26} \text{ kg/molecule})(1.63 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 6370 \text{ K}$

hydrogen

$T = \dfrac{2(3.354 \times 10^{-27} \text{ kg/molecule})(1.63 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 459 \text{ K}$

(d) EVALUATE: The "escape temperatures" are much less for the moon than for the earth. For the moon a larger fraction of the molecules at a given temperature will have speeds in the Maxwell-Boltzmann distribution larger than the escape speed. After a long time most of the molecules will have escaped from the moon.

18.78. **IDENTIFY:** $v_{rms} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $M_{H_2} = 2.02 \times 10^{-3} \text{ kg/mol}$. $M_{O_2} = 32.0 \times 10^{-3} \text{ kg/mol}$. For earth, $M = 5.97 \times 10^{24} \text{ kg}$ and $R = 6.38 \times 10^6 \text{ m}$. For Jupiter, $M = 1.90 \times 10^{27} \text{ kg}$ and $R = 6.91 \times 10^7 \text{ m}$. For a sphere, $M = \rho V = \rho \dfrac{4}{3}\pi r^3$.

The escape speed is $v_{escape} = \sqrt{\dfrac{2GM}{R}}$.

EXECUTE: **(a)** Jupiter: $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(140 \text{ K})/(2.02 \times 10^{-3} \text{ kg/mol})} = 1.31 \times 10^3 \text{ m/s}$.

$v_{escape} = 6.06 \times 10^4 \text{ m/s}$. $v_{rms} = 0.022 v_{escape}$.

Earth: $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(220 \text{ K})/(2.02 \times 10^{-3} \text{ kg/mol})} = 1.65 \times 10^3 \text{ m/s}$. $v_{escape} = 1.12 \times 10^4 \text{ m/s}$.

$v_{rms} = 0.15 v_{escape}$.

(b) Escape from Jupiter is not likely for any molecule, while escape from earth is much more probable.

(c) $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(200 \text{ K})/(32.0 \times 10^{-3} \text{kg/mol})} = 395$ m/s. The radius of the asteroid is

$R = (3M/4\pi\rho)^{1/3} = 4.68 \times 10^5$ m, and the escape speed is $v_{escape} = \sqrt{2GM/R} = 542$ m/s. Over time the O_2

molecules would essentially all escape and there can be no such atmosphere.

EVALUATE: As Figure 18.23 in the textbook shows, there are some molecules in the velocity distribution that have speeds greater than v_{rms}. But as the speed increases above v_{rms} the number with speeds in that range decreases.

18.79.   IDENTIFY: $v_{rms} = \sqrt{\dfrac{3kT}{m}}$. The number of molecules in an object of mass m is $N = nN_A = \dfrac{m}{M}N_A$.

SET UP: The volume of a sphere of radius r is $V = \dfrac{4}{3}\pi r^3$.

EXECUTE: (a) $m = \dfrac{3kT}{v_{rms}^2} = \dfrac{3(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}{(0.0010 \text{ m/s})^2} = 1.24 \times 10^{-14}$ kg.

(b) $N = mN_A/M = (1.24 \times 10^{-14} \text{ kg})(6.022 \times 10^{23} \text{ molecules/mol})/(18.0 \times 10^{-3} \text{ kg/mol})$

$N = 4.16 \times 10^{11}$ molecules.

(c) The diameter is $D = 2r = 2\left(\dfrac{3V}{4\pi}\right)^{1/3} = 2\left(\dfrac{3m/\rho}{4\pi}\right)^{1/3} = 2\left(\dfrac{3(1.24 \times 10^{-14} \text{ kg})}{4\pi(920 \text{ kg/m}^3)}\right)^{1/3} = 2.95 \times 10^{-6}$ m which is

too small to see.

EVALUATE: v_{rms} decreases as m increases.

18.80.   IDENTIFY: For a simple harmonic oscillator, $x = A\cos\omega t$ and $v_x = -\omega A\sin\omega t$, with $\omega = \sqrt{k/m}$.

SET UP: The average value of $\cos(2\omega t)$ over one period is zero, so $(\sin^2 \omega t)_{av} = (\cos^2 \omega t)_{av} = \frac{1}{2}$.

