11

EQUILIBRIUM AND ELASTICITY

11.1. **IDENTIFY:** Use Eq. (11.3) to calculate x_{cm}. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

SET UP: Use coordinates with the origin at the left end of the bar and the $+x$ axis along the bar. $m_1 = 0.120$ kg, $m_2 = 0.055$ kg, $m_3 = 0.110$ kg.

EXECUTE: $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(0.120\text{ kg})(0.250\text{ m}) + 0 + (0.110\text{ kg})(0.500\text{ m})}{0.120\text{ kg} + 0.055\text{ kg} + 0.110\text{ kg}} = 0.298\text{ m}.$ The fulcrum should be placed 29.8 cm to the right of the left-hand end.

EVALUATE: The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

11.2. **IDENTIFY:** Use Eq. (11.3) to calculate x_{cm} of the composite object.

SET UP: Use coordinates where the origin is at the original center of gravity of the object and $+x$ is to the right. With the 1.50 g mass added, $x_{cm} = -2.20$ cm, $m_1 = 5.00$ g and $m_2 = 1.50$ g. $x_1 = 0$.

EXECUTE: $x_{cm} = \dfrac{m_2x_2}{m_1 + m_2}$. $x_2 = \left(\dfrac{m_1 + m_2}{m_2}\right)x_{cm} = \left(\dfrac{5.00\text{ g} + 1.50\text{ g}}{1.50\text{ g}}\right)(-2.20\text{ cm}) = -9.53\text{ cm}.$

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

EVALUATE: The new center of gravity is somewhere between the added mass and the original center of gravity.

11.3. **IDENTIFY:** Treat the rod and clamp as point masses. The center of gravity of the rod is at its midpoint, and we know the location of the center of gravity of the rod-clamp system.

SET UP: $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$.

EXECUTE: $1.20\text{ m} = \dfrac{(1.80\text{ kg})(1.00\text{ m}) + (2.40\text{ kg})x_2}{1.80\text{ kg} + 2.40\text{ kg}}$.

$$x_2 = \frac{(1.20\text{ m})(1.80\text{ kg} + 2.40\text{ kg}) - (1.80\text{ kg})(1.00\text{ m})}{2.40\text{ kg}} = 1.35\text{ m}$$

EVALUATE: The clamp is to the right of the center of gravity of the system, so the center of gravity of the system lies between that of the rod and the clamp, which is reasonable.

11.4. **IDENTIFY:** Apply the first and second conditions for equilibrium to the trap door.

SET UP: For $\sum \tau_z = 0$ take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

EXECUTE: **(a)** The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

(b) With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N.

The hinges supply an upward force of $300\text{ N} - 150\text{ N} = 150\text{ N}$.

EVALUATE: Less force must be applied when it is applied farther from the hinges.

11.5. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the ladder.

SET UP: Take the axis to be at point A. The free-body diagram for the ladder is given in Figure 11.5. The torque due to F must balance the torque due to the weight of the ladder.

EXECUTE: $F(8.0\text{ m})\sin 40° = (2800\text{ N})(10.0\text{ m})$, so $F = 5.45\text{ kN}$.

EVALUATE: The force required is greater than the weight of the ladder, because the moment arm for F is less than the moment arm for w.

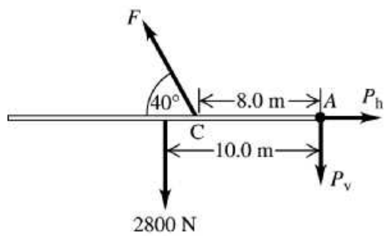


Figure 11.5

11.6. **IDENTIFY:** Apply the first and second conditions of equilibrium to the board.

SET UP: The free-body diagram for the board is given in Figure 11.6. Since the board is uniform its center of gravity is 1.50 m from each end. