17

TEMPERATURE AND HEAT

17.1. **IDENTIFY** and **SET UP:** $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: **(a)** $T_F = (9/5)(-62.8) + 32 = -81.0°F$

(b) $T_F = (9/5)(56.7) + 32 = 134.1°F$

(c) $T_F = (9/5)(31.1) + 32 = 88.0°F$

EVALUATE: Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.2. **IDENTIFY** and **SET UP:** To convert a temperature between °C and K use $T_C = T_K - 273.15$. To convert from °F to °C, subtract 32° and multiply by 5/9. To convert from °C to °F, multiply by 9/5 and add 32°. To convert a temperature difference, use that Celsius and Kelvin degrees are the same size and that $9\text{ F°} = 5\text{ C°}$.

EXECUTE: **(a)** $T_C = T_K - 273.15 = 310 - 273.15 = 36.9°C$; $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(36.9°) + 32° = 98.4°F$.

(b) $T_K = T_C + 273.15 = 40 + 273.15 = 313\text{ K}$; $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(40°) + 32° = 104°F$.

(c) 7 C° = 7 K; 7 C° = (7 C°)(9 F°/5 C°) = 13 F°.

(d) 4.0°C: $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(4.0°) + 32° = 39.2°F$; $T_K = T_C + 273.15 = 4.0 + 273.15 = 277\text{ K}$.

−160°C: $T_F = \frac{9}{5}T_C + 32° = \frac{9}{5}(-160°) + 32° = -256°F$; $T_K = T_C + 273.15 = -160 + 273.15 = 113\text{ K}$.

(e) $T_C = \frac{5}{9}(T_F - 32°) = \frac{5}{9}(105° - 32°) = 41°C$; $T_K = T_C + 273.15 = 41 + 273.15 = 314\text{ K}$.

EVALUATE: Celsius-Fahrenheit conversions do not involve simple proportions due to the additive constant of 32°, but Celsius-Kelvin conversions require only simple addition/subtraction of 273.15.

17.3. **IDENTIFY:** Convert ΔT between different scales.

SET UP: ΔT is the same on the Celsius and Kelvin scales. $180\text{ F°} = 100\text{ C°}$, so $1\text{ C°} = \frac{9}{5}\text{F°}$.

EXECUTE: **(a)** $\Delta T = 49.0\text{ F°}$. $\Delta T = (49.0\text{ F°})\left(\frac{1\text{ C°}}{\frac{9}{5}\text{ F°}}\right) = 27.2\text{ C°}$.

(b) $\Delta T = -100\text{ F°}$. $\Delta T = (-100.0\text{ F°})\left(\frac{1\text{ C°}}{\frac{9}{5}\text{ F°}}\right) = -55.6\text{ C°}$

EVALUATE: The magnitude of the temperature change is larger in F° than in C°.

17.4. **IDENTIFY:** Set $T_C = T_F$ and $T_F = T_K$.

SET UP: $T_F = \frac{9}{5}T_C + 32°C$ and $T_K = T_C + 273.15 = \frac{5}{9}(T_F - 32°) + 273.15$

EXECUTE: **(a)** $T_F = T_C = T$ gives $T = \frac{9}{5}T + 32°$ and $T = -40°$; $-40°C = -40°F$.

(b) $T_F = T_K = T$ gives $T = \frac{5}{9}(T - 32°) + 273.15$ and $T = \frac{9}{4}\left(-\left(\frac{5}{9}\right)(32°) + 273.15\right) = 575°$; $575°F = 575\text{ K}$.

EVALUATE: Since $T_K = T_C + 273.15$ there is no temperature at which Celsius and Kelvin thermometers agree.

17.5. **IDENTIFY:** Convert ΔT in kelvins to C° and to F°.

SET UP: $1\text{ K} = 1\text{ C}° = \frac{9}{5}\text{F}°$

EXECUTE: **(a)** $\Delta T_\text{F} = \frac{9}{5}\Delta T_\text{C} = \frac{9}{5}(-10.0\text{ C}°) = -18.0\text{ F}°$

(b) $\Delta T_\text{C} = \Delta T_\text{K} = -10.0\text{ C}°$

EVALUATE: Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given ΔT value.

17.6. **IDENTIFY:** Convert T_K to T_C and then convert T_C to T_F.

SET UP: $T_\text{K} = T_\text{C} + 273.15$ and $T_\text{F} = \frac{9}{5}T_\text{C} + 32°$.

EXECUTE: **(a)** $T_\text{C} = 400 - 273.15 = 127°\text{C}$, $T_\text{F} = (9/5)(126.85) + 32 = 260°\text{F}$

(b) $T_\text{C} = 95 - 273.15 = -178°\text{C}$, $T_\text{F} = (9/5)(-178.15) + 32 = -289°\text{F}$

(c) $T_\text{C} = 1.55\times10^7 - 273.15 = 1.55\times10^7°\text{C}$, $T_\text{F} = (9/5)(1.55\times10^7) + 32 = 2.79\times10^7°\text{F}$

EVALUATE: All temperatures on the Kelvin scale are positive. T_C is negative if the temperature is below the freezing point of water.

17.7. **IDENTIFY:** When the volume is constant, $\dfrac{T_2}{T_1} = \dfrac{p_2}{p_1}$, for T in kelvins.

SET UP: $T_\text{triple} = 273.16\text{ K}$. Figure 17.7 in the textbook gives that the temperature at which CO_2 solidifies is $T_{CO_2} = 195\text{ K}$.

EXECUTE: $$p_2 = p_1\left(\frac{T_2}{T_1}\right) = (1.35\text{ atm})\left(\frac{195\text{ K}}{273.16\text{ K}}\right) = 0.964\text{ atm}$$

EVALUATE: The pressure decreases when T decreases.

17.8. **IDENTIFY:** Apply Eq. (17.5) and solve for p.

SET UP: $p_\text{triple} = 325$ mm of mercury

EXECUTE: $$p = (325.0\text{ mm of mercury})\left(\frac{373.15\text{ K}}{273.16\text{ K}}\right) = 444\text{ mm of mercury}$$

EVALUATE: mm of mercury is a unit of pressure. Since Eq. (17.5) involves a ratio of pressures, it is not necessary to convert the pressure to units of Pa.

17.9. **IDENTIFY and SET UP:** Fit the data to a straight line for $p(T)$ and use this equation to find T when $p = 0$.

EXECUTE: **(a)** If the pressure varies linearly with temperature, then $p_2 = p_1 + \gamma(T_2 - T_1)$.

$$\gamma = \frac{p_2 - p_1}{T_2 - T_1} = \frac{6.50\times10^4\text{ Pa} - 4.80\times10^4\text{ Pa}}{100°\text{C} - 0.01°\text{C}} = 170.0\text{ Pa/C}°$$

Apply $p = p_1 + \gamma(T - T_1)$ with $T_1 = 0.01°\text{C}$ and $p = 0$ to solve for T.

$$0 = p_1 + \gamma(T - T_1)$$

$$T = T_1 - \frac{p_1}{\gamma} = 0.01°\text{C} - \frac{4.80\times10^4\text{ Pa}}{170\text{ Pa/C}°} = -282°\text{C}.$$

(b) Let $T_1 = 100°\text{C}$ and $T_2 = 0.01°\text{C}$; use Eq. (17.4) to calculate p_2. Eq. (17.4) says $T_2/T_1 = p_2/p_1$, where T is in kelvins.

$p_2 = p_1\left(\dfrac{T_2}{T_1}\right) = 6.50\times10^4\text{ Pa}\left(\dfrac{0.01 + 273.15}{100 + 273.15}\right) = 4.76\times10^4\text{ Pa}$; this differs from the 4.80×10^4 Pa that was measured so Eq. (17.4) is not precisely obeyed.

EVALUATE: The answer to part (a) is in reasonable agreement with the accepted value of $-273°\text{C}$.

17.10. **IDENTIFY:** $1\text{ K} = 1\text{ C}°$ and $1\text{ C}° = \frac{9}{5}\text{F}°$, so $1\text{ K} = \frac{9}{5}\text{R}°$.

SET UP: On the Kelvin scale, the triple point is 273.16 K.

EXECUTE: $T_\text{triple} = (9/5)273.16\text{ K} = 491.69°\text{R}$.

EVALUATE: One could also look at Figure 17.7 in the textbook and note that the Fahrenheit scale extends from $-460°\text{F}$ to $+32°\text{F}$ and conclude that the triple point is about $492°\text{R}$.

17.11. **IDENTIFY:** $\Delta L = L_0 \alpha \Delta T$

SET UP: For steel, $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$

EXECUTE: $\Delta L = (1.2\times10^{-5}(\text{C}°)^{-1})(1410\ \text{m})(18.0°\text{C} - (-5.0°\text{C})) = +0.39\ \text{m}$

EVALUATE: The length increases when the temperature increases. The fractional increase is very small, since $\alpha\Delta T$ is small.

17.12. **IDENTIFY:** Apply $\Delta L = \alpha L_0 \Delta T$ and calculate ΔT. Then $T_2 = T_1 + \Delta T$, with $T_1 = 15.5°\text{C}$.

SET UP: Table 17.1 gives $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$ for steel.

EXECUTE: $\Delta T = \dfrac{\Delta L}{\alpha L_0} = \dfrac{0.471\ \text{ft}}{[1.2\times10^{-5}\ (\text{C}°)^{-1}][1671\ \text{ft}]} = 23.5\ \text{C}°$. $T_2 = 15.5°\text{C} + 23.5\ \text{C}° = 39.0°\text{C}$.

EVALUATE: Since then the lengths enter in the ratio $\Delta L/L_0$, we can leave the lengths in ft.

17.13. **IDENTIFY:** Apply $L = L_0(1 + \alpha\Delta T)$ to the diameter D of the penny.

SET UP: $1\ \text{K} = 1\ \text{C}°$, so we can use temperatures in $°\text{C}$.

EXECUTE: Death Valley: $\alpha D_0 \Delta T = (2.6\times10^{-5}\ (\text{C}°)^{-1})(1.90\ \text{cm})(28.0\ \text{C}°) = 1.4\times10^{-3}\ \text{cm}$, so the diameter is 1.9014 cm. Greenland: $\alpha D_0 \Delta T = -3.6\times10^{-3}\ \text{cm}$, so the diameter is 1.8964 cm.

EVALUATE: When T increases the diameter increases and when T decreases the diameter decreases.

17.14. **IDENTIFY:** Apply $L = L_0(1 + \alpha\Delta T)$ to the diameter d of the rivet.

SET UP: For aluminum, $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$. Let d_0 be the diameter at –78.0°C and d be the diameter at 23.0°C.

EXECUTE: $d = d_0 + \Delta d = d_0(1 + \alpha\Delta T) = (0.4500\ \text{cm})(1 + (2.4\times10^{-5}(\text{C}°)^{-1})(23.0°\text{C} - [-78.0°\text{C}])$.

$d = 0.4511\ \text{cm} = 4.511\ \text{mm}$.

EVALUATE: We could have let d_0 be the diameter at $23.0°\text{C}$ and d be the diameter at $-78.0°\text{C}$. Then $\Delta T = -78.0°\text{C} - 23.0°\text{C}$.

17.15. **IDENTIFY:** Find the change ΔL in the diameter of the lid. The diameter of the lid expands according to Eq. (17.6).

SET UP: Assume iron has the same α as steel, so $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: $\Delta L = \alpha L_0 \Delta T = (1.2\times10^{-5}\ (\text{C}°)^{-1})(725\ \text{mm})(30.0\ \text{C}°) = 0.26\ \text{mm}$

EVALUATE: In Eq. (17.6), ΔL has the same units as L.

17.16. **IDENTIFY:** $\Delta V = \beta V_0 \Delta T$. Use the diameter at $-15°\text{C}$ to calculate the value of V_0 at that temperature.

SET UP: For a hemisphere of radius R, the volume is $V = \frac{2}{3}\pi R^3$. Table 17.2 gives $\beta = 7.2\times10^{-5}\ (\text{C}°)^{-1}$ for aluminum.

EXECUTE: $V_0 = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi(27.5\ \text{m})^3 = 4.356\times10^4\ \text{m}^3$.

$\Delta V = (7.2\times10^{-5}\ (\text{C}°)^{-1})(4.356\times10^4\ \text{m}^3)(35°\text{C} - [-15°\text{C}]) = 160\ \text{m}^3$

EVALUATE: We could also calculate $R = R_0(1 + \alpha\Delta T)$ and calculate the new V from R. The increase in volume is $V - V_0$, but we would have to be careful to avoid round-off errors when two large volumes of nearly the same size are subtracted.

17.17. **IDENTIFY:** Apply $\Delta V = V_0 \beta \Delta T$.

SET UP: For copper, $\beta = 5.1\times10^{-5}\ (\text{C}°)^{-1}$. $\Delta V/V_0 = 0.150\times10^{-2}$.

EXECUTE: $\Delta T = \dfrac{\Delta V/V_0}{\beta} = \dfrac{0.150\times10^{-2}}{5.1\times10^{-5}\ (\text{C}°)^{-1}} = 29.4\ \text{C}°$. $T_\text{f} = T_\text{i} + \Delta T = 49.4°\text{C}$.

EVALUATE: The volume increases when the temperature increases.

17.18. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$ to the tank and to the ethanol.

SET UP: For ethanol, $\beta_e = 75\times10^{-5}\ (\text{C}°)^{-1}$. For steel, $\beta_s = 3.6\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: The volume change for the tank is

$\Delta V_s = V_0\beta_s\Delta T = (2.80\ \text{m}^3)(3.6\times10^{-5}(\text{C}°)^{-1})(-14.0\ \text{C}°) = -1.41\times10^{-3}\ \text{m}^3 = -1.41\ \text{L}.$

The volume change for the ethanol is

$\Delta V_e = V_0\beta_e\Delta T = (2.80\ \text{m}^3)(75\times10^{-5}\ (\text{C}°)^{-1})(-14.0\ \text{C}°) = -2.94\times10^{-2}\ \text{m}^3 = -29.4\ \text{L}.$

The empty volume in the tank is $\Delta V_e - \Delta V_s = -29.4\ \text{L} - (-1.4\ \text{L}) = -28.0\ \text{L}$. 28.0 L of ethanol can be added to the tank.

EVALUATE: Both volumes decrease. But $\beta_e > \beta_s$, so the magnitude of the volume decrease for the ethanol is greater than it is for the tank.

17.19. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$ to the volume of the flask and to the mercury. When heated, both the volume of the flask and the volume of the mercury increase.

SET UP: For mercury, $\beta_{\text{Hg}} = 18\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: 8.95 cm^3 of mercury overflows, so $\Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = 8.95\ \text{cm}^3$.

EXECUTE: $\Delta V_{\text{Hg}} = V_0\beta_{\text{Hg}}\Delta T = (1000.00\ \text{cm}^3)(18\times10^{-5}\ (\text{C}°)^{-1})(55.0\ \text{C}°) = 9.9\ \text{cm}^3$.

$\Delta V_{\text{glass}} = \Delta V_{\text{Hg}} - 8.95\ \text{cm}^3 = 0.95\ \text{cm}^3$. $\beta_{\text{glass}} = \dfrac{\Delta V_{\text{glass}}}{V_0\Delta T} = \dfrac{0.95\ \text{cm}^3}{(1000.00\ \text{cm}^3)(55.0\ \text{C}°)} = 1.7\times10^{-5}\ (\text{C}°)^{-1}.$

EVALUATE: The coefficient of volume expansion for the mercury is larger than for glass. When they are heated, both the volume of the mercury and the inside volume of the flask increase. But the increase for the mercury is greater and it no longer all fits inside the flask.

17.20. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ to each linear dimension of the surface.

SET UP: The area can be written as $A = aL_1L_2$, where a is a constant that depends on the shape of the surface. For example, if the object is a sphere, $a = 4\pi$ and $L_1 = L_2 = r$. If the object is a cube, $a = 6$ and $L_1 = L_2 = L$, the length of one side of the cube. For aluminum, $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** $A_0 = aL_{01}L_{02}$. $L_1 = L_{01}(1+\alpha\Delta T)$. $L_2 = L_{02}(1+\alpha\Delta T)$.

$A = aL_1L_2 = aL_{01}L_{02}(1+\alpha\Delta T)^2 = A_0(1 + 2\alpha\Delta T + [\alpha\Delta T]^2)$. $\alpha\Delta T$ is very small, so $[\alpha\Delta T]^2$ can be neglected and $A = A_0(1+2\alpha\Delta T)$. $\Delta A = A - A_0 = (2\alpha)A_0\Delta T$

(b) $\Delta A = (2\alpha)A_0\Delta T = (2)(2.4\times10^{-5}\ (\text{C}°)^{-1})(\pi(0.275\ \text{m})^2)(12.5\ \text{C}°) = 1.4\times10^{-4}\ \text{m}^2$

EVALUATE: The derivation assumes the object expands uniformly in all directions.

17.21. **IDENTIFY** and **SET UP:** Apply the result of Exercise 17.20a to calculate ΔA for the plate, and then $A = A_0 + \Delta A$.

EXECUTE: **(a)** $A_0 = \pi r_0^2 = \pi(1.350\ \text{cm}/2)^2 = 1.431\ \text{cm}^2$

(b) Exercise 17.20 says $\Delta A = 2\alpha A_0\Delta T$, so

$\Delta A = 2(1.2\times10^{-5}\ \text{C}°^{-1})(1.431\ \text{cm}^2)(175°\text{C} - 25°\text{C}) = 5.15\times10^{-3}\ \text{cm}^2$

$A = A_0 + \Delta A = 1.436\ \text{cm}^2$

EVALUATE: A hole in a flat metal plate expands when the metal is heated just as a piece of metal the same size as the hole would expand.

17.22. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ to the diameter D_{ST} of the steel cylinder and the diameter D_{BR} of the brass piston.

SET UP: For brass, $\alpha_{\text{BR}} = 2.0\times10^{-5}\ (\text{C}°)^{-1}$. For steel, $\alpha_{\text{ST}} = 1.2\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** No, the brass expands more than the steel.

(b) Call D_0 the inside diameter of the steel cylinder at 20°C. At 150°C, $D_{\text{ST}} = D_{\text{BR}}$.

$D_0 + \Delta D_{\text{ST}} = 25.000\ \text{cm} + \Delta D_{\text{BR}}$. This gives $D_0 + \alpha_{\text{ST}}D_0\Delta T = 25.000\ \text{cm} + \alpha_{\text{BR}}(25.000\ \text{cm})\Delta T$.

$$D_0 = \frac{25.000\text{ cm}(1+\alpha_{\text{BR}}\Delta T)}{1+\alpha_{\text{ST}}\Delta T} = \frac{(25.000\text{ cm})[1+(2.0\times10^{-5}\ (\text{C°})^{-1})(130\text{ C°})]}{1+(1.2\times10^{-5}\ (\text{C°})^{-1})(130\text{ C°})} = 25.026\text{ cm.}$$

EVALUATE: The space inside the steel cylinder expands just like a solid piece of steel of the same size.

17.23. **IDENTIFY** and **SET UP:** For part (a), apply Eq. (17.6) to the linear expansion of the wire. For part (b), apply Eq. (17.12) and calculate F/A.

EXECUTE: **(a)** $\Delta L = \alpha L_0 \Delta T$

$$\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{1.9\times10^{-2}\text{ m}}{(1.50\text{ m})(420°\text{C} - 20°\text{C})} = 3.2\times10^{-5}\ (\text{C°})^{-1}$$

(b) Eq. (17.12): stress $F/A = -Y\alpha\Delta T$

$\Delta T = 20°\text{C} - 420°\text{C} = -400\text{ C°}$ (ΔT always means final temperature minus initial temperature)

$F/A = -(2.0\times10^{11}\text{ Pa})(3.2\times10^{-5}(\text{C°})^{-1})(-400\text{ C°}) = +2.6\times10^{9}\text{ Pa}$

EVALUATE: F/A is positive means that the stress is a tensile (stretching) stress. The answer to part (a) is consistent with the values of α for metals in Table 17.1. The tensile stress for this modest temperature decrease is huge.

17.24. **IDENTIFY:** Apply Eq. (17.12) and solve for F.

SET UP: For brass, $Y = 0.9\times10^{11}\text{ Pa}$ and $\alpha = 2.0\times10^{-5}\ (\text{C°})^{-1}$.

EXECUTE: $F = -Y\alpha\Delta T A = -(0.9\times10^{11}\text{ Pa})(2.0\times10^{-5}(\text{C°})^{-1})(-110\text{ C°})(2.01\times10^{-4}\text{ m}^2) = 4.0\times10^{4}\text{ N}$

EVALUATE: A large force is required. ΔT is negative and a positive tensile force is required.