EXECUTE: $x = A\cos\omega t$, $v_x = -\omega A\sin\omega t$, $U_{av} = \frac{1}{2}kA^2(\cos^2 \omega t)_{av}$, $K_{av} = \frac{1}{2}m\omega^2 A^2(\sin^2 \omega t)_{av}$. Using

$(\sin^2 \omega t)_{av} = (\cos^2 \omega t)_{av} = \frac{1}{2}$ and $m\omega^2 = k$ shows that $K_{av} = U_{av}$.

EVALUATE: In general, at any given instant of time $U \neq K$. It is only the values averaged over one period that are equal.

18.81.   IDENTIFY: The equipartition principle says that each atom has an average kinetic energy of $\frac{1}{2}kT$ for each

degree of freedom. There is an equal average potential energy.

SET UP: The atoms in a three-dimensional solid have three degrees of freedom and the atoms in a two-dimensional solid have two degrees of freedom.

EXECUTE: (a) In the same manner that Eq. (18.28) was obtained, the heat capacity of the two-dimensional solid would be $2R = 16.6$ J/mol·K.

(b) The heat capacity would behave qualitatively like those in Figure 18.21 in the textbook, and the heat capacity would decrease with decreasing temperature.

EVALUATE: At very low temperatures the equipartition theorem doesn't apply. Most of the atoms remain in their lowest energy states because the next higher energy level is not accessible.

18.82.   IDENTIFY: The equipartition principle says that each molecule has average kinetic energy of $\frac{1}{2}kT$ for

each degree of freedom. $I = 2m(L/2)^2$, where L is the distance between the two atoms in the molecule.

$K_{rot} = \frac{1}{2}I\omega^2$. $\omega_{rms} = \sqrt{(\omega^2)_{av}}$.

SET UP: The mass of one atom is $m = M/N_A = (16.0 \times 10^{-3} \text{ kg/mol})/(6.022 \times 10^{23} \text{ molecules/mol}) =$

2.66×10^{-26} kg.

EXECUTE: (a) The two degrees of freedom associated with the rotation for a diatomic molecule account for two-fifths of the total kinetic energy, so $K_{rot} = nRT = (1.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K}) = 2.49 \times 10^3$ J.

(b) $I = 2m(L/2)^2 = 2\left(\dfrac{16.0\times10^{-3}\text{ kg/mol}}{6.022\times10^{23}\text{ molecules/mol}}\right)(6.05\times10^{-11}\text{ m})^2 = 1.94\times10^{-46}\text{ kg}\cdot\text{m}^2$

(c) Since the result in part (b) is for one mole, the rotational kinetic energy for one atom is K_{rot}/N_A and

$\omega_{\text{rms}} = \sqrt{\dfrac{2K_{\text{rot}}/N_A}{I}} = \sqrt{\dfrac{2(2.49\times10^3\text{ J})}{(1.94\times10^{-46}\text{ kg}\cdot\text{m}^2)(6.022\times10^{23}\text{ molecules/mol})}} = 6.52\times10^{12}\text{ rad/s.}$ This is

much larger than the typical value for a piece of rotating machinery.

EVALUATE: The average rotational period, $T = \dfrac{2\pi\text{ rad}}{\omega_{\text{rms}}}$, for molecules is very short.

18.83.   **IDENTIFY:** $C_V = N\left(\frac{1}{2}R\right)$, where N is the number of degrees of freedom.

SET UP: There are three translational degrees of freedom.

EXECUTE: For CO_2, $N = 5$ and the contribution to C_V other than from vibration is

$\frac{5}{2}R = 20.79$ J/mol$\cdot$K and $C_V - \frac{5}{2}R = 0.270\, C_V$. So 27% of C_V is due to vibration. For both SO_2 and H_2S,

$N = 6$ and the contribution to C_V other than from vibration is $\frac{6}{2}R = 24.94$ J/mol$\cdot$K. The respective

fractions of C_V from vibration are 21% and 3.9%.

EVALUATE: The vibrational contribution is much less for H_2S. In H_2S the vibrational energy steps are

larger because the two hydrogen atoms have small mass and $\omega = \sqrt{k/m}$.

18.84.   **IDENTIFY:** Evaluate the integral, as specified in the problem.

SET UP: Use the integral formula given in Problem 18.85, with $\alpha = m/2kT$.