17.25. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ and stress $= F/A = -Y\alpha\Delta T$.

SET UP: For steel, $\alpha = 1.2\times10^{-5}\ (\text{C°})^{-1}$ and $Y = 2.0\times10^{11}\text{ Pa}$.

EXECUTE: **(a)** $\Delta L = L_0\alpha\Delta T = (12.0\text{ m})(1.2\times10^{-5}\ (\text{C°})^{-1})(35.0\text{ C°}) = 5.0\text{ mm}$

(b) stress $= -Y\alpha\Delta T = -(2.0\times10^{11}\text{ Pa})(1.2\times10^{-5}\ (\text{C°})^{-1})(35.0\text{ C°}) = -8.4\times10^{7}\text{ Pa}$. The minus sign means the stress is compressive.

EVALUATE: Commonly occurring temperature changes result in very small fractional changes in length but very large stresses if the length change is prevented from occurring.

17.26. **IDENTIFY:** The heat required is $Q = mc\Delta T$. $P = 200\text{ W} = 200\text{ J/s}$, which is energy divided by time.

SET UP: For water, $c = 4.19\times10^{3}\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = mc\Delta T = (0.320\text{ kg})(4.19\times10^{3}\text{ J/kg}\cdot\text{K})(60.0\text{ C°}) = 8.04\times10^{4}\text{ J}$

(b) $t = \dfrac{8.04\times10^{4}\text{ J}}{200.0\text{ J/s}} = 402\text{ s} = 6.7\text{ min}$

EVALUATE: 0.320 kg of water has volume 0.320 L. The time we calculated in part (b) is consistent with our everyday experience.

17.27. **IDENTIFY** and **SET UP:** Apply Eq. (17.13) to the kettle and water.

EXECUTE: kettle

$Q = mc\Delta T$, $c = 910\text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.50\text{ kg})(910\text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 8.873\times10^{4}\text{ J}$

water

$Q = mc\Delta T$, $c = 4190\text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.80\text{ kg})(4190\text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 4.902\times10^{5}\text{ J}$

Total $Q = 8.873\times10^{4}\text{ J} + 4.902\times10^{5}\text{ J} = 5.79\times10^{5}\text{ J}$

EVALUATE: Water has a much larger specific heat capacity than aluminum, so most of the heat goes into raising the temperature of the water.

17.28. **IDENTIFY** and **SET UP:** Use Eq. (17.13)

EXECUTE: **(a)** $Q = mc\Delta T$

$m = \frac{1}{2}(1.3\times10^{-3}\ \text{kg}) = 0.65\times10^{-3}\ \text{kg}$

$Q = (0.65\times10^{-3}\ \text{kg})(1020\ \text{J/kg}\cdot\text{K})(37°\text{C} - (-20°\text{C})) = 38\ \text{J}$

(b) 20 breaths/min (60 min/1 h) = 1200 breaths/h

So $Q = (1200)(38\ \text{J}) = 4.6\times10^{4}\ \text{J}.$

EVALUATE: The heat loss rate is $Q/t = 13\ \text{W}$.

17.29. **IDENTIFY:** Apply $Q = mc\Delta T$. $m = w/g$.

SET UP: The temperature change is $\Delta T = 18.0\ \text{K}$.

EXECUTE: $c = \dfrac{Q}{m\Delta T} = \dfrac{gQ}{w\Delta T} = \dfrac{(9.80\ \text{m/s}^2)(1.25\times10^4\ \text{J})}{(28.4\ \text{N})(18.0\ \text{K})} = 240\ \text{J/kg}\cdot\text{K}.$

EVALUATE: The value for c is similar to that for silver in Table 17.3, so it is a reasonable result.

17.30. **IDENTIFY:** The heat input increases the temperature of 2.5 gal/min of water from 10°C to 49°C.

SET UP: 1.00 L of water has a mass of 1.00 kg, so
$9.46\ \text{L/min} = (9.46\ \text{L/min})(1.00\ \text{kg/L})(1\ \text{min}/60\ \text{s}) = 0.158\ \text{kg/s}$. For water, $c = 4190\ \text{J/kg}\cdot\text{C°}$.

EXECUTE: $Q = mc\Delta T$ so $H = (Q/t) = (m/t)c\,\Delta T$. Putting in the numbers gives

$H = (0.158\ \text{kg/s})(4190\ \text{J/kg}\cdot\text{C°})(49°\text{C} - 10°\text{C}) = 2.6\times10^4\ \text{W} = 26\ \text{kW}.$

EVALUATE: The power requirement is large, the equivalent of 260 100-watt light bulbs, but this large power is needed only for short periods of time. The rest of the time, the unit uses no energy, unlike a conventional water heater which continues to replace lost heat even when hot water is not needed.

17.31. **IDENTIFY:** Apply $Q = mc\Delta T$ to find the heat that would raise the temperature of the student's body 7 C°.

SET UP: $1\ \text{W} = 1\ \text{J/s}$

EXECUTE: Find Q to raise the body temperature from 37°C to 44°C.

$Q = mc\Delta T = (70\ \text{kg})(3480\ \text{J/kg}\cdot\text{K})(7\ \text{C°}) = 1.7\times10^6\ \text{J}.$

$t = \dfrac{1.7\times10^6\ \text{J}}{1200\ \text{J/s}} = 1400\ \text{s} = 23\ \text{min}.$

EVALUATE: Heat removal mechanisms are essential to the well-being of a person.

17.32. **IDENTIFY** and **SET UP:** Set the change in gravitational potential energy equal to the quantity of heat added to the water.

EXECUTE: The change in mechanical energy equals the decrease in gravitational potential energy, $\Delta U = -mgh;\ |\Delta U| = mgh.\ Q = |\Delta U| = mgh$ implies $mc\Delta T = mgh$

$\Delta T = gh/c = (9.80\ \text{m/s}^2)(225\ \text{m})/(4190\ \text{J/kg}\cdot\text{K}) = 0.526\ \text{K} = 0.526\ \text{C°}$

EVALUATE: Note that the answer is independent of the mass of the object. Note also the small change in temperature that corresponds to this large change in height!

17.33. **IDENTIFY:** The work done by friction is the loss of mechanical energy. The heat input for a temperature change is $Q = mc\Delta T$.

SET UP: The crate loses potential energy mgh, with $h = (8.00\ \text{m})\sin 36.9°$, and gains kinetic energy $\frac{1}{2}mv_2^2$.

EXECUTE: **(a)**

$W_f = -mgh + \frac{1}{2}mv_2^2 = -(35.0\ \text{kg})((9.80\ \text{m/s}^2)(8.00\ \text{m})\sin 36.9° + \frac{1}{2}(2.50\ \text{m/s})^2) = -1.54\times10^3\ \text{J}.$

(b) Using the results of part (a) for Q gives $\Delta T = (1.54\times10^3\ \text{J})/((35.0\ \text{kg})(3650\ \text{J/kg}\cdot\text{K})) = 1.21\times10^{-2}\ \text{C°}.$

EVALUATE: The temperature rise is very small.

17.34. **IDENTIFY:** The work done by the brakes equals the initial kinetic energy of the train. Use the volume of the air to calculate its mass. Use $Q = mc\Delta T$ applied to the air to calculate ΔT for the air.

SET UP: $K = \frac{1}{2}mv^2$. $m = \rho V$.

EXECUTE: The initial kinetic energy of the train is $K = \frac{1}{2}(25{,}000 \text{ kg})(15.5 \text{ m/s})^2 = 3.00\times10^6$ J.

Therefore, Q for the air is 3.00×10^6 J. $m = \rho V = (1.20 \text{ kg/m}^3)(65.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 1.87\times10^4$ kg.

$Q = mc\Delta T$ gives $\Delta T = \dfrac{Q}{mc} = \dfrac{3.00\times10^6 \text{ J}}{(1.87\times10^4 \text{ kg})(1020 \text{ J/kg}\cdot\text{K})} = 0.157 \text{ C}°.$

EVALUATE: The mass of air in the station is comparable to the mass of the train and the temperature rise is small.

17.35. **IDENTIFY:** Set $K = \frac{1}{2}mv^2$ equal to $Q = mc\Delta T$ for the nail and solve for ΔT.

SET UP: For aluminum, $c = 0.91\times10^3$ J/kg · K.

EXECUTE: The kinetic energy of the hammer before it strikes the nail is

$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.80 \text{ kg})(7.80 \text{ m/s})^2 = 54.8$ J. Each strike of the hammer transfers $0.60(54.8 \text{ J}) = 32.9$ J,

and with 10 strikes $Q = 329$ J. $Q = mc\Delta T$ and $\Delta T = \dfrac{Q}{mc} = \dfrac{329 \text{ J}}{(8.00\times10^{-3} \text{ kg})(0.91\times10^3 \text{ J/kg}\cdot\text{K})} = 45.2 \text{ C}°.$

EVALUATE: This agrees with our experience that hammered nails get noticeably warmer.

17.36. **IDENTIFY** and **SET UP:** Use the power and time to calculate the heat input Q and then use Eq. (17.13) to calculate c.

(a) EXECUTE: $P = Q/t$, so the total heat transferred to the liquid is $Q = Pt = (65.0 \text{ W})(120 \text{ s}) = 7800$ J.

Then $Q = mc\Delta T$ gives $c = \dfrac{Q}{m\Delta T} = \dfrac{7800 \text{ K}}{0.780 \text{ kg}(22.54°\text{C} - 18.55°\text{C})} = 2.51\times10^3$ J/kg · K

(b) EVALUATE: Then the actual Q transferred to the liquid is less than 7800 J so the actual c is less than our calculated value; our result in part (a) is an overestimate.

17.37. **IDENTIFY:** Some of the kinetic energy of the bullet is transformed through friction into heat, which raises the temperature of the water in the tank.

SET UP: Set the loss of kinetic energy of the bullet equal to the heat energy Q transferred to the water. $Q = mc\Delta T$. From Table 17.3, the specific heat of water is 4.19×10^3 J/kg · C°.

SOLVE: The kinetic energy lost by the bullet is

$K_\text{i} - K_\text{f} = \frac{1}{2}m(v_\text{i}^2 - v_\text{f}^2) = \frac{1}{2}(15.0\times10^{-3} \text{ kg})[(865 \text{ m/s})^2 - (534 \text{ m/s})^2] = 3.47\times10^3$ J, so for the water

$Q = 3.47\times10^3$ J. $Q = mc\Delta T$ gives $\Delta T = \dfrac{Q}{mc} = \dfrac{3.47\times10^3 \text{ J}}{(13.5 \text{ kg})(4.19\times10^3 \text{ J/kg}\cdot\text{C}°)} = 0.0613 \text{ C}°.$

EVALUATE: The heat energy required to change the temperature of ordinary-size objects is very large compared to the typical kinetic energies of moving objects.

17.38. **IDENTIFY:** The latent heat of fusion L_f is defined by $Q = mL_\text{f}$ for the solid $\rightarrow$ liquid phase transition. For a temperature change, $Q = mc\Delta T$.

SET UP: At $t = 1$ min the sample is at its melting point and at $t = 2.5$ min all the sample has melted.

EXECUTE: **(a)** It takes 1.5 min for all the sample to melt once its melting point is reached and the heat input during this time interval is $(1.5 \text{ min})(10.0\times10^3 \text{ J/min}) = 1.50\times10^4$ J. $Q = mL_\text{f}$.

$$L_\text{f} = \frac{Q}{m} = \frac{1.50\times10^4 \text{ J}}{0.500 \text{ kg}} = 3.00\times10^4 \text{ J/kg}.$$

(b) The liquid's temperature rises 30 C° in 1.5 min. $Q = mc\Delta T$.

$$c_\text{liquid} = \frac{Q}{m\Delta T} = \frac{1.50\times10^4 \text{ J}}{(0.500 \text{ kg})(30 \text{ C}°)} = 1.00\times10^3 \text{ J/kg}\cdot\text{K}.$$

The solid's temperature rises 15 C° in 1.0 min. $c_\text{solid} = \dfrac{Q}{m\Delta T} = \dfrac{1.00\times10^4 \text{ J}}{(0.500 \text{ kg})(15 \text{ C}°)} = 1.33\times10^3$ J/kg · K.

EVALUATE: The specific heat capacities for the liquid and solid states are different. The values of c and L_f that we calculated are within the range of values in Tables 17.3 and 17.4.

17.39. **IDENTIFY** and **SET UP:** Heat comes out of the metal and into the water. The final temperature is in the range $0 < T < 100°\text{C}$, so there are no phase changes. $Q_{\text{system}} = 0.$

(a) EXECUTE: $Q_{\text{water}} + Q_{\text{metal}} = 0$

$m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}} = 0$

$(1.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(2.0\text{ C}°) + (0.500\text{ kg})(c_{\text{metal}})(-78.0\text{ C}°) = 0$

$c_{\text{metal}} = 215\text{ J/kg}\cdot\text{K}$

(b) EVALUATE: Water has a larger specific heat capacity so stores more heat per degree of temperature change.

(c) If some heat went into the styrofoam then Q_{metal} should actually be larger than in part (a), so the true c_{metal} is larger than we calculated; the value we calculated would be smaller than the true value.

17.40. **IDENTIFY:** The heat that comes out of the person goes into the ice-water bath and causes some of the ice to melt.

SET UP: Normal body temperature is 98.6°F = 37.0°C, so for the person $\Delta T = -5\text{ C}°$. The ice-water bath stays at 0°C. A mass m of ice melts and $Q_{\text{ice}} = mL_{\text{f}}$. From Table 17.4, for water $L_{\text{f}} = 334\times10^3\text{ J/kg}$.

EXECUTE: $Q_{\text{person}} = mc\Delta T = (70.0\text{ kg})(3480\text{ J/kg}\cdot\text{C}°)(-5.0\text{ C}°) = -1.22\times10^6\text{ J}$. Therefore, the amount of heat that goes into the ice is $1.22\times10^6\text{ J}$. $m_{\text{ice}}L_{\text{f}} = 1.22\times10^6\text{ J}$ and $m_{\text{ice}} = \dfrac{1.22\times10^6\text{ J}}{334\times10^3\text{ J/kg}} = 3.7\text{ kg}.$

EVALUATE: If less ice than this is used, all the ice melts and the temperature of the water in the bath rises above 0°C.

17.41. **IDENTIFY:** The heat lost by the cooling copper is absorbed by the water and the pot, which increases their temperatures.

SET UP: For copper, $c_{\text{c}} = 390\text{ J/kg}\cdot\text{K}$. For iron, $c_{\text{i}} = 470\text{ J/kg}\cdot\text{K}$. For water, $c_{\text{w}} = 4.19\times10^3\text{ J/kg}\cdot\text{K}$.

EXECUTE: For the copper pot,

$Q_{\text{c}} = m_{\text{c}}c_{\text{c}}\,\Delta T_{\text{c}} = (0.500\text{ kg})(390\text{ J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (195\text{ J/K})T - 3900\text{ J}$. For the block of iron,

$Q_{\text{i}} = m_{\text{i}}c_{\text{i}}\,\Delta T_{\text{i}} = (0.250\text{ kg})(470\text{ J/kg}\cdot\text{K})(T - 85.0°\text{C}) = (117.5\text{ J/K})T - 9988\text{ J}$. For the water,

$Q_{\text{w}} = m_{\text{w}}c_{\text{w}}\,\Delta T_{\text{w}} = (0.170\text{ kg})(4190\text{ J/kg}\cdot\text{K})(T - 20.0°\text{C}) = (712.3\text{ J/K})T - 1.425\times10^4\text{ J}$. $\Sigma Q = 0$ gives

$(195\text{ J/K})T - 3900\text{ J} + (117.5\text{ J/K})T - 9988\text{ J} + (712.3\text{ J/K})T - 1.425\times10^4\text{ J}$. $T = \dfrac{2.814\times10^4\text{ J}}{1025\text{ J/K}} = 27.5°\text{C}.$

EVALUATE: The basic principle behind this problem is conservation of energy: no energy is lost; it is only transferred.

17.42. **IDENTIFY:** The energy generated in the body is used to evaporate water, which prevents the body from overheating.

SET UP: Energy is (power)(time); calculate the heat energy Q produced in one hour. The mass m of water that vaporizes is related to Q by $Q = mL_{\text{v}}$. 1.0 kg of water has a volume of 1.0 L.

EXECUTE: **(a)** $Q = (0.80)(500\text{ W})(3600\text{ s}) = 1.44\times10^6\text{ J}$. The mass of water that evaporates each hour is

$$m = \frac{Q}{L_{\text{v}}} = \frac{1.44\times10^6\text{ J}}{2.42\times10^6\text{ J/kg}} = 0.60\text{ kg}.$$

(b) (0.60 kg/h)(1.0 L/kg) = 0.60 L/h. The number of bottles of water is $\dfrac{0.60\text{ L/h}}{0.750\text{ L/bottle}} = 0.80\text{ bottles/h}.$

EVALUATE: It is not unreasonable to drink 8/10 of a bottle of water per hour during vigorous exercise.

17.43. **IDENTIFY:** If it cannot be gotten rid of in some way, the metabolic energy transformed to heat will increase the temperature of the body.

SET UP: From Problem 17.42, $Q = 1.44\times10^6\text{ J}$ and $m = 70\text{ kg}$. $Q = mc\Delta T$. Convert the temperature change in C° to F° using that 9 F° = 5 C°.

EXECUTE: **(a)** $Q = mc\Delta T$ so $\Delta T = \dfrac{Q}{mc} = \dfrac{1.44\times10^6\text{ J}}{(70\text{ kg})(3500\text{ J/kg}\cdot\text{C}°)} = 5.9\text{ C}°.$

(b) $\Delta T = (5.9°\text{C})\left(\dfrac{9\ \text{F}°}{5\ \text{C}°}\right) = 10.6°\text{F}.$ $T = 98.6°\text{F} + 10.6\ \text{F}° = 109°\text{F}.$

EVALUATE: A temperature this high can cause heat stroke and be lethal.

17.44. **IDENTIFY:** By energy conservation, the heat lost by the water is gained by the ice. This heat must first increase the temperature of the ice from −40.0°C to the melting point of 0.00°C, then melt the ice, and finally increase its temperature to 20.0°C. The target variable is the mass of the water m.

SET UP: $Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{w}}\Delta T_{\text{melted ice}}$ and $Q_{\text{water}} = mc_{\text{w}}\Delta T_{\text{w}}$.

EXECUTE: Using $Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{w}}\Delta T_{\text{melted ice}}$, with the values given in the table in the text, we have $Q_{\text{ice}} = (0.200\ \text{kg})[2100\ \text{J/(kg}\cdot\text{C°)}](40.0\text{C°}) + (0.200\ \text{kg})(3.34\times10^5\ \text{J/kg})$

$+ (0.200\ \text{kg})[4190\ \text{J/(kg}\cdot\text{C°)}](20.0\text{C°}) = 1.004\times10^5\ \text{J}.$

$Q_{\text{water}} = mc_{\text{w}}\Delta T_{\text{w}} = m[4190\ \text{J/(kg}\cdot\text{C°)}](20.0\text{C°} - 80.0\text{C°}) = -(251{,}400\ \text{J/kg})m$. $Q_{\text{ice}} + Q_{\text{water}} = 0$ gives

$1.004\times10^5\ \text{J} = (251{,}400\ \text{J/kg})m.$ $m = 0.399\ \text{kg}.$

EVALUATE: There is about twice as much water as ice because the water must provide the heat not only to melt the ice but also to increase its temperature.

17.45. **IDENTIFY:** By energy conservation, the heat lost by the copper is gained by the ice. This heat must first increase the temperature of the ice from –20.0°C to the melting point of 0.00°C, then melt some of the ice. At the final thermal equilibrium state, there is ice and water, so the temperature must be 0.00°C. The target variable is the initial temperature of the copper.

SET UP: For temperature changes, $Q = mc\Delta T$ and for a phase change from solid to liquid $Q = mL_{\text{F}}$.

EXECUTE: For the ice,

$Q_{\text{ice}} = (2.00\ \text{kg})[2100\ \text{J/(kg}\cdot\text{C°)}](20.0\text{C°}) + (0.80\ \text{kg})(3.34\times10^5\ \text{J/kg}) = 3.512\times10^5\ \text{J}.$ For the copper, using the specific heat from the table in the text gives

$Q_{\text{copper}} = (6.00\ \text{kg})[390\ \text{J/(kg}\cdot\text{C°)}](0°\text{C} - T) = -(2.34\times10^3\ \text{J/C°})T.$ Setting the sum of the two heats equal to zero gives $3.512\times10^5\ \text{J} = (2.34\times10^3\ \text{J/C°})T,$ which gives $T = 150°\text{C}.$

EVALUATE: Since the copper has a smaller specific heat than that of ice, it must have been quite hot initially to provide the amount of heat needed.