EXECUTE: (a) $\int_0^\infty f(v)\,dv = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}\,dv = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\left(\dfrac{1}{4(m/2kT)}\right)\sqrt{\dfrac{\pi}{m/2kT}} = 1$

EVALUATE: (b) $f(v)\,dv$ is the probability that a particle has speed between v and $v + dv$; the probability

that the particle has some speed is unity, so the sum (integral) of $f(v)\,dv$ must be 1.

18.85.   **IDENTIFY and SET UP:** Evaluate the integral in Eq. (18.31) as specified in the problem.

EXECUTE: $\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}\int_0^\infty v^4 e^{-mv^2/2kT}\,dv$

The integral formula with $n = 2$ gives $\int_0^\infty v^4 e^{-av^2}\,dv = (3/8a^2)\sqrt{\pi/a}$.

Apply with $a = m/2kT$, $\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}(3/8)(2kT/m)^2\sqrt{2\pi kT/m} = (3/2)(2kT/m) = 3kT/m.$

EVALUATE: Eq. (18.16) says $\frac{1}{2}m(v^2)_{\text{av}} = 3kT/2,$ so $(v^2)_{\text{av}} = 3kT/m,$ in agreement with our calculation.

18.86.   **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: If $v^2 = x,$ then $dx = 2v\,dv.$

EXECUTE: $\int_0^\infty v f(v)\,dv = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^3 e^{-mv^2/2kT}\,dv.$ Making the suggested change of variable,

$v^2 = x.$ $2v\,dv = dx,$ $v^3\,dv = (1/2)x\,dx,$ and the integral becomes

$$\int_0^\infty v f(v)\,dv = 2\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\int_0^\infty x e^{-mx/2kT}\,dx = 2\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\left(\dfrac{2kT}{m}\right)^2 = \dfrac{2}{\sqrt{\pi}}\sqrt{\dfrac{2kT}{m}} = \sqrt{\dfrac{8kT}{\pi m}}$$

which is Eq. (18.35).

EVALUATE: The integral $\int_0^\infty v f(v)\,dv$ is the definition of $v_{\text{av}}.$

18.87. **IDENTIFY:** $f(v)dv$ is the probability that a particle has a speed between v and $v+dv$. Eq. (18.32) gives $f(v)$. v_{mp} is given by Eq. (18.34).

SET UP: For O_2, the mass of one molecule is $m = M/N_A = 5.32 \times 10^{-26}$ kg.

EXECUTE: **(a)** $f(v)dv$ is the fraction of the particles that have speed in the range from v to $v+dv$. The number of particles with speeds between v and $v+dv$ is therefore $dN = Nf(v)dv$ and

$$\Delta N = N \int_v^{v+\Delta v} f(v)dv.$$

(b) Setting $v = v_{mp} = \sqrt{\dfrac{2kT}{m}}$ in $f(v)$ gives $f(v_{mp}) = 4\pi \left(\dfrac{m}{2\pi kT}\right)^{3/2} \left(\dfrac{2kT}{m}\right) e^{-1} = \dfrac{4}{e\sqrt{\pi}v_{mp}}$. For oxygen

gas at 300 K, $v_{mp} = 3.95 \times 10^2$ m/s and $f(v)\Delta v = 0.0421$.

(c) Increasing v by a factor of 7 changes f by a factor of $7^2 e^{-48}$, and $f(v)\Delta v = 2.94 \times 10^{-21}$.

(d) Multiplying the temperature by a factor of 2 increases the most probable speed by a factor of $\sqrt{2}$, and the answers are decreased by $\sqrt{2}$: 0.0297 and 2.08×10^{-21}.

(e) Similarly, when the temperature is one-half what it was in parts (b) and (c), the fractions increase by $\sqrt{2}$ to 0.0595 and 4.15×10^{-21}.

EVALUATE: **(f)** At lower temperatures, the distribution is more sharply peaked about the maximum (the most probable speed), as is shown in Figure 18.23a in the textbook.

18.88. **IDENTIFY:** Apply the definition of relative humidity given in the problem. $pV = nRT = \dfrac{m_{tot}}{M} RT$.

SET UP: $M = 18.0 \times 10^{-3}$ kg/mol.