17.46. **IDENTIFY:** Apply $Q = mc\Delta T$ to each object. The net heat flow Q_{system} for the system (man, soft drink) is zero.

SET UP: The mass of 1.00 L of water is 1.00 kg. Let the man be designated by the subscript m and the "'water" by w. T is the final equilibrium temperature. $c_{\text{w}} = 4190\ \text{J/kg}\cdot\text{K}.$ $\Delta T_{\text{K}} = \Delta T_{\text{C}}.$

EXECUTE: **(a)** $Q_{\text{system}} = 0$ gives $m_{\text{m}}c_{\text{m}}\Delta T_{\text{m}} + m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} = 0.$ $m_{\text{m}}c_{\text{m}}(T - T_{\text{m}}) + m_{\text{w}}c_{\text{w}}(T - T_{\text{w}}) = 0.$

$m_{\text{m}}c_{\text{m}}(T_{\text{m}} - T) = m_{\text{w}}c_{\text{w}}(T - T_{\text{w}}).$ Solving for T, $T = \dfrac{m_{\text{m}}c_{\text{m}}T_{\text{m}} + m_{\text{w}}c_{\text{w}}T_{\text{w}}}{m_{\text{m}}c_{\text{m}} + m_{\text{w}}c_{\text{w}}}.$

$$T = \frac{(70.0\ \text{kg})(3480\ \text{J/kg}\cdot\text{K})(37.0°\text{C}) + (0.355\ \text{kg})(4190\ \text{J/kg}\cdot\text{C°})(12.0°\text{C})}{(70.0\ \text{kg})(3480\ \text{J/kg}\cdot\text{C°}) + (0.355\ \text{kg})(4190\ \text{J/kg}\cdot\text{C°})} = 36.85°\text{C}$$

(b) It is possible a sensitive digital thermometer could measure this change since they can read to $0.1°\text{C}$. It is best to refrain from drinking cold fluids prior to orally measuring a body temperature due to cooling of the mouth.

EVALUATE: Heat comes out of the body and its temperature falls. Heat goes into the soft drink and its temperature rises.

17.47. **IDENTIFY:** For the man's body, $Q = mc\Delta T$.

SET UP: From Exercise 17.46, $\Delta T = 0.15\ \text{C°}$ when the body returns to $37.0°\text{C}$.

EXECUTE: The rate of heat loss is Q/t. $\dfrac{Q}{t} = \dfrac{mc\Delta T}{t}$ and $t = \dfrac{mc\Delta T}{(Q/t)}$.

$$t = \frac{(70.355\ \text{kg})(3480\ \text{J/kg}\cdot\text{C°})(0.15\ \text{C°})}{7.00\times10^6\ \text{J/day}} = 0.00525\ \text{d} = 7.6\ \text{minutes}.$$

EVALUATE: Even if all the BMR energy stays in the body, it takes the body several minutes to return to its normal temperature.

17.48. **IDENTIFY:** For a temperature change $Q = mc\Delta T$ and for the liquid to solid phase change $Q = -mL_f$.

SET UP: For water, $c = 4.19\times10^3$ J/kg·K and $L_f = 3.34\times10^5$ J/kg.

EXECUTE: $Q = mc\Delta T - mL_f = (0.350\text{ kg})([4.19\times10^3\text{ J/kg}\cdot\text{K}][-18.0\text{ C°}] - 3.34\times10^5\text{ J/kg}) = -1.43\times10^5\text{ J}.$

The minus sign says 1.43×10^5 J must be removed from the water.

$$(1.43\times10^5\text{ J})\left(\frac{1\text{ cal}}{4.186\text{ J}}\right) = 3.42\times10^4\text{ cal} = 34.2\text{ kcal}.$$

$$(1.43\times10^5\text{ J})\left(\frac{1\text{ Btu}}{1055\text{ J}}\right) = 136\text{ Btu}.$$

EVALUATE: $Q < 0$ when heat comes out of an object. The equation $Q = mc\Delta T$ puts in the correct sign automatically, from the sign of $\Delta T = T_f - T_i$. But in $Q = \pm L$ we must select the correct sign.

17.49. **IDENTIFY** and **SET UP:** Use Eq. (17.13) for the temperature changes and Eq. (17.20) for the phase changes.

EXECUTE: Heat must be added to do the following:

ice at $-10.0°\text{C} \rightarrow$ ice at $0°\text{C}$

$Q_{ice} = mc_{ice}\Delta T = (12.0\times10^{-3}\text{ kg})(2100\text{ J/kg}\cdot\text{K})(0°\text{C} - (-10.0°\text{C})) = 252\text{ J}$

phase transition ice $(0°\text{C}) \rightarrow$ liquid water $(0°\text{C})$(melting)

$Q_{melt} = +mL_f = (12.0\times10^{-3}\text{ kg})(334\times10^3\text{ J/kg}) = 4.008\times10^3\text{ J}$

water at $0°\text{C}$ (from melted ice) $\rightarrow$ water at $100°\text{C}$

$Q_{water} = mc_{water}\Delta T = (12.0\times10^{-3}\text{ kg})(4190\text{ J/kg}\cdot\text{K})(100°\text{C} - 0°\text{C}) = 5.028\times10^3\text{ J}$

phase transition water $(100°\text{C}) \rightarrow$ steam $(100°\text{C})$(boiling)

$Q_{boil} = +mL_v = (12.0\times10^{-3}\text{ kg})(2256\times10^3\text{ J/kg}) = 2.707\times10^4\text{ J}$

The total Q is $Q = 252\text{ J} + 4.008\times10^3\text{ J} + 5.028\times10^3\text{ J} + 2.707\times10^4\text{ J} = 3.64\times10^4\text{ J}$

$(3.64\times10^4\text{ J})(1\text{ cal}/4.186\text{ J}) = 8.70\times10^3\text{ cal}$

$(3.64\times10^4\text{ J})(1\text{ Btu}/1055\text{ J}) = 34.5\text{ Btu}$

EVALUATE: Q is positive and heat must be added to the material. Note that more heat is needed for the liquid to gas phase change than for the temperature changes.

17.50. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change and $Q = +mL_f$ for the solid to liquid phase transition. The ice starts to melt when its temperature reaches $0.0°\text{C}$. The system stays at $0.00°\text{C}$ until all the ice has melted.

SET UP: For ice, $c = 2.10\times10^3$ J/kg·K. For water, $L_f = 3.34\times10^5$ J/kg.

EXECUTE: **(a)** Q to raise the temperature of ice to $0.00°\text{C}$:

$$Q = mc\Delta T = (0.550\text{ kg})(2.10\times10^3\text{ J/kg}\cdot\text{K})(15.0\text{ C°}) = 1.73\times10^4\text{ J}.\quad t = \frac{1.73\times10^4\text{ J}}{800.0\text{ J/min}} = 21.7\text{ min}.$$

(b) To melt all the ice requires $Q = mL_f = (0.550\text{ kg})(3.34\times10^5\text{ J/kg}) = 1.84\times10^5\text{ J}.$

$t = \dfrac{1.84\times10^5\text{ J}}{800.0\text{ J/min}} = 230\text{ min}.$ The total time after the start of the heating is 252 min.

(c) A graph of T versus t is sketched in Figure 17.50.

EVALUATE: It takes much longer for the ice to melt than it takes the ice to reach the melting point.

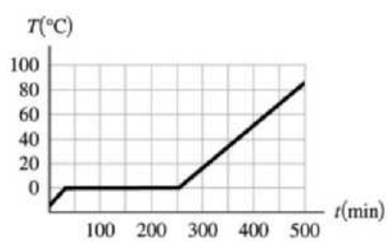


Figure 17.50

17.51. **IDENTIFY** and **SET UP:** The heat that must be added to a lead bullet of mass *m* to melt it is $Q = mc\Delta T + mL_\text{f}$ ($mc\Delta T$ is the heat required to raise the temperature from 25°C to the melting point of 327.3°C; mL_f is the heat required to make the solid $\rightarrow$ liquid phase change.) The kinetic energy of the bullet if its speed is *v* is $K = \frac{1}{2}mv^2$.

EXECUTE: $K = Q$ says $\frac{1}{2}mv^2 = mc\Delta T + mL_\text{f}$

$v = \sqrt{2(c\Delta T + L_\text{f})}$

$v = \sqrt{2[(130 \text{ J/kg}\cdot\text{K})(327.3°\text{C} - 25°\text{C}) + 24.5\times10^3 \text{ J/kg}]} = 357 \text{ m/s}$

EVALUATE: This is a typical speed for a rifle bullet. A bullet fired into a block of wood does partially melt, but in practice not all of the initial kinetic energy is converted to heat that remains in the bullet.

17.52. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_\text{v}$.

SET UP: For water, $L_\text{v} = 2.256\times10^6$ J/kg and $c = 4.19\times10^3$ J/kg $\cdot$ K.

EXECUTE: **(a)** $Q = +m(-L_\text{v} + c\Delta T)$

$Q = +(25.0\times10^{-3} \text{ kg})(-2.256\times10^6 \text{ J/kg} + [4.19\times10^3 \text{ J/kg}\cdot\text{K}][-66.0 \text{ C°}]) = -6.33\times10^4 \text{ J}$

(b) $Q = mc\Delta T = (25.0\times10^{-3} \text{ kg})(4.19\times10^3 \text{ J/kg}\cdot\text{K})(-66.0 \text{ C°}) = -6.91\times10^3 \text{ J}$.

(c) The total heat released by the water that starts as steam is nearly a factor of ten larger than the heat released by water that starts at 100°C. Steam burns are much more severe than hot-water burns.

EVALUATE: For a given amount of material, the heat for a phase change is typically much more than the heat for a temperature change.

17.53. **IDENTIFY:** Use $Q = Mc\Delta T$ to find Q for a temperature rise from 34.0°C to 40.0°C. Set this equal to $Q = mL_\text{v}$ and solve for *m*, where *m* is the mass of water the camel would have to drink.

SET UP: $c = 3480$ J/kg $\cdot$ K and $L_\text{v} = 2.42\times10^6$ J/kg. For water, 1.00 kg has a volume 1.00 L. $M = 400$ kg is the mass of the camel.

EXECUTE: The mass of water that the camel saves is

$$m = \frac{Mc\Delta T}{L_\text{v}} = \frac{(400 \text{ kg})(3480 \text{ J/kg}\cdot\text{K})(6.0 \text{ K})}{(2.42\times10^6 \text{ J/kg})} = 3.45 \text{ kg} \text{ which is a volume of 3.45 L.}$$

EVALUATE: This is nearly a gallon of water, so it is an appreciable savings.

17.54. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the liquid $\rightarrow$ vapor phase change, $Q = +mL_\text{v}$.

SET UP: The density of water is 1000 kg/m^3.

EXECUTE: **(a)** The heat that goes into mass *m* of water to evaporate it is $Q = +mL_\text{v}$. The heat flow for the man is $Q = m_\text{man}c\Delta T$, where $\Delta T = -1.00$ C°. $\Sigma Q = 0$ so $mL_\text{v} + m_\text{man}c\Delta T = 0$ and

$$m = -\frac{m_\text{man}c\Delta T}{L_\text{v}} = -\frac{(70.0 \text{ kg})(3480 \text{ J/kg}\cdot\text{K})(-1.00 \text{ C°})}{2.42\times10^6 \text{ J/kg}} = 0.101 \text{ kg} = 101 \text{ g}.$$

(b) $V = \frac{m}{\rho} = \frac{0.101 \text{ kg}}{1000 \text{ kg/m}^3} = 1.01\times10^{-4} \text{ m}^3 = 101 \text{ cm}^3$. This is about 35% of the volume of a soft-drink can.

EVALUATE: Fluid loss by evaporation from the skin can be significant.

17.55. **IDENTIFY:** The asteroid's kinetic energy is $K = \frac{1}{2}mv^2$. To boil the water, its temperature must be raised to 100.0°C and the heat needed for the phase change must be added to the water.

SET UP: For water, $c = 4190$ J/kg $\cdot$ K and $L_\text{v} = 2256\times10^3$ J/kg.

EXECUTE: $K = \frac{1}{2}(2.60\times10^{15} \text{ kg})(32.0\times10^3 \text{ m/s})^2 = 1.33\times10^{24}$ J. $Q = mc\Delta T + mL_\text{v}$.

$$m = \frac{Q}{c\Delta T + L_\text{v}} = \frac{1.33\times10^{22} \text{ J}}{(4190 \text{ J/kg}\cdot\text{K})(90.0 \text{ K}) + 2256\times10^3 \text{ J/kg}} = 5.05\times10^{15} \text{ kg}.$$

EVALUATE: The mass of water boiled is 2.5 times the mass of water in Lake Superior.

17.56. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change. The net Q for the system (sample, can and water) is zero.

SET UP: For water, $c_w = 4.19\times10^3$ J/kg·K. For copper, $c_c = 390$ J/kg·K.

EXECUTE: For the water, $Q_w = m_w c_w \Delta T_w = (0.200\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(7.1\text{ C}°) = 5.95\times10^3$ J.

For the copper can, $Q_c = m_c c_c \Delta T_c = (0.150\text{ kg})(390\text{ J/kg}\cdot\text{K})(7.1\text{ C}°) = 415$ J.

For the sample, $Q_s = m_s c_s \Delta T_s = (0.085\text{ kg})c_s(-73.9\text{ C}°)$.

$\Sigma Q = 0$ gives $(0.085\text{ kg})(-73.9\text{ C}°)c_s + 415\text{ J} + 5.95\times10^3\text{ J} = 0$. $c_s = 1.01\times10^3$ J/kg·K.

EVALUATE: Heat comes out of the sample and goes into the water and the can. The value of c_s we calculated is consistent with the values in Table 17.3.

17.57. **IDENTIFY** and **SET UP:** Heat flows out of the water and into the ice. The net heat flow for the system is zero. The ice warms to 0°C, melts, and then the water from the melted ice warms from 0°C to the final temperature.

EXECUTE: $Q_{system} = 0$; calculate Q for each component of the system: (Beaker has small mass says that $Q = mc\Delta T$ for beaker can be neglected.)

0.250 kg of water: cools from 75.0°C to 40.0°C

$Q_{water} = mc\Delta T = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})(40.0°\text{C} - 75.0°\text{C}) = -3.666\times10^4$ J.

ice: warms to 0°C; melts; water from melted ice warms to 40.0°C

$Q_{ice} = mc_{ice}\Delta T + mL_f + mc_{water}\Delta T.$

$Q_{ice} = m[(2100\text{ J/kg}\cdot\text{K})(0°\text{C} - (-20.0°\text{C})) + 334\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(40.0°\text{C} - 0°\text{C})].$

$Q_{ice} = (5.436\times10^5\text{ J/kg})m$. $Q_{system} = 0$ says $Q_{water} + Q_{ice} = 0$. $-3.666\times10^4\text{ J} + (5.436\times10^5\text{ J/kg})m = 0$.

$$m = \frac{3.666\times10^4\text{ J}}{5.436\times10^5\text{ J/kg}} = 0.0674\text{ kg}.$$

EVALUATE: Since the final temperature is 40.0°C we know that all the ice melts and the final system is all liquid water. The mass of ice added is much less than the mass of the 75°C water; the ice requires a large heat input for the phase change.

17.58. **IDENTIFY:** For a temperature change $Q = mc\Delta T$. For a melting phase transition $Q = mL_f$. The net Q for the system (sample, vial and ice) is zero.

SET UP: Ice remains, so the final temperature is 0.0°C. For water, $L_f = 3.34\times10^5$ J/kg.

EXECUTE: For the sample, $Q_s = m_s c_s \Delta T_s = (16.0\times10^{-3}\text{ kg})(2250\text{ J/kg}\cdot\text{K})(-19.5\text{ C}°) = -702$ J. For the vial, $Q_v = m_v c_v \Delta T_v = (6.0\times10^{-3}\text{ kg})(2800\text{ J/kg}\cdot\text{K})(-19.5\text{ C}°) = -328$ J. For the ice that melts, $Q_i = mL_f$.

$\Sigma Q = 0$ gives $mL_f - 702\text{ J} - 328\text{ J} = 0$ and $m = 3.08\times10^{-3}\text{ kg} = 3.08$ g.

EVALUATE: Only a small fraction of the ice melts. The water for the melted ice remains at 0°C and has no heat flow.

17.59. **IDENTIFY** and **SET UP:** Large block of ice implies that ice is left, so $T_2 = 0°\text{C}$ (final temperature). Heat comes out of the ingot and into the ice. The net heat flow is zero. The ingot has a temperature change and the ice has a phase change.

EXECUTE: $Q_{system} = 0$; calculate Q for each component of the system:

ingot

$Q_{ingot} = mc\Delta T = (4.00\text{ kg})(234\text{ J/kg}\cdot\text{K})(0°\text{C} - 750°\text{C}) = -7.02\times10^5$ J

ice

$Q_{ice} = +mL_f$, where m is the mass of the ice that changes phase (melts)

$Q_{system} = 0$ says $Q_{ingot} + Q_{ice} = 0$

$-7.02\times10^5\text{ J} + m(334\times10^3\text{ J/kg}) = 0$

$$m = \frac{7.02\times10^5\ \text{J}}{334\times10^3\ \text{J/kg}} = 2.10\ \text{kg}$$

EVALUATE: The liquid produced by the phase change remains at 0°C since it is in contact with ice.

17.60. IDENTIFY: The initial temperature of the ice and water mixture is $0.0°\text{C}$. Assume all the ice melts. We will know that assumption is incorrect if the final temperature we calculate is less than $0.0°\text{C}$. The net Q for the system (can, water, ice and lead) is zero.

SET UP: For copper, $c_\text{c} = 390\ \text{J/kg}\cdot\text{K}$. For lead, $c_\text{l} = 130\ \text{J/kg}\cdot\text{K}$. For water, $c_\text{w} = 4.19\times10^3\ \text{J/kg}\cdot\text{K}$ and $L_\text{f} = 3.34\times10^5\ \text{J/kg}$.

EXECUTE: For the copper can, $Q_\text{c} = m_\text{c}c_\text{c}\Delta T_\text{c} = (0.100\ \text{kg})(390\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (39.0\ \text{J/K})T$.

For the water, $Q_\text{w} = m_\text{w}c_\text{w}\Delta T_\text{w} = (0.160\ \text{kg})(4.19\times10^3\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (670.4\ \text{J/K})T$.

For the ice, $Q_\text{i} = m_\text{i}L_\text{f} + m_\text{i}c_\text{w}\Delta T_\text{w}$

$Q_\text{i} = (0.018\ \text{kg})(3.34\times10^5\ \text{J/kg}) + (0.018\ \text{kg})(4.19\times10^3\ \text{J/kg}\cdot\text{K})(T - 0.0°\text{C}) = 6012\ \text{J} + (75.4\ \text{J/K})T$

For the lead, $Q_\text{l} = m_\text{l}c_\text{l}\Delta T_\text{l} = (0.750\ \text{kg})(130\ \text{J/kg}\cdot\text{K})(T - 255°\text{C}) = (97.5\ \text{J/K})T - 2.486\times10^4\ \text{J}$

$\Sigma Q = 0$ gives $(39.0\ \text{J/K})T + (670.4\ \text{J/K})T + 6012\ \text{J} + (75.4\ \text{J/K})T + (97.5\ \text{J/K})T - 2.486\times10^4\ \text{J} = 0$.

$$T = \frac{1.885\times10^4\ \text{J}}{882.3\ \text{J/K}} = 21.4°\text{C}.$$

EVALUATE: $T > 0.0°\text{C}$, which confirms that all the ice melts.

17.61. IDENTIFY: Set $Q_\text{system} = 0$, for the system of water, ice and steam. $Q = mc\Delta T$ for a temperature change and $Q = \pm mL$ for a phase transition.

SET UP: For water, $c = 4190\ \text{J/kg}\cdot\text{K}$, $L_\text{f} = 334\times10^3\ \text{J/kg}$ and $L_\text{v} = 2256\times10^3\ \text{J/kg}$.