EXECUTE: **(a)** The pressure due to water vapor is $(0.60)(2.34 \times 10^3 \text{ Pa}) = 1.40 \times 10^3$ Pa.

(b) $m_{tot} = \dfrac{MpV}{RT} = \dfrac{(18.0 \times 10^{-3} \text{ kg/mol})(1.40 \times 10^3 \text{ Pa})(1.00 \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} = 10$ g

EVALUATE: The vapor pressure of water vapor at this temperature is much less than the total atmospheric pressure of 1.0×10^5 Pa.

18.89. **IDENTIFY:** The measurement gives the dew point. Relative humidity is defined in Problem 18.88.

SET UP: relative humidity $= \dfrac{\text{partial pressure of water vapor at temperature } T}{\text{vapor pressure of water at temperature } T}$

EXECUTE: The experiment shows that the dew point is 16.0°C, so the partial pressure of water vapor at 30.0°C is equal to the vapor pressure at 16.0°C, which is 1.81×10^3 Pa.

Thus the relative humidity $= \dfrac{1.81 \times 10^3 \text{ Pa}}{4.25 \times 10^3 \text{ Pa}} = 0.426 = 42.6\%$.

EVALUATE: The lower the dew point is compared to the air temperature, the smaller the relative humidity.

18.90. **IDENTIFY:** Use the definition of relative humidity in Problem 18.88 and the vapor pressure table in Problem 18.89.

SET UP: At 28.0°C the vapor pressure of water is 3.78×10^3 Pa.

EXECUTE: For a relative humidity of 35%, the partial pressure of water vapor is $(0.35)(3.78 \times 10^3 \text{ Pa}) = 1.323 \times 10^3$ Pa. This is close to the vapor pressure at 12°C, which would be at an altitude $(30°C - 12°C)/(0.6 \text{ C}°/100 \text{ m}) = 3$ km above the ground. For a relative humidity of 80%, the vapor pressure will be the same as the water pressure at around 24°C, corresponding to an altitude of about 1 km.

EVALUATE: Clouds form at a lower height when the relative humidity at the surface is larger.

18.91.   IDENTIFY: Eq. (18.21) gives the mean free path λ. In Eq. (18.20) use $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ in place of v.

$pV = nRT = NkT$. The escape speed is $v_{escape} = \sqrt{\dfrac{2GM}{R}}$.

SET UP: For atomic hydrogen, $M = 1.008 \times 10^{-3}$ kg/mol.

EXECUTE: (a) From Eq. (18.21),

$\lambda = (4\pi\sqrt{2}r^2(N/V))^{-1} = (4\pi\sqrt{2}(5.0\times10^{-11}\text{ m})^2(50\times10^6\text{ m}^{-3}))^{-1} = 4.5\times10^{11}$ m.

(b) $v_{rms} = \sqrt{3RT/M} = \sqrt{3(8.3145\text{ J/mol}\cdot\text{K})(20\text{ K})/(1.008\times10^{-3}\text{ kg/mol})} = 703$ m/s, and the time between collisions is then $(4.5\times10^{11}\text{ m})/(703\text{ m/s}) = 6.4\times10^8$ s, about 20 yr. Collisions are not very important.

(c) $p = (N/V)kT = (50/1.0\times10^{-6}\text{ m}^3)(1.381\times10^{-23}\text{ J/K})(20\text{ K}) = 1.4\times10^{-14}$ Pa.

(d) $v_{escape} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G(Nm/V)(4\pi R^3/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR^2}$

$v_{escape} = \sqrt{(8\pi/3)(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(50\times10^6\text{ m}^{-3})(1.67\times10^{-27}\text{ kg})(10\times9.46\times10^{15}\text{ m})^2}$

$v_{escape} = 650$ m/s. This is lower than v_{rms} and the cloud would tend to evaporate.

(e) In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$,

$kT_{ISM}(N/V)_{ISM} = kT_{nebula}(N/V)_{nebula}$ and the result follows.

(f) With the result of part (e),

$$T_{ISM} = T_{nebula}\left(\frac{(N/V)_{nebula}}{(N/V)_{ISM}}\right) = (20\text{ K})\left(\frac{50\times10^6\text{ m}^3}{(200\times10^{-6}\text{ m}^3)^{-1}}\right) = 2\times10^5\text{ K},$$

more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

EVALUATE: The temperature of a gas is determined by the average kinetic energy per atom of the gas. The energy density for the gas also depends on the number of atoms per unit volume, and this is very small for the ISM.