EXECUTE: The steam both condenses and cools, and the ice melts and heats up along with the original water. $m_\text{i}L_\text{f} + m_\text{i}c(28.0\ \text{C}°) + m_\text{w}c(28.0\ \text{C}°) - m_\text{steam}L_\text{v} + m_\text{steam}c(-72.0\ \text{C}°) = 0$. The mass of steam needed is

$$m_\text{steam} = \frac{(0.450\ \text{kg})(334\times10^3\ \text{J/kg}) + (2.85\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(28.0\ \text{C}°)}{2256\times10^3\ \text{J/kg} + (4190\ \text{J/kg}\cdot\text{K})(72.0\ \text{C}°)} = 0.190\ \text{kg}.$$

EVALUATE: Since the final temperature is greater than $0.0°\text{C}$, we know that all the ice melts.

17.62. IDENTIFY: At steady state, the rate of heat flow is the same throughout both rods, as well as out of the boiling water and into the ice-water mixture. The heat that flows into the ice-water mixture goes only into melting ice since the temperature remains at 0.00°C.

SET UP: For steady state heat flow, $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$. The heat to melt ice is $Q = mL_\text{f}$.

EXECUTE: **(a)** $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$ is the same for both of the rods. Using the physical properties of brass and copper from the tables in the text, we have

$$\frac{[109.0\ \text{W/(m}\cdot\text{K)}](100.0°\text{C} - T)}{0.200\ \text{m}} = \frac{[385.0\ \text{W/(m}\cdot\text{K)}](T - 0.0°\text{C})}{0.800\ \text{m}}.$$

$436.0(100 - T) = 385.0T$. Solving for T gives $T = 53.1°\text{C}$.

(b) The heat entering the ice-water mixture is

$Q = \dfrac{kAt\Delta T}{L} = \dfrac{[109.0\ \text{W/(m}\cdot\text{K)}](0.00500\ \text{m}^2)(300.0\ \text{s})(100.0°\text{C} - 53.1°\text{C})}{0.200\ \text{m}}$. $Q = 3.834\times10^4\ \text{J}$. Then

$Q = mL_\text{f}$ so $m = \dfrac{3.834\times10^4\ \text{J}}{3.34\times10^5\ \text{J/kg}} = 0.115\ \text{kg}$.

EVALUATE: The temperature of the interface between the two rods is between the two extremes (0°C and 100°C), but not midway between them.

17.63. **IDENTIFY** and **SET UP:** The temperature gradient is $(T_H - T_C)/L$ and can be calculated directly. Use Eq. (17.21) to calculate the heat current *H*. In part (c) use *H* from part (b) and apply Eq. (17.21) to the 12.0-cm section of the left end of the rod. $T_2 = T_H$ and $T_1 = T$, the target variable.

EXECUTE: **(a)** temperature gradient $= (T_H - T_C)/L = (100.0°\text{C} - 0.0°\text{C})/0.450\text{ m} = 222\text{ C°/m} = 222\text{ K/m}$

(b) $H = kA(T_H - T_C)/L.$ From Table 17.5, $k = 385\text{ W/m}\cdot\text{K}$, so

$H = (385\text{ W/m}\cdot\text{K})(1.25\times10^{-4}\text{ m}^2)(222\text{ K/m}) = 10.7\text{ W}$

(c) $H = 10.7\text{ W}$ for all sections of the rod.

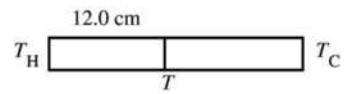


Figure 17.63

Apply $H = kA\Delta T/L$ to the 12.0 cm section (Figure 17.63): $T_H - T = LH/kA$ and

$$T = T_H - LH/Ak = 100.0°\text{C} - \frac{(0.120\text{ m})(10.7\text{ W})}{(1.25\times10^{-4}\text{ m}^2)(385\text{ W/m}\cdot\text{K})} = 73.3°\text{C}$$

EVALUATE: *H* is the same at all points along the rod, so $\Delta T/\Delta x$ is the same for any section of the rod with length Δx. Thus $(T_H - T)/(12.0\text{ cm}) = (T_H - T_C)/(45.0\text{ cm})$ gives that $T_H - T = 26.7\text{ C°}$ and $T = 73.3°\text{C}$, as we already calculated.

17.64. **IDENTIFY:** For a melting phase transition, $Q = mL_f$. The rate of heat conduction is $\dfrac{Q}{t} = \dfrac{kA(T_H - T_C)}{L}$.

SET UP: For water, $L_f = 3.34\times10^5\text{ J/kg}$.

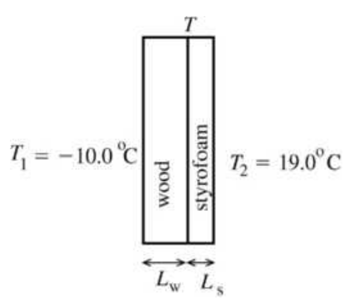
EXECUTE: The heat conducted by the rod in 10.0 min is

$Q = mL_f = (8.50\times10^{-3}\text{ kg})(3.34\times10^5\text{ J/kg}) = 2.84\times10^3\text{ J}$. $\dfrac{Q}{t} = \dfrac{2.84\times10^3\text{ J}}{600\text{ s}} = 4.73\text{ W}$.

$$k = \frac{(Q/t)L}{A(T_H - T_C)} = \frac{(4.73\text{ W})(0.600\text{ m})}{(1.25\times10^{-4}\text{ m}^2)(100\text{ C°})} = 227\text{ W/m}\cdot\text{K}.$$

EVALUATE: The heat conducted by the rod is the heat that enters the ice and produces the phase change.

17.65. **IDENTIFY** and **SET UP:** Call the temperature at the interface between the wood and the styrofoam *T*. The heat current in each material is given by $H = kA(T_H - T_C)/L$.



See Figure 17.65.

Heat current through the wood: $H_w = k_w A(T - T_1)L_w$

Heat current through the styrofoam: $H_s = k_s A(T_2 - T)/L_s$

Figure 17.65

In steady-state heat does not accumulate in either material. The same heat has to pass through both materials in succession, so $H_w = H_s$.

EXECUTE: **(a)** This implies $k_w A(T - T_1)/L_w = k_s A(T_2 - T)/L_s$

$k_w L_s (T - T_1) = k_s L_w (T_2 - T)$

$$T = \frac{k_w L_s T_1 + k_s L_w T_2}{k_w L_s + k_s L_w} = \frac{-0.0176\text{ W}\cdot°\text{C/K} + 00057\text{ W}\cdot°\text{C/K}}{0.00206\text{ W/K}} = -5.8°\text{C}$$

EVALUATE: The temperature at the junction is much closer in value to T_1 than to T_2. The styrofoam has a very small k, so a larger temperature gradient is required for than for wood to establish the same heat current.

(b) IDENTIFY and **SET UP:** Heat flow per square meter is $\frac{H}{A}=k\left(\frac{T_H-T_C}{L}\right)$. We can calculate this either for the wood or for the styrofoam; the results must be the same.

EXECUTE: <u>wood</u>

$$\frac{H_w}{A}=k_w\frac{T-T_1}{L_w}=(0.080\text{ W/m}\cdot\text{K})\frac{(-5.8°\text{C}-(-10.0°\text{C}))}{0.030\text{ m}}=11\text{ W/m}^2.$$

<u>styrofoam</u>

$$\frac{H_s}{A}=k_s\frac{T_2-T}{L_s}=(0.010\text{ W/m}\cdot\text{K})\frac{(19.0°\text{C}-(-5.8°\text{C}))}{0.022\text{ m}}=11\text{ W/m}^2.$$

EVALUATE: H must be the same for both materials and our numerical results show this. Both materials are good insulators and the heat flow is very small.

17.66. **IDENTIFY:** $\frac{Q}{t}=\frac{kA(T_H-T_C)}{L}$

SET UP: $T_H-T_C=175°\text{C}-35°\text{C}$. $1\text{ K}=1\text{ C}°$, so there is no need to convert the temperatures to kelvins.

EXECUTE: **(a)** $\frac{Q}{t}=\frac{(0.040\text{ W/m}\cdot\text{K})(1.40\text{ m}^2)(175°\text{C}-35°\text{C})}{4.0\times10^{-2}\text{ m}}=196\text{ W}.$

(b) The power input must be 196 W, to replace the heat conducted through the walls.

EVALUATE: The heat current is small because k is small for fiberglass.

17.67. **IDENTIFY:** There is a temperature difference across the skin, so we have heat conduction through the skin.

SET UP: Apply $H=kA\frac{T_H-T_C}{L}$ and solve for k.

EXECUTE: $k=\frac{HL}{A(T_H-T_C)}=\frac{(75\text{ W})(0.75\times10^{-3}\text{ m})}{(2.0\text{ m}^2)(37°\text{C}-30.0°\text{C})}=4.0\times10^{-3}\text{ W/m}\cdot\text{C}°.$

EVALUATE: This is a small value; skin is a poor conductor of heat. But the thickness of the skin is small, so the rate of heat conduction through the skin is not small.

17.68. **IDENTIFY:** $\frac{Q}{t}=\frac{kA\Delta T}{L}$. Q/t is the same for both sections of the rod.

SET UP: For copper, $k_c=385\text{ W/m}\cdot\text{K}$. For steel, $k_s=50.2\text{ W/m}\cdot\text{K}$.

EXECUTE: **(a)** For the copper section, $\frac{Q}{t}=\frac{(385\text{ W/m}\cdot\text{K})(4.00\times10^{-4}\text{ m}^2)(100°\text{C}-65.0°\text{C})}{1.00\text{ m}}=5.39\text{ J/s}.$

(b) For the steel section, $L=\frac{kA\Delta T}{(Q/t)}=\frac{(50.2\text{ W/m}\cdot\text{K})(4.00\times10^{-4}\text{ m}^2)(65.0°\text{C}-0°\text{C})}{5.39\text{ J/s}}=0.242\text{ m}.$

EVALUATE: The thermal conductivity for steel is much less than that for copper, so for the same ΔT and A a smaller L for steel would be needed for the same heat current as in copper.

17.69. **IDENTIFY** and **SET UP:** The heat conducted through the bottom of the pot goes into the water at 100°C to convert it to steam at 100°C. We can calculate the amount of heat flow from the mass of material that changes phase. Then use Eq. (17.21) to calculate T_H, the temperature of the lower surface of the pan.

EXECUTE: $Q=mL_v=(0.390\text{ kg})(2256\times10^3\text{ J/kg})=8.798\times10^5\text{ J}$

$H=Q/t=8.798\times10^5\text{ J/180 s}=4.888\times10^3\text{ J/s}$

Then $H=kA(T_H-T_C)/L$ says that $T_H-T_C=\frac{HL}{kA}=\frac{(4.888\times10^3\text{ J/s})(8.50\times10^{-3}\text{ m})}{(50.2\text{ W/m}\cdot\text{K})(0.150\text{ m}^2)}=5.52\text{ C}°$

$T_H=T_C+5.52\text{ C}°=100°\text{C}+5.52\text{ C}°=105.5°\text{C}$

EVALUATE: The larger T_H-T_C is the larger H is and the faster the water boils.

17.70. **IDENTIFY:** Apply Eq. (17.21) and solve for A.

SET UP: The area of each circular end of a cylinder is related to the diameter D by $A = \pi R^2 = \pi(D/2)^2$. For steel, $k = 50.2\ \text{W/m}\cdot\text{K}$. The boiling water has $T = 100°\text{C}$, so $\Delta T = 300\ \text{K}$.

EXECUTE: $\frac{Q}{t} = kA\frac{\Delta T}{L}$ and $150\ \text{J/s} = (50.2\ \text{W/m}\cdot\text{K})A\left(\frac{300\ \text{K}}{0.500\ \text{m}}\right)$. This gives $A = 4.98\times10^{-3}\ \text{m}^2$, and

$D = \sqrt{4A/\pi} = \sqrt{4(4.98\times10^{-3}\ \text{m}^2)/\pi} = 8.0\times10^{-2}\ \text{m} = 8.0\ \text{cm}$.

EVALUATE: H increases when A increases.

17.71. **IDENTIFY:** Assume the temperatures of the surfaces of the window are the outside and inside temperatures. Use the concept of thermal resistance. For part (b) use the fact that when insulating materials are in layers, the R values are additive.

SET UP: From Table 17.5, $k = 0.8\ \text{W/m}\cdot\text{K}$ for glass. $R = L/k$.

EXECUTE: **(a)** For the glass, $R_{\text{glass}} = \frac{5.20\times10^{-3}\ \text{m}}{0.8\ \text{W/m}\cdot\text{K}} = 6.50\times10^{-3}\ \text{m}^2\cdot\text{K/W}$.

$$H = \frac{A(T_\text{H} - T_\text{C})}{R} = \frac{(1.40\ \text{m})(2.50\ \text{m})(39.5\ \text{K})}{6.50\times10^{-3}\ \text{m}^2\cdot\text{K/W}} = 2.1\times10^4\ \text{W}$$

(b) For the paper, $R_{\text{paper}} = \frac{0.750\times10^{-3}\ \text{m}}{0.05\ \text{W/m}\cdot\text{K}} = 0.015\ \text{m}^2\cdot\text{K/W}$. The total R is

$R = R_{\text{glass}} + R_{\text{paper}} = 0.0215\ \text{m}^2\cdot\text{K/W}$. $H = \frac{A(T_\text{H} - T_\text{C})}{R} = \frac{(1.40\ \text{m})(2.50\ \text{m})(39.5\ \text{K})}{0.0215\ \text{m}^2\cdot\text{K/W}} = 6.4\times10^3\ \text{W}$.

EVALUATE: The layer of paper decreases the rate of heat loss by a factor of about 3.

17.72. **IDENTIFY:** The rate of energy radiated per unit area is $\frac{H}{A} = e\sigma T^4$.

SET UP: A blackbody has $e = 1$.

EXECUTE: **(a)** $\frac{H}{A} = (1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(273\ \text{K})^4 = 315\ \text{W/m}^2$

(b) $\frac{H}{A} = (1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(2730\ \text{K})^4 = 3.15\times10^6\ \text{W/m}^2$

EVALUATE: When the Kelvin temperature increases by a factor of 10 the rate of energy radiation increases by a factor of 10^4.

17.73. **IDENTIFY:** Use Eq. (17.25) to calculate A.

SET UP: $H = Ae\sigma T^4$ so $A = H/e\sigma T^4$

150-W and all electrical energy consumed is radiated says $H = 150\ \text{W}$

EXECUTE: $A = \frac{150\ \text{W}}{(0.35)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(2450\ \text{K})^4} = 2.1\times10^{-4}\ \text{m}^2(1\times10^4\ \text{cm}^2/1\ \text{m}^2) = 2.1\ \text{cm}^2$

EVALUATE: Light bulb filaments are often in the shape of a tightly wound coil to increase the surface area; larger A means a larger radiated power H.

17.74. **IDENTIFY:** The net heat current is $H = Ae\sigma(T^4 - T_\text{s}^4)$. A power input equal to H is required to maintain constant temperature of the sphere.

SET UP: The surface area of a sphere is $4\pi r^2$.

EXECUTE: $H = 4\pi(0.0150\ \text{m})^2(0.35)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)([3000\ \text{K}]^4 - [290\ \text{K}]^4) = 4.54\times10^3\ \text{W}$

EVALUATE: Since $3000\ \text{K} > 290\ \text{K}$ and H is proportional to T^4, the rate of emission of heat energy is much greater than the rate of absorption of heat energy from the surroundings.

17.75. **IDENTIFY:** Apply $H = Ae\sigma T^4$ and calculate A.

SET UP: For a sphere of radius R, $A = 4\pi R^2$. $\sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. The radius of the earth is $R_\text{E} = 6.38\times10^6\ \text{m}$, the radius of the sun is $R_{\text{sun}} = 6.96\times10^8\ \text{m}$, and the distance between the earth and the sun is $r = 1.50\times10^{11}\ \text{m}$.

EXECUTE: The radius is found from $R=\sqrt{\dfrac{A}{4\pi}}=\sqrt{\dfrac{H/(\sigma T^4)}{4\pi}}=\sqrt{\dfrac{H}{4\pi\sigma}\dfrac{1}{T^2}}.$

(a) $R_a=\sqrt{\dfrac{(2.7\times10^{32}\ \text{W})}{4\pi(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\dfrac{1}{(11{,}000\ \text{K})^2}}=1.61\times10^{11}\ \text{m}$

(b) $R_b=\sqrt{\dfrac{(2.10\times10^{23}\ \text{W})}{4\pi(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\dfrac{1}{(10{,}000\ \text{K})^2}}=5.43\times10^{6}\ \text{m}$

EVALUATE: **(c)** The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.76. **IDENTIFY:** Apply $\Delta L=L_0\alpha\Delta T$ to the radius of the hoop. The thickness of the space equals the increase in radius of the hoop.

SET UP: The earth has radius $R_E=6.38\times10^6$ m and this is the initial radius R_0 of the hoop. For steel, $\alpha=1.2\times10^{-5}\ \text{K}^{-1}$. $1\ \text{K}=1\ \text{C}°$.

EXECUTE: The increase in the radius of the hoop would be

$$\Delta R=R\alpha\Delta T=(6.38\times10^6\ \text{m})(1.2\times10^{-5}\ \text{K}^{-1})(0.5\ \text{K})=38\ \text{m}.$$

EVALUATE: Even though ΔR is large, the fractional change in radius, $\Delta R/R_0$, is very small.

17.77. **IDENTIFY** and **SET UP:** Use the temperature difference in M° and in C° between the melting and boiling points of mercury to relate M° to C°. Also adjust for the different zero points on the two scales to get an equation for T_M in terms of T_C.

(a) EXECUTE: normal melting point of mercury: $-39°\text{C}=0.0°\text{M}$
normal boiling point of mercury: $357°\text{C}=100.0°\text{M}$
$100.0\ \text{M}°=396\ \text{C}°$ so $1\ \text{M}°=3.96\ \text{C}°$

Zero on the M scale is -39 on the C scale, so to obtain T_C multiply T_M by 3.96 and then subtract 39°:
$T_C=3.96T_M-39°$

Solving for T_M gives $T_M=\frac{1}{3.96}(T_C+39°)$

The normal boiling point of water is 100°C; $T_M=\frac{1}{3.96}(100°+39°)=35.1°\text{M}$

(b) $10.0\ \text{M}°=39.6\ \text{C}°$

EVALUATE: A M° is larger than a C° since it takes fewer of them to express the difference between the boiling and melting points for mercury.

17.78. **IDENTIFY:** $v=\sqrt{F/\mu}=\sqrt{FL/m}$. For the fundamental, $\lambda=2L$ and $f=\dfrac{v}{\lambda}=\dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. F, v and λ change when T changes because L changes. $\Delta L=L\alpha\Delta T$, where L is the original length.

SET UP: For copper, $\alpha=1.7\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** We can use differentials to find the frequency change because all length changes are small percents. $\Delta f\approx\dfrac{\partial f}{\partial L}\Delta L$ (only L changes due to heating).

$\Delta f=\frac{1}{2}\frac{1}{2}(F/mL)^{-1/2}(F/m)(-1/L^2)\Delta L=-\frac{1}{2}\left(\frac{1}{2}\sqrt{\dfrac{F}{mL}}\right)\dfrac{\Delta L}{L}=-\frac{1}{2}f\dfrac{\Delta L}{L}.$

$\Delta f=-\frac{1}{2}(\alpha\Delta T)f=-\frac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°)(440\ \text{Hz})=-0.15\ \text{Hz}.$ The frequency decreases since the length increases.

(b) $\Delta v=\dfrac{\partial v}{\partial L}\Delta L.$

$\dfrac{\Delta v}{v}=\dfrac{\frac{1}{2}(FL/m)^{-1/2}(F/m)\Delta L}{\sqrt{FL/m}}=\dfrac{\Delta L}{2L}=\dfrac{\alpha\Delta T}{2}=\dfrac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°)=3.4\times10^{-4}=0.034\%.$

(c) $\lambda = 2L$ so $\Delta\lambda = 2\Delta L \rightarrow \dfrac{\Delta\lambda}{\lambda} = \dfrac{2\Delta L}{2L} = \dfrac{\Delta L}{L} = \alpha\Delta T.$

$\dfrac{\Delta\lambda}{\lambda} = (1.7\times10^{-5}\ (\text{C°})^{-1})(40\ \text{C°}) = 6.8\times10^{-4} = 0.068\%.$ λ increases.

EVALUATE: The wave speed and wavelength increase when the length increases and the frequency decreases. The percentage change in the frequency is -0.034%. The fractional change in all these quantities is very small.