18.92.   IDENTIFY: Follow the procedure of Example 18.4, but use $T = T_0 - \alpha y$.

SET UP: $\ln(1+x) \approx x$ when x is very small.

EXECUTE: (a) $\dfrac{dp}{dy} = -\dfrac{pM}{RT}$, which in this case becomes $\dfrac{dp}{p} = -\dfrac{Mg}{R}\dfrac{dy}{T_0 - \alpha y}$. This integrates to

$\ln\left(\dfrac{p}{p_0}\right) = \dfrac{Mg}{R\alpha}\ln\left(1 - \dfrac{\alpha y}{T_0}\right)$, or $p = p_0\left(1 - \dfrac{\alpha y}{T_0}\right)^{Mg/R\alpha}$.

(b) For sufficiently small α, $\ln\left(1 - \dfrac{\alpha y}{T_0}\right) \approx -\dfrac{\alpha y}{T_0}$, and this gives the expression derived in Example 18.4.

(c) $\left(1 - \dfrac{(0.6\times10^{-2}\text{ C°/m})(8863\text{ m})}{(288\text{ K})}\right) = 0.8154$, $\dfrac{Mg}{R\alpha} = \dfrac{(28.8\times10^{-3})(9.80\text{ m/s}^2)}{(8.3145\text{ J/mol}\cdot\text{K})(0.6\times10^{-2}\text{ C°/m})} = 5.6576$ and

$p_0(0.8154)^{5.6576} = 0.315$ atm, which is 0.95 of the result found in Example 18.4.

EVALUATE: The pressure is calculated to decrease more rapidly with altitude when we assume that T also decreases with altitude.

18.93.   IDENTIFY and SET UP: For N particles, $v_{av} = \dfrac{\Sigma v_i}{N}$ and $v_{rms} = \sqrt{\dfrac{\Sigma v_i^2}{N}}$.

EXECUTE: (a) $v_{av} = \frac{1}{2}(v_1 + v_2)$, $v_{rms} = \dfrac{1}{\sqrt{2}}\sqrt{v_1^2 + v_2^2}$ and

$$v_{rms}^2 - v_{av}^2 = \frac{1}{2}(v_1^2 + v_2^2) - \frac{1}{4}(v_1^2 + v_2^2 + 2v_1v_2) = \frac{1}{4}(v_1^2 + v_2^2 - 2v_1v_2) = \frac{1}{4}(v_1 - v_2)^2$$

This shows that $v_{rms} \geq v_{av}$, with equality holding if and only if the particles have the same speeds.

(b) $v'^2_{rms} = \frac{1}{N+1}(Nv_{rms}^2 + u^2), v'_{av} = \frac{1}{N+1}(Nv_{av} + u),$ and the given forms follow immediately.

(c) The algebra is similar to that in part (a); it helps somewhat to express

$$v'^2_{av} = \frac{1}{(N+1)^2}(N((N+1) - 1)v_{av}^2 + 2Nv_{av}u + ((N+1) - N)u^2).$$

$$v'^2_{av} = \frac{N}{N+1}v_{av}^2 + \frac{N}{(N+1)^2}(-v_{av}^2 + 2v_{av}u - u^2) + \frac{1}{N+1}u^2$$

Then,

$$v'^2_{rms} - v'^2_{av} = \frac{N}{(N+1)}(v_{rms}^2 - v_{av}^2) + \frac{N}{(N+1)^2}(v_{av}^2 - 2v_{av}u + u^2) = \frac{N}{N+1}(v_{rms}^2 - v_{av}^2) + \frac{N}{(N+1)^2}(v_{av} - u)^2.$$

If $v_{rms} > v_{av}$, then this difference is necessarily positive, and $v'_{rms} > v'_{av}$.

(d) The result has been shown for $N = 1$, and it has been shown that validity for N implies validity for $N + 1$; by induction, the result is true for all N.

EVALUATE: $v_{rms} > v_{av}$ because v_{rms} gives more weight to particles that have greater speed.