17.79. **IDENTIFY** and **SET UP:** Use Eq. (17.8) for the volume expansion of the oil and of the cup. Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but β is larger for the oil so it expands more. When the oil starts to overflow, $\Delta V_{\text{oil}} = \Delta V_{\text{glass}} + (2.00\times10^{-3}\ \text{m})A,$ where A is the cross-sectional area of the cup.

EXECUTE: $\Delta V_{\text{oil}} = V_{0,\text{oil}}\beta_{\text{oil}}\Delta T = (9.8\ \text{cm})A\beta_{\text{oil}}\Delta T.$ $\Delta V_{\text{glass}} = V_{0,\text{glass}}\beta_{\text{glass}}\Delta T = (10.0\ \text{cm})A\beta_{\text{glass}}\Delta T.$

$(9.8\ \text{cm})A\beta_{\text{oil}}\Delta T = (10.0\ \text{cm})A\beta_{\text{glass}}\Delta T + (0.200\ \text{cm})A.$ The A divides out. Solving for ΔT gives $\Delta T = 31.3\ \text{C°}.$ $T_2 = T_1 + \Delta T = 53.3\text{°C}.$

EVALUATE: If the expansion of the cup is neglected, the olive oil will have expanded to fill the cup when $(0.200\ \text{cm})A = (9.8\ \text{cm})A\beta_{\text{oil}}\Delta T,$ so $\Delta T = 30.0\ \text{C°}$ and $T_2 = 52.0\text{°C}.$ Our result is slightly higher than this. The cup also expands but not very much since $\beta_{\text{glass}} << \beta_{\text{oil}}.$

17.80. **IDENTIFY:** As the tape changes temperature, the distances between the markings will increase, thus making the readings inaccurate.

SET UP: For steel, $\alpha = 1.2\times10^{-5}\,(\text{C°})^{-1}.$ The two points that match the length of the object are 25.970 m apart at 20.0°C. Find the distance between them at 5.00°C. For linear expansion, $L = L_0(1+\alpha\Delta T).$

EXECUTE: $L = L_0(1+\alpha\Delta T) = (25.970\ \text{m})(1+[1.2\times10^{-5}\,(\text{C°})^{-1}][5.00\text{°C} - 20.0\text{°C}]) = 25.965\ \text{m}.$ The true distance between the points is 25.965 m.

EVALUATE: The error in measurement is 25.970 m – 25.965 m = 0.005 m = 5 mm. This is not likely to be a very serious error in a measurement of nearly 30 m. If greater precision is needed, some sort of laser measuring device would probably be used.

17.81. **IDENTIFY:** Use Eq. (17.6) to find the change in diameter of the sphere and the change in length of the cable. Set the sum of these two increases in length equal to 2.00 mm.

SET UP: $\alpha_{\text{brass}} = 2.0\times10^{-5}\ \text{K}^{-1}$ and $\alpha_{\text{steel}} = 1.2\times10^{-5}\ \text{K}^{-1}.$

EXECUTE: $\Delta L = (\alpha_{\text{brass}}L_{0,\text{brass}} + \alpha_{\text{steel}}L_{0,\text{steel}})\Delta T.$

$$\Delta T = \frac{2.00\times10^{-3}\ \text{m}}{(2.0\times10^{-5}\ \text{K}^{-1})(0.350\ \text{m}) + (1.2\times10^{-5}\ \text{K}^{-1})(10.5\ \text{m})} = 15.0\ \text{C°}.\quad T_2 = T_1 + \Delta T = 35.0\text{°C}.$$

EVALUATE: The change in diameter of the brass sphere is 0.10 mm. This is small, but should not be neglected.

17.82. **IDENTIFY:** Conservation of energy says $Q_e + Q_c = 0,$ where Q_e and Q_c are the heat changes for the ethanol and cylinder. To find the volume of ethanol that overflows calculate ΔV for the ethanol and for the cylinder.

SET UP: For ethanol, $c_e = 2428\ \text{J/kg}\cdot\text{K}$ and $\beta_e = 75\times10^{-5}\ \text{K}^{-1}.$

EXECUTE: **(a)** $Q_e + Q_c = 0$ gives $m_e c_e(T_f - [-10.0\text{°C}]) + m_c c_c(T_f - 20.0\text{°C}) = 0.$

$$T_f = \frac{(20.0\text{°C})m_c c_c - (10.0\text{°C})m_e c_e}{m_e c_e + m_c c_c}.$$

$$T_f = \frac{(20.0\text{°C})(0.110\ \text{kg})(840\ \text{J/kg}\cdot\text{K}) - (10.0\text{°C})(0.0873\ \text{kg})(2428\ \text{J/kg}\cdot\text{K})}{(0.0873\ \text{kg})(2428\ \text{J/kg}\cdot\text{K}) + (0.110\ \text{kg})(840\ \text{J/kg}\cdot\text{K})}.$$

$$T_f = \frac{-271.6\text{°C}}{304.4} = -0.892\text{°C}.$$

(b) $\Delta V_e = \beta_e V_e \Delta T = (75\times10^{-5}\ \text{K}^{-1})(108\ \text{cm}^3)(-0.892\text{°C} - [-10.0\text{°C}]) = +0.738\ \text{cm}^3.$

$\Delta V_c = \beta_c V_c \Delta T = (1.2\times10^{-5}\ \text{K}^{-1})(108\ \text{cm}^3)(-0.892°\text{C} - 20.0°\text{C}) = -0.0271\ \text{cm}^3$. The volume that overflows is $0.738\ \text{cm}^3 - (-0.0271\ \text{cm}^3) = 0.765\ \text{cm}^3$.

EVALUATE: The cylinder cools so its volume decreases. The ethanol warms, so its volume increases. The sum of the magnitudes of the two volume changes gives the volume that overflows.

17.83. **IDENTIFY** and **SET UP:** Call the metals *A* and *B*. Use the data given to calculate α for each metal.

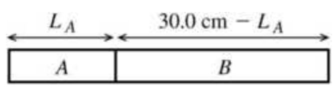
EXECUTE: $\Delta L = L_0\alpha\Delta T$ so $\alpha = \Delta L/(L_0\Delta T)$

metal *A*: $\alpha_A = \dfrac{\Delta L}{L_0\Delta T} = \dfrac{0.0650\ \text{cm}}{(30.0\ \text{cm})(100\ \text{C}°)} = 2.167\times10^{-5}\ (\text{C}°)^{-1}$

metal *B*: $\alpha_B = \dfrac{\Delta L}{L_0\Delta T} = \dfrac{0.0350\ \text{cm}}{(30.0\ \text{cm})(100\ \text{C}°)} = 1.167\times10^{-5}\ (\text{C}°)^{-1}$

EVALUATE: L_0 and ΔT are the same, so the rod that expands the most has the larger α.

IDENTIFY and **SET UP:** Now consider the composite rod (Figure 17.83). Apply Eq. (17.6). The target variables are L_A and L_B, the lengths of the metals *A* and *B* in the composite rod.



$\Delta T = 100\ \text{C}°$
$\Delta L = 0.058\ \text{cm}$

Figure 17.83

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = (\alpha_A L_A + \alpha_B L_B)\Delta T$

$\Delta L/\Delta T = \alpha_A L_A + \alpha_B(0.300\ \text{m} - L_A)$

$$L_A = \frac{\Delta L/\Delta T - (0.300\ \text{m})\alpha_B}{\alpha_A - \alpha_B} = \frac{(0.058\times10^{-2}\ \text{m}/100\ \text{C}°) - (0.300\ \text{m})(1.167\times10^{-5}(\text{C}°)^{-1})}{1.00\times10^{-5}\ (\text{C}°)^{-1}} = 23.0\ \text{cm}$$

$L_B = 30.0\ \text{cm} - L_A = 30.0\ \text{cm} - 23.0\ \text{cm} = 7.0\ \text{cm}$

EVALUATE: The expansion of the composite rod is similar to that of rod *A*, so the composite rod is mostly metal *A*.

17.84. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$ to the gasoline and to the volume of the tank.

SET UP: For aluminum, $\beta = 7.2\times10^{-5}\ \text{K}^{-1}$. $1\ \text{L} = 10^{-3}\ \text{m}^3$.

EXECUTE: **(a)** The lost volume, 2.6 L, is the difference between the expanded volume of the fuel and the tanks, and the maximum temperature difference is

$$\Delta T = \frac{\Delta V}{(\beta_{\text{fuel}} - \beta_{\text{A1}})V_0} = \frac{(2.6\times10^{-3}\ \text{m}^3)}{(9.5\times10^{-4}\ (\text{C}°)^{-1} - 7.2\times10^{-5}\ (\text{C}°)^{-1})(106.0\times10^{-3}\ \text{m}^3)} = 28\ \text{C}°.$$

The maximum temperature was 32°C.

(b) No fuel can spill if the tanks are filled just before takeoff.

EVALUATE: Both the volume of the gasoline and the capacity of the tanks increased when *T* increased. But β is larger for gasoline than for aluminum so the volume of the gasoline increased more. When the tanks have returned to 4.0°C on Sunday morning there is 2.6 L of air space in the tanks.

17.85. **IDENTIFY:** The change in length due to heating is $\Delta L_T = L_0\alpha\Delta T$ and this need not equal ΔL. The change in length due to the tension is $\Delta L_F = \dfrac{FL_0}{AY}$. Set $\Delta L = \Delta L_F + \Delta L_T$.

SET UP: $\alpha_{\text{brass}} = 2.0\times10^{-5}\ (\text{C}°)^{-1}$. $\alpha_{\text{steel}} = 1.5\times10^{-5}\ (\text{C}°)^{-1}$. $Y_{\text{steel}} = 20\times10^{10}\ \text{Pa}$.

EXECUTE: **(a)** The change in length is due to the tension and heating. $\dfrac{\Delta L}{L_0} = \dfrac{F}{AY} + \alpha\Delta T$. Solving for *F*/*A*,

$\dfrac{F}{A} = Y\left(\dfrac{\Delta L}{L_0} - \alpha\Delta T\right)$.

(b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that ΔL is not zero, but is the amount $\alpha_{\text{brass}} L_0 \Delta T$ that the brass expands, and so

$$\frac{F}{A} = Y_{\text{steel}}(\alpha_{\text{brass}} - \alpha_{\text{steel}})\Delta T = (20\times10^{10}\ \text{Pa})(2.0\times10^{-5}\ (\text{C}°)^{-1} - 1.2\times10^{-5}\ (\text{C}°)^{-1})(120\ \text{C}°) = 1.92\times10^{8}\ \text{Pa}.$$

EVALUATE: The length of the brass bar increases more than the length of the steel wires. The wires remain taut and are under tension when the temperature of the system is raised above 20°C.

17.86. IDENTIFY and **SET UP:** $v = \sqrt{F/\mu}$. The coefficient of linear expansion α is defined by $\Delta L = L_0 \alpha \Delta T$. This can be combined with $Y = \dfrac{F/A}{\Delta L/L_0}$ to give $\Delta F = -Y\alpha A \Delta T$ for the change in tension when the temperature changes by ΔT. Combine the two equations and solve for α.

EXECUTE: $v_1 = \sqrt{F/\mu}$, $v_1^2 = F/\mu$ and $F = \mu v_1^2$

The length and hence μ stay the same but the tension decreases by $\Delta F = -Y\alpha A \Delta T$.

$$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A \Delta T)/\mu}$$

$$v_2^2 = F/\mu - Y\alpha A\,\Delta T/\mu = v_1^2 - Y\alpha A\,\Delta T/\mu$$

And $\mu = m/L$ so $A/\mu = AL/m = V/m = 1/\rho$. ($A$ is the cross-sectional area of the wire, V is the volume of a length L.) Thus $v_1^2 - v_2^2 = \alpha(Y\,\Delta T/\rho)$ and $\alpha = \dfrac{v_1^2 - v_2^2}{(Y/\rho)\,\Delta T}$.

EVALUATE: When T increases the tension decreases and v decreases.

17.87. IDENTIFY: For a string, $f_n = \dfrac{n}{2L}\sqrt{\dfrac{F}{\mu}}$.

SET UP: For the fundamental, $n = 1$. Solving for F gives $F = \mu 4L^2 f^2$. Note that $\mu = \pi r^2 \rho$, so $\mu = \pi(0.203\times10^{-3}\ \text{m})^2(7800\ \text{kg/m}^3) = 1.01\times10^{-3}\ \text{kg/m}$.

EXECUTE: **(a)** $F = (1.01\times10^{-3}\ \text{kg/m})4(0.635\ \text{m})^2(247.0\ \text{Hz})^2 = 99.4\ \text{N}$

(b) To find the fractional change in the frequency we must take the ratio of Δf to f: $f = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}}$ and

$$\Delta f = \Delta\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \Delta\left(\frac{1}{2L\sqrt{\mu}}F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\Delta\left(F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}$$

Now divide both sides by the original equation for f and cancel terms: $\dfrac{\Delta f}{f} = \dfrac{\dfrac{1}{2L\sqrt{\mu}}\dfrac{1}{2}\dfrac{\Delta F}{\sqrt{F}}}{\dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}}} = \dfrac{1}{2}\dfrac{\Delta F}{F}$.

(c) The coefficient of thermal expansion α is defined by $\Delta l = l_0 \alpha \Delta T$. Combining this with $Y = \dfrac{F/A}{\Delta l/l_0}$ gives $\Delta F = -Y\alpha A \Delta T$. $\Delta F = -(2.00\times10^{11}\ \text{Pa})(1.20\times10^{-5}/\text{C}°)\pi(0.203\times10^{-3}\ \text{m})^2(11\ \text{C}°) = -3.4\ \text{N}$. Then $\Delta F/F - 0.034$, $\Delta f/f = -0.017$ and $\Delta f = -4.2\ \text{Hz}$. The pitch falls. This also explains the constant tuning in the string sections of symphonic orchestras.

EVALUATE: An increase in temperature causes a decrease in tension of the string, and this lowers the frequency of each standing wave.

17.88. IDENTIFY: Apply the equation derived in part (a) of Problem 17.85 to the steel and aluminum sections. The sum of the ΔL values of the two sections must be zero.

SET UP: For steel, $Y = 20\times10^{10}$ Pa and $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$. For aluminum, $Y = 7.0\times10^{10}$ Pa and $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: In deriving Eq. (17.12), it was assumed that $\Delta L = 0$; if this is not the case when there are both thermal and tensile stresses, Eq. (17.12) becomes $\Delta L = L_0\left(\alpha\Delta T + \dfrac{F}{AY}\right)$. (See Problem 17.85.) For the situation in this problem, there are two length changes which must sum to zero, and so Eq. (17.12) may be extended to two materials a and b in the form $L_{0a}\left(\alpha_a\,\Delta T + \dfrac{F}{AY_a}\right) + L_{0b}\left(\alpha_b\,\Delta T + \dfrac{F}{AY_b}\right) = 0.$ Note that in the above, ΔT, F and A are the same for the two rods. Solving for the stress F/A,

$\dfrac{F}{A} = -\dfrac{\alpha_a L_{0a} + \alpha_b L_{0b}}{L_{0a}/Y_a + L_{0b}/Y_b}\Delta T.$ Putting in the numbers gives

$$\frac{F}{A} = -\frac{(1.2\times10^{-5}\ (\text{C}°)^{-1})(0.450\ \text{m}) + (2.4\times10^{-5}\ (\text{C}°)^{-1})(0.250\ \text{m})}{(0.450\ \text{m})/(20\times10^{10}\ \text{Pa}) + (0.250\ \text{m})/(7\times10^{10}\ \text{Pa})}(60.0\ \text{C}°) = -1.2\times10^{8}\ \text{Pa}.$$

EVALUATE: F/A is negative and the stress is compressive. If the steel rod was considered alone and its length was held fixed, the stress would be $-Y_{\text{steel}}\alpha_{\text{steel}}\Delta T = -1.4\times10^{8}$ Pa. For the aluminum rod alone the stress would be $-Y_{\text{aluminum}}\alpha_{\text{aluminum}}\Delta T = -1.0\times10^{8}$ Pa. The stress for the combined rod is the average of these two values.

17.89. **(a) IDENTIFY** and **SET UP:** The diameter of the ring undergoes linear expansion (increases with T) just like a solid steel disk of the same diameter as the hole in the ring. Heat the ring to make its diameter equal to 2.5020 in.

EXECUTE: $\Delta L = \alpha L_0 \Delta T$ so $\Delta T = \dfrac{\Delta L}{L_0\alpha} = \dfrac{0.0020\ \text{in.}}{(2.5000\ \text{in.})(1.2\times10^{-5}(\text{C}°)^{-1})} = 66.7\ \text{C}°$

$T = T_0 + \Delta T = 20.0°\text{C} + 66.7\ \text{C}° = 87°\text{C}$

(b) IDENTIFY and **SET UP:** Apply the linear expansion equation to the diameter of the brass shaft and to the diameter of the hole in the steel ring.

EXECUTE: $L = L_0(1 + \alpha\Delta T)$

Want L_s (steel) $= L_b$ (brass) for the same ΔT for both materials: $L_{0s}(1 + \alpha_s\Delta T) = L_{0b}(1 + \alpha_b\Delta T)$ so

$L_{0s} + L_{0s}\alpha_s\Delta T = L_{0b} + L_{0b}\alpha_b\Delta T$

$$\Delta T = \frac{L_{0b} - L_{0s}}{L_{0s}\alpha_s - L_{0b}\alpha_b} = \frac{2.5020\ \text{in.} - 2.5000\ \text{in.}}{(2.5000\ \text{in.})(1.2\times10^{-5}(\text{C}°)^{-1}) - (2.5050\ \text{in.})(2.0\times10^{-5}(\text{C}°)^{-1})}$$

$$\Delta T = \frac{0.0020}{3.00\times10^{-5} - 5.00\times10^{-5}}\ \text{C}° = -100\ \text{C}°$$

$T = T_0 + \Delta T = 20.0°\text{C} - 100\ \text{C}° = -80°\text{C}$

EVALUATE: Both diameters decrease when the temperature is lowered but the diameter of the brass shaft decreases more since $\alpha_b > \alpha_s$; $|\Delta L_b| - |\Delta L_s| = 0.0020$ in.

17.90. **IDENTIFY:** Follow the derivation of Eq. (17.12).

SET UP: For steel, the bulk modulus is $B = 1.6\times10^{11}$ Pa and the volume expansion coefficient is $\beta = 3.6\times10^{-5}\ \text{K}^{-1}$.

EXECUTE: **(a)** The change in volume due to the temperature increase is $\beta V\Delta T$, and the change in volume due to the pressure increase is $-\dfrac{V}{B}\Delta p$. Setting the net change equal to zero,

$\beta V\Delta T = V\dfrac{\Delta p}{B}$, or $\Delta p = B\beta\Delta T$.

(b) From the above, $\Delta p = (1.6\times10^{11}\ \text{Pa})(3.6\times10^{-5}\ \text{K}^{-1})(15.0\ \text{K}) = 8.6\times10^{7}$ Pa.

EVALUATE: Δp in part (b) is about 850 atm. A small temperature increase corresponds to a very large pressure increase.

17.91. **IDENTIFY:** Apply Eq. (11.14) to the volume increase of the liquid due to the pressure decrease. Eq. (17.8) gives the volume decrease of the cylinder and liquid when they are cooled. Can think of the liquid expanding when the pressure is reduced and then contracting to the new volume of the cylinder when the temperature is reduced.

SET UP: Let β_1 and β_m be the coefficients of volume expansion for the liquid and for the metal. Let ΔT be the (negative) change in temperature when the system is cooled to the new temperature.

EXECUTE: Change in volume of cylinder when cool: $\Delta V_m = \beta_m V_0 \Delta T$ (negative)

Change in volume of liquid when cool: $\Delta V_1 = \beta_1 V_0 \Delta T$ (negative)

The difference $\Delta V_1 - \Delta V_m$ must be equal to the negative volume change due to the increase in pressure, which is $-\Delta p V_0/B = -k\Delta p V_0$. Thus $\Delta V_1 - \Delta V_m = -k\Delta p V_0$.

$$\Delta T = -\frac{k\Delta p}{\beta_1 - \beta_m}$$

$$\Delta T = -\frac{(8.50\times10^{-10}\ \text{Pa}^{-1})(50.0\ \text{atm})(1.013\times10^5\ \text{Pa/1 atm})}{4.80\times10^{-4}\ \text{K}^{-1} - 3.90\times10^{-5}\ \text{K}^{-1}} = -9.8\ \text{C}°$$

$T = T_0 + \Delta T = 30.0°\text{C} - 9.8\ \text{C}° = 20.2°\text{C}$.

EVALUATE: A modest temperature change produces the same volume change as a large change in pressure; $B >> \beta$ for the liquid.

17.92. **IDENTIFY:** $Q_{\text{system}} = 0$. Assume that the normal melting point of iron is above 745°C so the iron initially is solid.

SET UP: For water, $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_v = 2256\times10^3\ \text{J/kg}$. For solid iron, $c = 470\ \text{J/kg}\cdot\text{K}$.

EXECUTE: The heat released when the iron slug cools to 100°C is $Q = mc\Delta T = (0.1000\ \text{kg})(470\ \text{J/kg}\cdot\text{K})(645\ \text{K}) = 3.03\times10^4\ \text{J}$. The heat absorbed when the temperature of the water is raised to 100°C is $Q = mc\Delta T = (0.0850\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(80.0\ \text{K}) = 2.85\times10^4\ \text{J}$. This is less than the heat released from the iron and $3.03\times10^4\ \text{J} - 2.85\times10^4\ \text{J} = 1.81\times10^3\ \text{J}$ of heat is available for converting some of the liquid water at 100°C to vapor. The mass m of water that boils is

$$m = \frac{1.81\times10^3\ \text{J}}{2256\times10^3\ \text{J/kg}} = 8.01\times10^{-4}\ \text{kg} = 0.801\ \text{g}.$$

(a) The final temperature is 100°C.

(b) There is $85.0\ \text{g} - 0.801\ \text{g} = 84.2\ \text{g}$ of liquid water remaining, so the final mass of the iron and remaining water is 184.2 g.

EVALUATE: If we ignore the phase change of the water and write $m_{\text{iron}}c_{\text{iron}}(T - 745°\text{C}) + m_{\text{water}}c_{\text{water}}(T - 20.0°\text{C}) = 0$, when we solve for T we will get a value slightly larger than 100°C. That result is unphysical and tells us that some of the water changes phase.

17.93. **(a) IDENTIFY:** Calculate K/Q. We don't know the mass m of the spacecraft, but it divides out of the ratio.

SET UP: The kinetic energy is $K = \frac{1}{2}mv^2$. The heat required to raise its temperature by 600 C° (but not to melt it) is $Q = mc\Delta T$.

EXECUTE: The ratio is $\dfrac{K}{Q} = \dfrac{\frac{1}{2}mv^2}{mc\Delta T} = \dfrac{v^2}{2c\Delta T} = \dfrac{(7700\ \text{m/s})^2}{2(910\ \text{J/kg}\cdot\text{K})(600\ \text{C}°)} = 54.3.$

(b) EVALUATE: The heat generated when friction work (due to friction force exerted by the air) removes the kinetic energy of the spacecraft during reentry is very large, and could melt the spacecraft. Manned space vehicles must have heat shields made of very high melting temperature materials, and reentry must be made slowly.

17.94. **IDENTIFY:** The rate at which thermal energy is being generated equals the rate at which the net torque due to the rope is doing work. The energy input associated with a temperature change is $Q = mc\Delta T$.

SET UP: The rate at which work is being done is $P = \tau\omega$. For iron, $c = 470\ \text{J/kg}\cdot\text{K}$. $1\ \text{C}° = 1\ \text{K}$

EXECUTE: (a) The net torque that the rope exerts on the capstan, and hence the net torque that the capstan exerts on the rope, is the difference between the forces of the ends of the rope times the radius of the capstan. The capstan is doing work on the rope at a rate

$P = \tau\omega = F_{\text{net}} r \dfrac{2\pi \text{ rad}}{T} = (520 \text{ N})(5.0\times 10^{-2} \text{ m})\dfrac{2\pi \text{ rad}}{(0.90 \text{ s})} = 182 \text{ W},$ or 180 W to two figures. A larger number of turns might increase the force, but for given forces, the torque is independent of the number of turns.

(b) $\dfrac{\Delta T}{t} = \dfrac{Q/t}{mc} = \dfrac{P}{mc} = \dfrac{(182 \text{ W})}{(6.00 \text{ kg})(470 \text{ J/kg}\cdot\text{K})} = 0.064 \text{ C°/s}.$

EVALUATE: The rate of temperature rise is proportional to the difference in tension between the ends of the rope and to the rate at which the capstan is rotating.

17.95. IDENTIFY and **SET UP:** To calculate Q, use Eq. (17.18) in the form $dQ = nC\,dT$ and integrate, using $C(T)$ given in the problem. C_{av} is obtained from Eq. (17.19) using the finite temperature range instead of an infinitesimal dT.

EXECUTE: (a) $dQ = nCdT$

$$Q = n\int_{T_1}^{T_2} C\,dT = n\int_{T_1}^{T_2} k(T^3/\Theta^3)dT = (nk/\Theta^3)\int_{T_1}^{T_2} T^3\,dt = (nk/\Theta^3)\left(\tfrac{1}{4}T^4\Big|_{T_1}^{T_2}\right)$$

$$Q = \frac{nk}{4\Theta^3}(T_2^4 - T_1^4) = \frac{(1.50 \text{ mol})(1940 \text{ J/mol}\cdot\text{K})}{4(281 \text{ K})^3}((40.0 \text{ K})^4 - (10.0 \text{ K})^4) = 83.6 \text{ J}$$

(b) $C_{\text{av}} = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T} = \dfrac{1}{1.50 \text{ mol}}\left(\dfrac{83.6 \text{ J}}{40.0 \text{ K} - 10.0 \text{ K}}\right) = 1.86 \text{ J/mol}\cdot\text{K}$

(c) $C = k(T/\Theta)^3 = (1940 \text{ J/mol}\cdot\text{K})(40.0 \text{ K}/281 \text{ K})^3 = 5.60 \text{ J/mol}\cdot\text{K}$

EVALUATE: C is increasing with T, so C at the upper end of the temperature integral is larger than its average value over the interval.

17.96. IDENTIFY: For a temperature change, $Q = mc\Delta T$, and for the liquid $\rightarrow$ solid phase change, $Q = -mL_{\text{f}}$.

SET UP: The volume V_{w} of the water determines its mass. $m_{\text{w}} = \rho_{\text{w}} V_{\text{w}}$. For water, $\rho_{\text{w}} = 1000 \text{ kg/m}^3$, $c = 4190 \text{ J/kg}\cdot\text{K}$ and $L_{\text{f}} = 334\times 10^3 \text{ J/kg}$.

EXECUTE: Set the heat energy that flows into the water equal to the final gravitational potential energy. $L_{\text{f}}\rho_{\text{w}}V_{\text{w}} + c_{\text{w}}\rho_{\text{w}}V_{\text{w}}\Delta T = mgh$. Solving for h gives

$$h = \frac{(1000 \text{ kg/m}^3)(1.9\times 0.80\times 0.160 \text{ m}^3)[334\times 10^3 \text{ J/kg} + (4190 \text{ J/kg}\cdot\text{K})(37 \text{ C°})]}{(70 \text{ kg})(9.8 \text{ m/s}^2)}.$$

$h = 1.73\times 10^5 \text{ m} = 173 \text{ km}.$

EVALUATE: The heat associated with temperature and phase changes corresponds to a very large amount of mechanical energy.

17.97. IDENTIFY: Apply $Q = mc\Delta T$ to the air in the room.

SET UP: The mass of air in the room is $m = \rho V = (1.20 \text{ kg/m}^3)(3200 \text{ m}^3) = 3840 \text{ kg}$. $1 \text{ W} = 1 \text{ J/s}$.

EXECUTE: (a) $Q = (3000 \text{ s})(90 \text{ students})(100 \text{ J/s}\cdot\text{student}) = 2.70\times 10^7 \text{ J}.$

(b) $Q = mc\Delta T$. $\Delta T = \dfrac{Q}{mc} = \dfrac{2.70\times 10^7 \text{ J}}{(3840 \text{ kg})(1020 \text{ J/kg}\cdot\text{K})} = 6.89 \text{ C°}$

(c) $\Delta T = (6.89 \text{ C°})\left(\dfrac{280 \text{ W}}{100 \text{ W}}\right) = 19.3 \text{ C°}.$

EVALUATE: In the absence of a cooling mechanism for the air, the air temperature would rise significantly.

17.98. IDENTIFY: $dQ = nCdT$ so for the temperature change $T_1 \rightarrow T_2$, $Q = n\int_{T_1}^{T_2} C(T)dT$.

SET UP: $\int dT = T$ and $\int TdT = \tfrac{1}{2}T^2$. Express T_1 and T_2 in kelvins: $T_1 = 300 \text{ K}$, $T_2 = 500 \text{ K}$.

EXECUTE: Denoting C by $C = a + bT$, a and b independent of temperature, integration gives

$Q = n(a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)).$

$Q = (3.00 \text{ mol})[(29.5 \text{ J/mol}\cdot\text{K})(500 \text{ K} - 300 \text{ K}) + (4.10\times10^{-3} \text{ J/mol}\cdot\text{K}^2)((500 \text{ K})^2 - (300 \text{ K})^2)].$

$Q = 1.97\times10^4 \text{ J}.$

EVALUATE: If C is assumed to have the constant value $29.5 \text{ J/mol}\cdot\text{K}$, then $Q = 1.77\times10^4 \text{ J}$ for this temperature change. At $T_1 = 300 \text{ K}$, $C = 32.0 \text{ J/mol}\cdot\text{K}$ and at $T_2 = 500 \text{ K}$, $C = 33.6 \text{ J/mol}\cdot\text{K}$. The average value of C is $32.8 \text{ J/mol}\cdot\text{K}$. If C is assumed to be constant and to have this average value, then $Q = 1.97\times10^4 \text{ J}$, which is equal to the correct value.

17.99. **IDENTIFY:** Use $Q = mL_f$ to find the heat that goes into the ice to melt it. This amount of heat must be conducted through the walls of the box; $Q = Ht$. Assume the surfaces of the styrofoam have temperatures of 5.00°C and 21.0°C.

SET UP: For water $L_f = 334\times10^3 \text{ J/kg}$. For styrofoam $k = 0.01 \text{ W/m}\cdot\text{K}$. One week is 6.048×10^5 s. The surface area of the box is $4(0.500 \text{ m})(0.800 \text{ m}) + 2(0.500 \text{ m})^2 = 2.10 \text{ m}^2$.

EXECUTE: $Q = mL_f = (24.0 \text{ kg})(334\times10^3 \text{ J/kg}) = 8.016\times10^6 \text{ J}$. $H = kA\frac{T_H - T_C}{L}$. $Q = Ht$ gives

$$L = \frac{tkA(T_H - T_C)}{Q} = \frac{(6.048\times10^5 \text{ s})(0.01 \text{ W/m}\cdot\text{K})(2.10 \text{ m}^2)(21.0°\text{C} - 5.00°\text{C})}{8.016\times10^6 \text{ J}} = 2.5 \text{ cm}$$

EVALUATE: We have assumed that the liquid water that is produced by melting the ice remains in thermal equilibrium with the ice so has a temperature of 0°C. The interior of the box and the ice are not in thermal equilibrium, since they have different temperatures.

17.100. **IDENTIFY:** For a temperature change $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_v$.

SET UP: For water, $c = 4190 \text{ J/kg}\cdot\text{K}$ and $L_v = 2256\times10^3 \text{ J/kg}$.

EXECUTE: The requirement that the heat supplied in each case is the same gives $m_w c_w \Delta T_w = m_s(c_w \Delta T_s + L_v)$, where $\Delta T_w = 42.0 \text{ K}$ and $\Delta T_s = 65.0 \text{ K}$. The ratio of the masses is

$$\frac{m_s}{m_w} = \frac{c_w \Delta T_w}{c_w \Delta T_s + L_v} = \frac{(4190 \text{ J/kg}\cdot\text{K})(42.0 \text{ K})}{(4190 \text{ J/kg}\cdot\text{K})(65.0 \text{ K}) + 2256\times10^3 \text{ J/kg}} = 0.0696,$$

so 0.0696 kg of steam supplies the same heat as 1.00 kg of water.

EVALUATE: Note the heat capacity of water is used to find the heat lost by the condensed steam, since the phase transition produces liquid water at an initial temperature of 100°C.

17.101. **(a) IDENTIFY** and **SET UP:** Assume that all the ice melts and that all the steam condenses. If we calculate a final temperature T that is outside the range 0°C to 100°C then we know that this assumption is incorrect. Calculate Q for each piece of the system and then set the total $Q_{system} = 0$.

EXECUTE: <u>copper can</u> (changes temperature from 0.0° to T; no phase change)

$Q_{can} = mc\Delta T = (0.446 \text{ kg})(390 \text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (173.9 \text{ J/K})T$

<u>ice</u> (melting phase change and then the water produced warms to T)

$Q_{ice} = +mL_f + mc\Delta T = (0.0950 \text{ kg})(334\times10^3 \text{ J/kg}) + (0.0950 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(T - 0.0°\text{C})$

$Q_{ice} = 3.173\times10^4 \text{ J} + (398.0 \text{ J/K})T.$

<u>steam</u> (condenses to liquid and then water produced cools to T)

$Q_{steam} = -mL_v + mc\Delta T = -(0.0350 \text{ kg})(2256\times10^3 \text{ J/kg}) + (0.0350 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(T - 100.0°\text{C})$

$Q_{steam} = -7.896\times10^4 \text{ J} + (146.6 \text{ J/K})T - 1.466\times10^4 \text{ J} = -9.362\times10^4 \text{ J} + (146.6 \text{ J/K})T$

$Q_{system} = 0$ implies $Q_{can} + Q_{ice} + Q_{steam} = 0.$

$(173.9 \text{ J/K})T + 3.173\times10^4 \text{ J} + (398.0 \text{ J/K})T - 9.362\times10^4 \text{ J} + (146.6 \text{ J/K})T = 0$

$(718.5 \text{ J/K})T = 6.189 \times 10^4 \text{ J}$

$$T = \frac{6.189 \times 10^4 \text{ J}}{718.5 \text{ J/K}} = 86.1°\text{C}$$

EVALUATE: This is between 0°C and 100°C so our assumptions about the phase changes being complete were correct.

(b) No ice, no steam and $0.0950 \text{ kg} + 0.0350 \text{ kg} = 0.130 \text{ kg}$ of liquid water.

17.102. IDENTIFY: The final amount of ice is less than the initial mass of water, so water remains and the final temperature is 0°C. The ice added warms to 0°C and heat comes out of water to convert that water to ice. Conservation of energy says $Q_\text{i} + Q_\text{w} = 0,$ where Q_i and Q_w are the heat flows for the ice that is added and for the water that freezes.

SET UP: Let m_i be the mass of ice that is added and m_w is the mass of water that freezes. The mass of ice increases by 0.418 kg, so $m_\text{i} + m_\text{w} = 0.418 \text{ kg}.$ For water, $L_\text{f} = 334 \times 10^3 \text{ J/kg}$ and for ice $c_\text{i} = 2100 \text{ J/kg} \cdot \text{K}.$ Heat comes out of the water when it freezes, so $Q_\text{w} = -mL_\text{f}.$

EXECUTE: $Q_\text{i} + Q_\text{w} = 0$ gives $m_\text{i}c_\text{i}(15.0 \text{ C°}) + (-m_\text{w}L_\text{f}) = 0,$ $m_\text{w} = 0.418 \text{ kg} - m_\text{i},$ so $m_\text{i}c_\text{i}(15.0 \text{ C°}) + (-0.418 \text{ kg} + m_\text{i})L_\text{f} = 0.$

$$m_\text{i} = \frac{(0.418 \text{ kg})L_\text{f}}{c_\text{i}(15.0 \text{ C°}) + L_\text{f}} = \frac{(0.418 \text{ kg})(334 \times 10^3 \text{ J/kg})}{(2100 \text{ J/kg} \cdot \text{K})(15.0 \text{ K}) + 334 \times 10^3 \text{ J/kg}} = 0.382 \text{ kg}.$$ 0.382 kg of ice was added.

EVALUATE: The mass of water that froze when the ice at -15.0C° was added was $0.868 \text{ kg} - 0.450 \text{ kg} - 0.382 \text{ kg} = 0.036 \text{ kg}.$

17.103. IDENTIFY and **SET UP:** Heat comes out of the steam when it changes phase and heat goes into the water and causes its temperature to rise. $Q_\text{system} = 0.$ First determine what phases are present after the system has come to a uniform final temperature.

(a) EXECUTE: Heat that must be removed from steam if all of it condenses is

$Q = -mL_\text{v} = -(0.0400 \text{ kg})(2256 \times 10^3 \text{ J/kg}) = -9.02 \times 10^4 \text{ J}$

Heat absorbed by the water if it heats all the way to the boiling point of 100°C:

$Q = mc\Delta T = (0.200 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(50.0 \text{ C°}) = 4.19 \times 10^4 \text{ J}$

EVALUATE: The water can't absorb enough heat for all the steam to condense. Steam is left and the final temperature then must be 100°C.

(b) EXECUTE: Mass of steam that condenses is $m = Q/L_\text{v} = 4.19 \times 10^4 \text{ J}/2256 \times 10^3 \text{ J/kg} = 0.0186 \text{ kg}.$ Thus there is $0.0400 \text{ kg} - 0.0186 \text{ kg} = 0.0214 \text{ kg}$ of steam left. The amount of liquid water is $0.0186 \text{ kg} + 0.200 \text{ kg} = 0.219 \text{ kg}.$

17.104. IDENTIFY: Heat is conducted out of the body. At steady state, the rate of heat flow is the same in both layers (fat and fur).

SET UP: Let the temperature of the fat-air boundary be T. A section of the two layers is sketched in Figure 17.104. A Kelvin degree is the same size as a Celsius degree, so $\text{W/m} \cdot \text{K}$ and $\text{W/m} \cdot \text{C°}$ are equivalent units. At steady state the heat current through each layer is equal to 50 W. The area of each layer is $A = 4\pi r^2,$ with $r = 0.75 \text{ m}.$

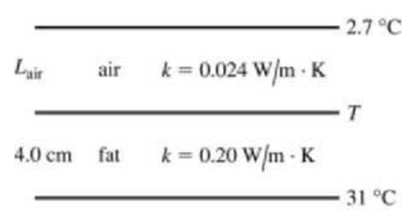


Figure 17.104

EXECUTE: **(a)** Apply $H = kA\dfrac{T_H - T_C}{L}$ to the fat layer and solve for $T_C = T$. For the fat layer $T_H = 31°C$.

$$T = T_H - \frac{HL}{kA} = 31°C - \frac{(50\ W)(4.0\times10^{-2}\ m)}{(0.20\ W/m\cdot K)(4\pi)(0.75\ m)^2} = 31°C - 1.4°C = 29.6°C.$$

(b) Apply $H = kA\dfrac{T_H - T_C}{L}$ to the air layer and solve for $L = L_{air}$. For the air layer $T_H = T = 29.6°C$ and $T_C = 2.7°C$. $L = \dfrac{kA(T_H - T_C)}{H} = \dfrac{(0.024\ W/m\cdot K)(4\pi)(0.75\ m)^2(29.6°C - 2.7°C)}{50\ W} = 9.1\ cm.$

EVALUATE: The thermal conductivity of air is much lass than the thermal conductivity of fat, so the temperature gradient for the air must be much larger to achieve the same heat current. So, most of the temperature difference is across the air layer.

17.105. **IDENTIFY:** Heat Q_l comes out of the lead when it solidifies and the solid lead cools to T_f. If mass m_s of steam is produced, the final temperature is $T_f = 100°C$ and the heat that goes into the water is $Q_w = m_w c_w (25.0\ C°) + m_s L_{v,w}$, where $m_w = 0.5000$ kg. Conservation of energy says $Q_l + Q_w = 0$. Solve for m_s. The mass that remains is $1.250\ kg + 0.5000\ kg - m_s$.

SET UP: For lead, $L_{f,l} = 24.5\times10^3$ J/kg, $c_l = 130$ J/kg · K and the normal melting point of lead is 327.3°C. For water, $c_w = 4190$ J/kg · K and $L_{v,w} = 2256\times10^3$ J/kg.

EXECUTE: $Q_l + Q_w = 0$. $-m_l L_{f,l} + m_l c_l(-227.3\ C°) + m_w c_w (25.0\ C°) + m_s L_{v,w} = 0$.

$$m_s = \frac{m_l L_{f,l} + m_l c_l(+227.3\ C°) - m_w c_w (25.0\ C°)}{L_{v,w}}.$$

$$m_s = \frac{+(1.250\ kg)(24.5\times10^3\ J/kg) + (1.250\ kg)(130\ J/kg\cdot K)(227.3\ K) - (0.5000\ kg)(4190\ J/kg\cdot K)(25.0\ K)}{2256\times10^3\ J/kg}$$

$m_s = \dfrac{1.519\times10^4\ J}{2256\times10^3\ J/kg} = 0.0067$ kg. The mass of water and lead that remains is 1.743 kg.

EVALUATE: The magnitude of heat that comes out of the lead when it goes from liquid at 327.3°C to solid at 100.0°C is 6.76×10^4 J. The heat that goes into the water to warm it to 100°C is 5.24×10^4 J. The additional heat that goes into the water, $6.76\times10^4\ J - 5.24\times10^4\ J = 1.52\times10^4\ J$ converts 0.0067 kg of water at 100°C to steam.

17.106. **IDENTIFY:** Apply $H = kA\dfrac{\Delta T}{L}$ and solve for k.

SET UP: H equals the power input required to maintain a constant interior temperature.

EXECUTE: $k = H\dfrac{L}{A\Delta T} = (180\ W)\dfrac{(3.9\times10^{-2}\ m)}{(2.18\ m^2)(65.0\ K)} = 5.0\times10^{-2}\ W/m\cdot K.$

EVALUATE: Our result is consistent with the values for insulating solids in Table 17.5.

17.107. **IDENTIFY:** Apply $H = kA\dfrac{\Delta T}{L}$.

SET UP: For the glass use $L = 12.45$ cm, to account for the thermal resistance of the air films on either side of the glass.

EXECUTE: **(a)** $H = (0.120\ W/m\cdot K)\ (2.00\times0.95\ m^2)\left(\dfrac{28.0\ C°}{5.0\times10^{-2}\ m + 1.8\times10^{-2}\ m}\right) = 93.9\ W.$

(b) The heat flow through the wood part of the door is reduced by a factor of $1-\dfrac{(0.50)^2}{(2.00\times 0.95)}=0.868,$ so it becomes 81.5 W. The heat flow through the glass is

$H_{\text{glass}}=(0.80\text{ W/m}\cdot\text{K})(0.50\text{ m})^2\left(\dfrac{28.0\text{ C}°}{12.45\times 10^{-2}\text{ m}}\right)=45.0\text{ W},$ and so the ratio is $\dfrac{81.5+45.0}{93.9}=1.35.$

EVALUATE: The single-pane window produces a significant increase in heat loss through the door. (See Problem 17.109).

17.108. IDENTIFY: Apply Eq. (17.23).

SET UP: Let $\Delta T_1=\dfrac{HR_1}{A}$ be the temperature difference across the wood and let $\Delta T_2=\dfrac{HR_2}{A}$ be the temperature difference across the insulation. The temperature difference across the combination is $\Delta T=\Delta T_1+\Delta T_2.$ The effective thermal resistance R of the combination is defined by $\Delta T=\dfrac{HR}{A}.$

EXECUTE: $\Delta T=\Delta T_1+\Delta T_2$ gives $\dfrac{H}{A}(R_1+R_2)=\dfrac{H}{A}R,$ and $R=R_1+R_2.$

EVALUATE: A good insulator has a large value of R. R for the combination is larger than the R for any one of the layers.

17.109. IDENTIFY and **SET UP:** Use H written in terms of the thermal resistance R: $H=A\Delta T/R,$ where $R=L/k$ and $R=R_1+R_2+\ldots$ (additive).

EXECUTE: <u>single pane</u> $R_\text{s}=R_{\text{glass}}+R_{\text{film}},$ where $R_{\text{film}}=0.15\text{ m}^2\cdot\text{K/W}$ is the combined thermal resistance of the air films on the room and outdoor surfaces of the window.

$R_{\text{glass}}=L/k=(4.2\times 10^{-3}\text{ m})/(0.80\text{ W/m}\cdot\text{K})=0.00525\text{ m}^2\cdot\text{K/W}$

Thus $R_\text{s}=0.00525\text{ m}^2\cdot\text{K/W}+0.15\text{ m}^2\cdot\text{K/W}=0.1553\text{ m}^2\cdot\text{K/W}.$

<u>double pane</u> $R_\text{d}=2R_{\text{glass}}+R_{\text{air}}+R_{\text{film}},$ where R_{air} is the thermal resistance of the air space between the panes. $R_{\text{air}}=L/k=(7.0\times 10^{-3}\text{ m})/(0.024\text{ W/m}\cdot\text{K})=0.2917\text{ m}^2\cdot\text{K/W}$

Thus $R_\text{d}=2(0.00525\text{ m}^2\cdot\text{K/W})+0.2917\text{ m}^2\cdot\text{K/W}+0.15\text{ m}^2\cdot\text{K/W}=0.4522\text{ m}^2\cdot\text{K/W}$

$H_\text{s}=A\Delta T/R_\text{s}, H_\text{d}=A\Delta T/R_\text{d},$ so $H_\text{s}/H_\text{d}=R_\text{d}/R_\text{s}$ (since A and ΔT are same for both)

$H_\text{s}/H_\text{d}=(0.4522\text{ m}^2\cdot\text{K/W})/(0.1553\text{ m}^2\cdot\text{K/W})=2.9$

EVALUATE: The heat loss is about a factor of 3 less for the double-pane window. The increase in R for a double-pane is due mostly to the thermal resistance of the air space between the panes.

17.110. IDENTIFY: Apply $H=\dfrac{kA\Delta T}{L}$ to each rod. Conservation of energy requires that the heat current through the copper equals the sum of the heat currents through the brass and the steel.

SET UP: Denote the quantities for copper, brass and steel by 1, 2 and 3, respectively, and denote the temperature at the junction by T_0.

EXECUTE: (a) $H_1=H_2+H_3.$ Using Eq. (17.21) and dividing by the common area gives $\dfrac{k_1}{L_1}(100°\text{C}-T_0)=\dfrac{k_2}{L_2}T_0+\dfrac{k_3}{L_3}T_0.$ Solving for T_0 gives $T_0=\dfrac{(k_1/L_1)}{(k_1/L_1)+(k_2/L_2)+(k_3/L_3)}(100°\text{C}).$ Substitution of numerical values gives $T_0=78.4°\text{C}.$

(b) Using $H=\frac{kA}{L}\Delta T$ for each rod, with $\Delta T_1=21.6\text{ C}°,$ $\Delta T_2=\Delta T_3=78.4\text{ C}°$ gives

$H_1=12.8\text{ W}, H_2=9.50\text{ W}$ and $H_3=3.30\text{ W}.$

EVALUATE: In part (b), H_1 is seen to be the sum of H_2 and H_3.

17.111. **(a) EXECUTE:** Heat must be conducted from the water to cool it to 0°C and to cause the phase transition. The entire volume of water is not at the phase transition temperature, just the upper surface that is in contact with the ice sheet.

(b) IDENTIFY: The heat that must leave the water in order for it to freeze must be conducted through the layer of ice that has already been formed.

SET UP: Consider a section of ice that has area A. At time t let the thickness be h. Consider a short time interval t to $t+dt$. Let the thickness that freezes in this time be dh. The mass of the section that freezes in the time interval dt is $dm = \rho\, dV = \rho A\, dh$. The heat that must be conducted away from this mass of water to freeze it is $dQ = dmL_\text{f} = (\rho A L_\text{f})dh$. $H = dQ/dt = kA(\Delta T/h)$, so the heat dQ conducted in time dt throughout the thickness h that is already there is $dQ = kA\left(\dfrac{T_\text{H}-T_\text{C}}{h}\right)dt$. Solve for dh in terms of dt and integrate to get an expression relating h and t.

EXECUTE: Equate these expressions for dQ.

$$\rho A L_\text{f} dh = kA\left(\frac{T_\text{H}-T_\text{C}}{h}\right)dt$$

$$h\, dh = \left(\frac{k(T_\text{H}-T_\text{C})}{\rho L_\text{f}}\right)dt$$

Integrate from $t=0$ to time t. At $t=0$ the thickness h is zero.

$$\int_0^h h\, dh = [k(T_\text{H}-T_\text{C})/\rho L_\text{f}]\int_0^t dt$$

$$\tfrac{1}{2}h^2 = \frac{k(T_\text{H}-T_\text{C})}{\rho L_\text{f}}t \text{ and } h = \sqrt{\frac{2k(T_\text{H}-T_\text{C})}{\rho L_\text{f}}}\sqrt{t}$$

The thickness after time t is proportional to $\sqrt{t}$.

(c) The expression in part (b) gives $t = \dfrac{h^2\rho L_\text{f}}{2k(T_\text{H}-T_\text{C})} = \dfrac{(0.25\text{ m})^2(920\text{ kg/m}^3)(334\times10^3\text{ J/kg})}{2(1.6\text{ W/m}\cdot\text{K})(0°\text{C}-(-10°\text{C}))} = 6.0\times10^5\text{ s}$

$t = 170$ h.

(d) Find t for $h = 40$ m. t is proportional to h^2, so $t = (40\text{ m}/0.25\text{ m})^2(6.00\times10^5\text{ s}) = 1.5\times10^{10}\text{ s}$. This is about 500 years. With our current climate this will not happen.

EVALUATE: As the ice sheet gets thicker, the rate of heat conduction through it decreases. Part (d) shows that it takes a very long time for a moderately deep lake to totally freeze.

17.112. **IDENTIFY:** Apply Eq. (17.22) at each end of the short element. In part (b) use the fact that the net heat current into the element provides the Q for the temperature increase, according to $Q = mc\Delta T$.

SET UP: dT/dx is the temperature gradient.

EXECUTE: **(a)** $H = (380\text{ W/m}\cdot\text{K})(2.50\times10^{-4}\text{ m}^2)(140\text{ C°/m}) = 13.3\text{ W}$.

(b) Denoting the two ends of the element as 1 and 2, $H_2 - H_1 = \dfrac{Q}{t} = mc\dfrac{\Delta T}{t}$, where $\dfrac{\Delta T}{t} = 0.250\text{ C°/s}$.

$kA\left.\dfrac{dT}{dx}\right|_2 - kA\left.\dfrac{dT}{dx}\right|_1 = mc\left(\dfrac{\Delta T}{t}\right)$. The mass m is $\rho A\Delta x$, so $\left.\dfrac{dT}{dx}\right|_2 = \left.\dfrac{dT}{dx}\right|_1 + \dfrac{\rho c\Delta x}{k}\left(\dfrac{\Delta T}{t}\right)$.

$$kA\left.\frac{dT}{dx}\right|_2 = 140\text{ C°/m} + \frac{(1.00\times10^4\text{ kg/m}^3)(520\text{ J/kg}\cdot\text{K})(1.00\times10^{-2}\text{ m})(0.250\text{ C°/s})}{380\text{ W/m}\cdot\text{K}} = 174\text{ C°/m}.$$

EVALUATE: At steady-state the temperature of the short element is no longer changing and $H_1 = H_2$.

17.113. **IDENTIFY:** The rate of heat conduction through the walls is 1.25 kW. Use the concept of thermal resistance and the fact that when insulating materials are in layers, the R values are additive.

SET UP: The total area of the four walls is $2(3.50\text{ m})(2.50\text{ m}) + 2(3.00\text{ m})(2.50\text{ m}) = 32.5\text{ m}^2$

EXECUTE: $H = A\dfrac{T_H - T_C}{R}$ gives $R = \dfrac{A(T_H - T_C)}{H} = \dfrac{(32.5\text{ m}^2)(17.0\text{ K})}{1.25\times10^3\text{ W}} = 0.442\text{ m}^2\cdot\text{K/W}$. For the wood, $R_w = \dfrac{L}{k} = \dfrac{1.80\times10^{-2}\text{ m}}{0.060\text{ W/m}\cdot\text{K}} = 0.300\text{ m}^2\cdot\text{K/W}$. For the insulating material, $R_{in} = R - R_w = 0.142\text{ m}^2\cdot\text{K/W}$.

$R_{in} = \dfrac{L_{in}}{k_{in}}$ and $k_{in} = \dfrac{L_{in}}{R_{in}} = \dfrac{1.50\times10^{-2}\text{ m}}{0.142\text{ m}^2\cdot\text{K/W}} = 0.106\text{ W/m}\cdot\text{K}$.

EVALUATE: The thermal conductivity of the insulating material is larger than that of the wood, the thickness of the insulating material is less than that of the wood, and the thermal resistance of the wood is about three times that of the insulating material.

17.114. **IDENTIFY:** $I_1 r_1^2 = I_2 r_2^2$. Apply $H = Ae\sigma T^4$ (Eq. 17.25) to the sun.

SET UP: $I_1 = 1.50\times10^3\text{ W/m}^2$ when $r = 1.50\times10^{11}\text{ m}$.

EXECUTE: **(a)** The energy flux at the surface of the sun is

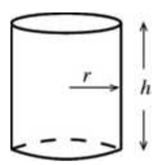
$$I_2 = (1.50\times10^3\text{ W/m}^2)\left(\frac{1.50\times10^{11}\text{ m}}{6.96\times10^8\text{ m}}\right)^2 = 6.97\times10^7\text{ W/m}^2.$$

(b) Solving Eq. (17.25) with $e = 1$, $T = \left[\dfrac{H}{A}\dfrac{1}{\sigma}\right]^{\frac{1}{4}} = \left[\dfrac{6.97\times10^7\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right]^{\frac{1}{4}} = 5920\text{ K}$.

EVALUATE: The total power output of the sun is $P = 4\pi r_2^2 I_2 = 4\times10^{26}\text{ W}$.

17.115. **IDENTIFY** and **SET UP:** Use Eq. (17.26) to find the net heat current into the can due to radiation. Use $Q = Ht$ to find the heat that goes into the liquid helium, set this equal to mL and solve for the mass m of helium that changes phase.

EXECUTE: Calculate the net rate of radiation of heat from the can. $H_{net} = Ae\sigma(T^4 - T_s^4)$.



The surface area of the cylindrical can is $A = 2\pi rh + 2\pi r^2$. (See Figure 17.115.)

Figure 17.115

$A = 2\pi r(h + r) = 2\pi(0.045\text{ m})(0.250\text{ m} + 0.045\text{ m}) = 0.08341\text{ m}^2$.

$H_{net} = (0.08341\text{ m}^2)(0.200)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)((4.22\text{ K})^4 - (77.3\text{ K})^4)$

$H_{net} = -0.0338\text{ W}$ (the minus sign says that the net heat current is into the can). The heat that is put into the can by radiation in one hour is $Q = -(H_{net})t = (0.0338\text{ W})(3600\text{ s}) = 121.7\text{ J}$. This heat boils a mass m of helium according to the equation $Q = mL_f$, so $m = \dfrac{Q}{L_f} = \dfrac{121.7\text{ J}}{2.09\times10^4\text{ J/kg}} = 5.82\times10^{-3}\text{ kg} = 5.82\text{ g}$.

EVALUATE: In the expression for the net heat current into the can the temperature of the surroundings is raised to the fourth power. The rate at which the helium boils away increases by about a factor of $(293/77)^4 = 210$ if the walls surrounding the can are at room temperature rather than at the temperature of the liquid nitrogen.

17.116. **IDENTIFY:** The nonmechanical part of the basal metabolic rate (i.e., the heat) leaves the body by radiation from the surface.

SET UP: In the radiation equation, $H_{net} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins; $e = 1.0$, $T = 30°\text{C} = 303\text{ K}$, and $T_s = 18°\text{C} = 291\text{ K}$. Call the basal metabolic rate BMR.

EXECUTE: **(a)** $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$.

$H_{\text{net}} = (2.0\text{ m}^2)(1.0)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)([303\text{ K}]^4 - [291\text{ K}]^4) = 140\text{ W}.$

(b) $(0.80)\text{BMR} = 140\text{ W}$, so $\text{BMR} = 180\text{ W}$.

EVALUATE: If the emissivity of the skin were less than 1.0, the body would radiate less so the BMR would have to be lower than we found in (b).

17.117. **IDENTIFY:** The jogger radiates heat but the air radiates heat back into the jogger.

SET UP: The emissivity of a human body is taken to be 1.0. In the equation for the radiation heat current, $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins.

EXECUTE: **(a)** $P_{\text{jog}} = (0.80)(1300\text{ W}) = 1.04\times10^3\text{ J/s}$.

(b) $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$, which gives

$H_{\text{net}} = (1.85\text{ m}^2)(1.00)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)([306\text{ K}]^4 - [313\text{ K}]^4) = -87.1\text{ W}$. The person gains 87.1 J of heat each second by radiation.

(c) The total excess heat per second is $1040\text{ J/s} + 87\text{ J/s} = 1130\text{ J/s}$.

(d) In $1\text{ min} = 60\text{ s}$, the runner must dispose of $(60\text{ s})(1130\text{ J/s}) = 6.78\times10^4\text{ J}$. If this much heat goes to evaporate water, the mass m of water that evaporates in one minute is given by $Q = mL_v$, so

$$m = \frac{Q}{L_v} = \frac{6.78\times10^4\text{ J}}{2.42\times10^6\text{ J/kg}} = 0.028\text{ kg} = 28\text{ g}.$$

(e) In a half-hour, or 30 minutes, the runner loses $(30\text{ min})(0.028\text{ kg/min}) = 0.84\text{ kg}$. The runner must drink 0.84 L, which is $\dfrac{0.84\text{ L}}{0.750\text{ L/bottle}} = 1.1$ bottles.

EVALUATE: The person *gains* heat by radiation since the air temperature is greater than his skin temperature.

17.118. **IDENTIFY:** The heat generated will remain in the runner's body, which will increase his body temperature.

SET UP: Problem 17.117 calculates that the net rate of heat input to the person is 1130 W. $Q = mc\Delta T$. $9\text{ F}° = 5\text{ C}°$.

EXECUTE: **(a)** $Q = Pt = (1130\text{ W})(1800\text{ s}) = 2.03\times10^6\text{ J}$. $Q = mc\Delta T$ so

$$\Delta T = \frac{Q}{mc} = \frac{2.03\times10^6\text{ J}}{(68\text{ kg})(3480\text{ J/kg}\cdot\text{C}°)} = 8.6\text{ C}°.$$

(b) $\Delta T = (8.6\text{ C}°)(9\text{ F}°/5\text{ C}°) = 15.5\text{ F}°$. $T = 98.6°\text{F} + 15.5\text{ F}° = 114°\text{F}$.

EVALUATE: This body temperature is lethal.

17.119. **IDENTIFY:** For the water, $Q = mc\Delta T$.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** At steady state, the input power all goes into heating the water, so $P = \dfrac{Q}{t} = \dfrac{mc\Delta T}{t}$ and

$\Delta T = \dfrac{Pt}{cm} = \dfrac{(1800\text{ W})(60\text{ s/min})}{(4190\text{ J/kg}\cdot\text{K})(0.500\text{ kg/min})} = 51.6\text{ K}$, and the output temperature is $18.0°\text{C} + 51.6\text{ C}° = 69.6°\text{C}$.

EVALUATE: **(b)** At steady state, the temperature of the apparatus is constant and the apparatus will neither remove heat from nor add heat to the water.

17.120. **IDENTIFY:** For the air the heat input is related to the temperature change by $Q = mc\Delta T$.

SET UP: The rate P at which heat energy is generated is related to the rate P_0 at which food energy is consumed by the hamster by $P = 0.10P_0$.

EXECUTE: **(a)** The heat generated by the hamster is the heat added to the box;

$$P = \frac{Q}{t} = mc\frac{\Delta T}{t} = (1.20\text{ kg/m}^3)(0.0500\text{ m}^3)(1020\text{ J/kg}\cdot\text{K})(1.60\text{ C}°/\text{h}) = 97.9\text{ J/h}.$$

(b) Taking the efficiency into account, the mass M of seed that must be eaten in time t is

$$\frac{M}{t} = \frac{P_0}{L_c} = \frac{P/(10\%)}{L_c} = \frac{979\ \text{J/h}}{24\ \text{J/g}} = 40.8\ \text{g/h}.$$

EVALUATE: This is about 1.5 ounces of seed consumed in one hour.

17.121. IDENTIFY: Heat Q_i goes into the ice when it warms to $0°\text{C}$, melts, and the resulting water warms to the final temperature T_f. Heat Q_{ow} comes out of the ocean water when it cools to T_f. Conservation of energy gives $Q_i + Q_{ow} = 0$.

SET UP: For ice, $c_i = 2100\ \text{J/kg} \cdot \text{K}$. For water, $L_f = 334 \times 10^3\ \text{J/kg}$ and $c_w = 4190\ \text{J/kg} \cdot \text{K}$. Let m be the total mass of the water on the earth's surface. So $m_i = 0.0175m$ and $m_{ow} = 0.975m$.

EXECUTE: $Q_i + Q_{ow} = 0$ gives $m_i c_i (30\ \text{C}°) + m_i L_f + m_i c_w T_f + m_{ow} c_w (T_f - 5.00°\text{C}) = 0$.

$$T_f = \frac{-m_i c_i (30\ \text{C}°) - m_i L_f + m_{ow} c_w (5.00\ \text{C}°)}{(m_i + m_{ow}) c_w}.$$

$$T_f = \frac{-(0.0175m)(2100\ \text{J/kg} \cdot \text{K})(30\ \text{K}) - (0.0175m)(334 \times 10^3\ \text{J/kg}) + (0.975m)(4190\ \text{J/kg} \cdot \text{K})(5.00\ \text{K})}{(0.0175m + 0.975m)(4190\ \text{J/kg} \cdot \text{K})}$$

$$T_f = \frac{1.348 \times 10^4\ \text{J/kg}}{4.159 \times 10^3\ \text{J/kg} \cdot \text{K}} = 3.24°\text{C}.$$ The temperature decrease is $1.76\ \text{C}°$.

EVALUATE: The mass of ice in the icecaps is much less than the mass of the water in the oceans, but much more heat is required to change the phase of 1 kg of ice than to change the temperature of 1 kg of water $1\ \text{C}°$, so the lowering of the temperature of the oceans would be appreciable.

17.122. IDENTIFY: The oceans take time to increase (or decrease) in temperature because they contain a large mass of water which has a high specific heat.

SET UP: The radius of the earth is $R_E = 6.38 \times 10^6\ \text{m}$. Since an ocean depth of 100 m is much less than the radius of the earth, we can calculate the volume of the water to this depth as the surface area of the oceans times 100 m. The total surface area of the earth is $A_{earth} = 4\pi R_E^{\,2}$. The density of seawater is $1.03 \times 10^3\ \text{kg/m}^3$ (Table12.1).

EXECUTE: The surface area of the oceans is $(2/3)A_{earth} = (2/3)(4\pi)(6.38 \times 10^6\ \text{m})^2 = 3.41 \times 10^{14}\ \text{m}^2$. The total rate of solar energy incident on the oceans is $(1050\ \text{W/m}^2)(3.41 \times 10^{14}\ \text{m}^2) = 3.58 \times 10^{17}\ \text{W}$. 12 hours per day for 30 days is $(12)(30)(3600)\ \text{s} = 1.30 \times 10^6\ \text{s}$, so the total solar energy input to the oceans in one month is $(1.30 \times 10^6\ \text{s})(3.58 \times 10^{17}\ \text{W}) = 4.65 \times 10^{23}\ \text{J}$. The volume of the seawater absorbing this energy is $(100\ \text{m})(3.41 \times 10^{14}\ \text{m}^2) = 3.41 \times 10^{16}\ \text{m}^3$. The mass of this water is $m = \rho V = (1.03 \times 10^3\ \text{kg/m}^3)(3.41 \times 10^{16}\ \text{m}^3) = 3.51 \times 10^{19}\ \text{kg}$. $Q = mc\Delta T$, so

$$\Delta T = \frac{Q}{mc} = \frac{4.65 \times 10^{23}\ \text{J}}{(3.51 \times 10^{19}\ \text{kg})(3890\ \text{J/kg} \cdot \text{C}°)} = 3.4\ \text{C}°.$$

EVALUATE: A temperature rise of 3.4 C° is significant. The solar energy input is a very large number, but so is the total mass of the top 100 m of seawater in the oceans.

17.123. IDENTIFY: Apply Eq. (17.22) to different points along the rod, where $\frac{dT}{dx}$ is the temperature gradient at each point.

SET UP: For copper, $k = 385\ \text{W/m} \cdot \text{K}$.

EXECUTE: **(a)** The initial temperature distribution, $T = (100°\text{C})\sin \pi x/L$, is shown in Figure 17.123a.

(b) After a very long time, no heat will flow, and the entire rod will be at a uniform temperature which must be that of the ends, 0°C.

(c) The temperature distribution at successively greater times $T_1 < T_2 < T_3$ is sketched in Figure 17.123b.

(d) $\dfrac{dT}{dx} = (100°\text{C})(\pi/L)\cos \pi x/L.$ At the ends, $x = 0$ and $x = L,$ the cosine is ± 1 and the temperature gradient is $\pm(100°\text{C})(\pi/0.100\ \text{m}) = \pm 3.14\times 10^3\ \text{C}°/\text{m}.$

(e) Taking the phrase "into the rod" to mean an absolute value, the heat current will be
$k\,A\dfrac{dT}{dx} = (385.0\ \text{W/m}\cdot\text{K})(1.00\times 10^{-4}\ \text{m}^2)(3.14\times 10^3\ \text{C}°/\text{m}) = 121\ \text{W}.$

(f) Either by evaluating $\dfrac{dT}{dx}$ at the center of the rod, where $\pi x/L = \pi/2$ and $\cos(\pi/2) = 0,$ or by checking the figure in part (a), the temperature gradient is zero, and no heat flows through the center; this is consistent with the symmetry of the situation. There will not be any heat current at the center of the rod at any later time.

(g) $\dfrac{k}{\rho c} = \dfrac{(385\ \text{W/m}\cdot\text{K})}{(8.9\times 10^3\ \text{kg/m}^3)(390\ \text{J/kg}\cdot\text{K})} = 1.1\times 10^{-4}\ \text{m}^2/\text{s}.$

(h) Although there is no net heat current, the temperature of the center of the rod is decreasing; by considering the heat current at points just to either side of the center, where there is a non-zero temperature gradient, there must be a net flow of heat out of the region around the center. Specifically,

$$H((L/2)+\Delta x) - H((L/2)-\Delta x) = \rho A(\Delta x)c\frac{\partial T}{\partial t} = kA\left(\left.\frac{\partial T}{\partial x}\right|_{(L/2+\Delta x)} - \left.\frac{\partial T}{\partial x}\right|_{(L/2-\Delta x)}\right) = kA\frac{\partial^2 T}{\partial x^2}\Delta x, \text{ from}$$

which the Heat Equation, $\dfrac{\partial T}{\partial t} = \dfrac{k}{\rho c}\dfrac{\partial^2 T}{\partial x^2}$ is obtained. At the center of the rod,

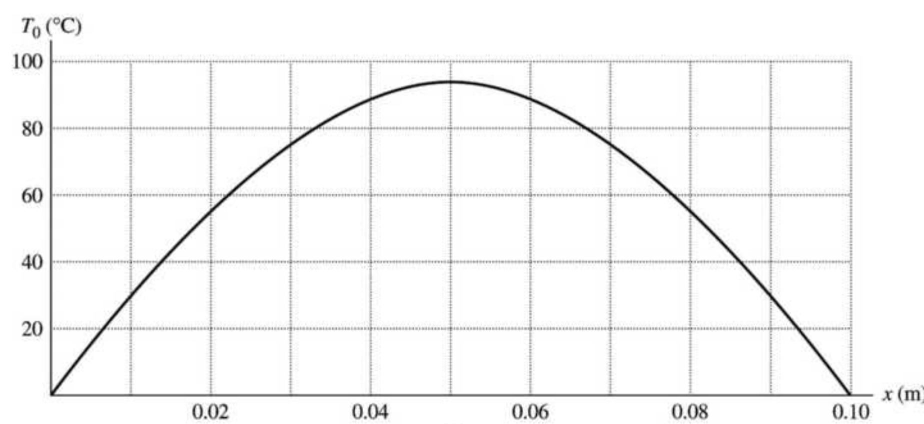
$\dfrac{\partial^2 T}{\partial x^2} = -(100\ \text{C}°)(\pi/L)^2,$ and so $\dfrac{\partial T}{\partial t} = -(1.11\times 10^{-4}\ \text{m}^2/\text{s})(100\ \text{C}°)\left(\dfrac{\pi}{0.100\ \text{m}}\right)^2 = -10.9\ \text{C}°/\text{s},$ or $-11\ \text{C}°/\text{s}$ to two figures.

(i) $\dfrac{100\ \text{C}°}{10.9\ \text{C}°/\text{s}} = 9.17\ \text{s}$

(j) Decrease (that is, become less negative), since as T decreases, $\dfrac{\partial^2 T}{\partial x^2}$ decreases. This is consistent with the graphs, which correspond to equal time intervals.

(k) At the point halfway between the end and the center, at any given time $\dfrac{\partial^2 T}{\partial x^2}$ is a factor of $\sin(\pi/4) = 1/\sqrt{2}$ less than at the center, and so the initial rate of change of temperature is $-7.71\ \text{C}°/\text{s}.$

EVALUATE: A plot of temperature as a function of both position and time for $0 \le t \le 50$ s is shown in Figure 17.123c.



(a)

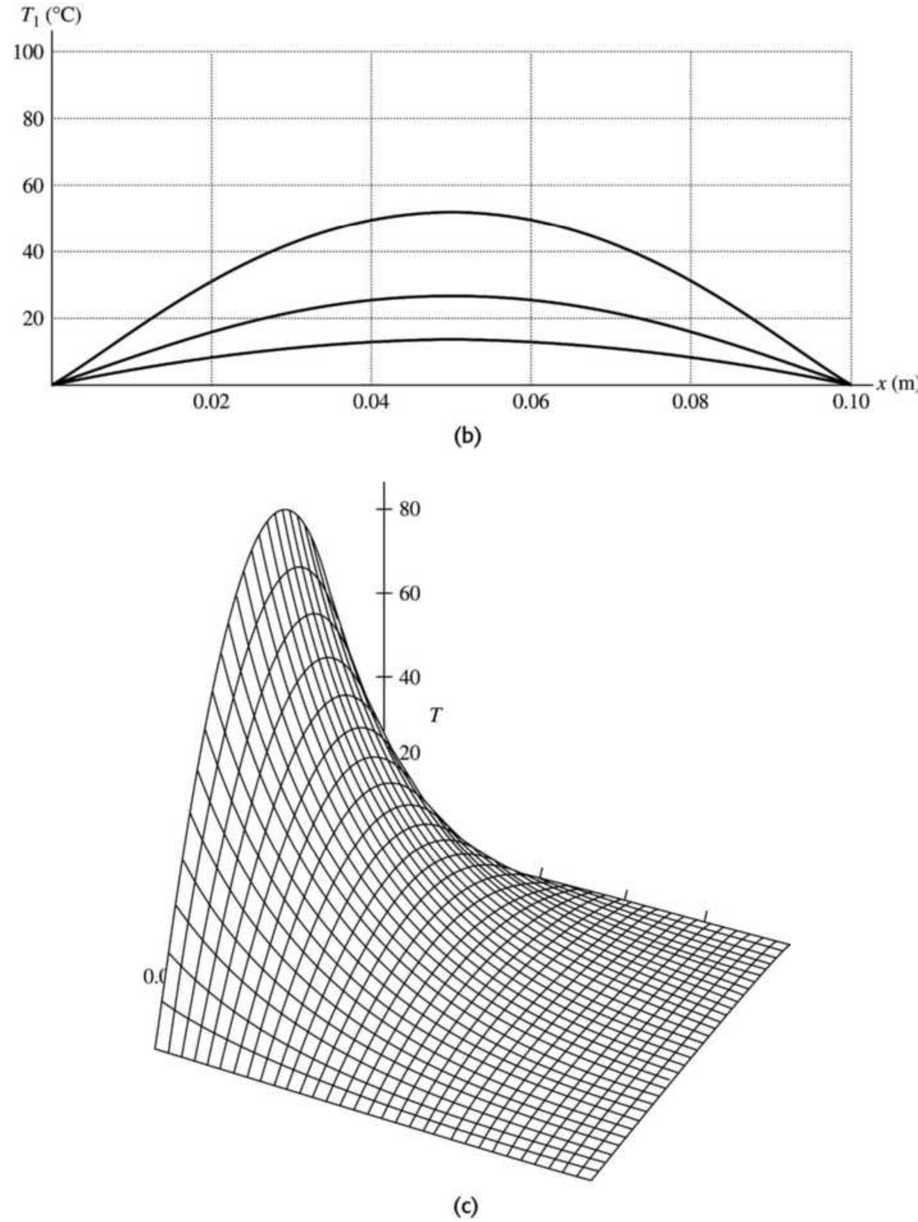


Figure 17.123

17.124. **IDENTIFY:** Apply Eq. (17.21). For a spherical or cylindrical surface, the area A in Eq. (17.21) is not constant, and the material must be considered to consist of shells with thickness dr and a temperature difference between the inside and outside of the shell dT. The heat current will be a constant, and must be found by integrating a differential equation.

SET UP: The surface area of a sphere is $4\pi r^2$. The surface area of the curved side of a cylinder is $2\pi rl$. $\ln(1+\varepsilon) \approx \varepsilon$ when $\varepsilon \ll 1$.

(a) Equation (17.21) becomes $H = k(4\pi r^2)\dfrac{dT}{dr}$ or $\dfrac{H\ dr}{4\pi r^2} = k\ dT$. Integrating both sides between the appropriate limits, $\dfrac{H}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = k(T_2 - T_1)$. In this case the "appropriate limits" have been chosen so that if the inner temperature T_2 is at the higher temperature T_1, the heat flows outward; that is, $\frac{dT}{dr} < 0$.

Solving for the heat current, $H = \frac{k4\pi ab(T_2 - T_1)}{b - a}$.

(b) The rate of change of temperature with radius is of the form $\frac{dT}{dr} = \frac{B}{r^2}$, with B a constant. Integrating from $r = a$ to r and from $r = a$ to $r = b$ gives $T(r) - T_2 = B\left(\frac{1}{a} - \frac{1}{r}\right)$ and $T_1 - T_2 = B\left(\frac{1}{a} - \frac{1}{b}\right)$. Using the second of these to eliminate B and solving for $T(r)$ gives $T(r) = T_2 - (T_2 - T_1)\left(\frac{r-a}{b-a}\right)\left(\frac{b}{r}\right)$. There are, of course, many equivalent forms. As a check, note that at $r = a$, $T = T_2$ and at $r = b$, $T = T_1$.

(c) As in part (a), the expression for the heat current is $H = k(2\pi rL)\frac{dT}{dr}$ or $\frac{H}{2\pi r} = kLdT$, which integrates, with the same condition on the limits, to $\frac{H}{2\pi}\ln(b/a) = kL(T_2 - T_1)$, or $H = \frac{2\pi kL(T_2 - T_1)}{\ln(b/a)}$.

(d) A method similar to that used in part (b) gives $T(r) = T_2 + (T_1 - T_2)\frac{\ln(r/a)}{\ln(b/a)}$.

EVALUATE: (e) For the sphere: Let $b - a = l$, and approximate $b \sim a$, with a the common radius. Then the surface area of the sphere is $A = 4\pi a^2$, and the expression for H is that of Eq. (17.21) (with l instead of L, which has another use in this problem). For the cylinder: with the same notation, consider $\ln\left(\frac{b}{a}\right) = \ln\left(1 + \frac{l}{a}\right) \sim \frac{l}{a}$, where the approximation for $\ln(1+\varepsilon)$ for small ε has been used. The expression for H then reduces to $k(2\pi La)(\Delta T/l)$, which is Eq. (17.21) with $A = 2\pi La$.

17.125. IDENTIFY: From the result of Problem 17.124, the heat current through each of the jackets is related to the temperature difference by $H = \frac{2\pi lk}{\ln(b/a)}\Delta T$, where l is the length of the cylinder and b and a are the inner and outer radii of the cylinder.

SET UP: Let the temperature across the cork be ΔT_1 and the temperature across the styrofoam be ΔT_2, with similar notation for the thermal conductivities and heat currents.

EXECUTE: (a) $\Delta T_1 + \Delta T_2 = \Delta T = 125\ \text{C}°$. Setting $H_1 = H_2 = H$ and canceling the common factors, $\frac{\Delta T_1 k_1}{\ln 2} = \frac{\Delta T_2 k_2}{\ln 1.5}$. Eliminating ΔT_2 and solving for ΔT_1 gives $\Delta T_1 = \Delta T\left(1 + \frac{k_1}{k_2}\frac{\ln 1.5}{\ln 2}\right)^{-1}$. Substitution of numerical values gives $\Delta T_1 = 37\ \text{C}°$, and the temperature at the radius where the layers meet is $140°\text{C} - 37°\text{C} = 103°\text{C}$.

(b) Substitution of this value for ΔT_1 into the above expression for $H_1 = H$ gives

$$H = \frac{2\pi(2.00\ \text{m})(0.0400\ \text{W/m}\cdot\text{K})}{\ln 2}(37\ \text{C}°) = 27\ \text{W}.$$

EVALUATE: $\Delta T = 103°\text{C} - 15°\text{C} = 88\ \text{C}°$. $H_2 = \frac{2\pi(2.00\ \text{m})(0.0100\ \text{W/m}\cdot\text{K})}{\ln(6.00/4.00)}(88\ \text{C}°) = 27\ \text{W}$. This is the same as H_1, as it should be.

17.126. IDENTIFY: Apply the concept of thermal expansion. In part (b) the object can be treated as a simple pendulum.

SET UP: For steel $\alpha = 1.2 \times 10^{-5}\ (\text{C}°)^{-1}$. $1\ \text{yr} = 86,400\ \text{s}$.

EXECUTE: (a) In hot weather, the moment of inertia I and the length d in Eq. (14.39) will both increase by the same factor, and so the period will be longer and the clock will run slow (lose time). Similarly, the clock will run fast (gain time) in cold weather.

(b) $\dfrac{\Delta L}{L_0} = \alpha \Delta T = (1.2\times10^{-5}\ (\text{C}°)^{-1})(10.0\ \text{C}°) = 1.2\times10^{-4}.$

(c) To avoid possible confusion, denote the pendulum period by τ. For this problem, $\dfrac{\Delta\tau}{\tau} = \dfrac{1}{2}\dfrac{\Delta L}{L} = 6.0\times10^{-5}$ so in one day the clock will gain $(86,400\ \text{s})(6.0\times10^{-5}) = 5.2\ \text{s}$.

(d) $\left|\dfrac{\Delta\tau}{\tau}\right| = \tfrac{1}{2}\alpha\Delta T.$ $\left|\dfrac{\Delta\tau}{\tau}\right| = \dfrac{1.0\ \text{s}}{86,400\ \text{s}}$ gives $\Delta T = 2[(1.2\times10^{-5}(\text{C}°)^{-1})(86,400)]^{-1} = 1.9\ \text{C}°$. T must be controlled to within $1.9\ \text{C}°$.

EVALUATE: In part (d) the answer does not depend on the period of the pendulum. It depends only on the fractional change in the period.

17.127. IDENTIFY: The rate in (iv) is given by Eq. (17.26), with $T = 309\ \text{K}$ and $T_s = 320\ \text{K}$. The heat absorbed in the evaporation of water is $Q = mL$.

SET UP: $m = \rho V$, so $\dfrac{m}{V} = \rho.$

EXECUTE: **(a)** The rates are: (i) 280 W,

(ii) $(54\ \text{J/h}\cdot\text{C}°\cdot\text{m}^2)(1.5\ \text{m}^2)(11\ \text{C}°)/(3600\ \text{s/h}) = 0.248\ \text{W}$,

(iii) $(1400\ \text{W/m}^2)(1.5\ \text{m}^2) = 2.10\times10^3\ \text{W}$,

(iv) $(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.5\ \text{m}^2)((320\ \text{K})^4 - (309\ \text{K})^4) = 116\ \text{W}$.

The total is $2.50\ \text{kW}$, with the largest portion due to radiation from the sun.

(b) $\dfrac{P}{\rho L_\text{v}} = \dfrac{2.50\times10^3\ \text{W}}{(1000\ \text{kg/m}^3)(2.42\times10^6\ \text{J/kg}\cdot\text{K})} = 1.03\times10^{-6}\ \text{m}^3/\text{s}$. This is equal to $3.72\ \text{L/h}$.

(c) Redoing the above calculations with $e = 0$ and the decreased area gives a power of 945 W and a corresponding evaporation rate of 1.4 L/h. Wearing reflective clothing helps a good deal. Large areas of loose-weave clothing also facilitate evaporation.

EVALUATE: The radiant energy from the sun absorbed by the area covered by clothing is assumed to be zero, since $e \approx 0$ for the clothing and the clothing reflects almost all the radiant energy incident on it. For the same reason, the exposed skin area is the area used in Eq. (17.26).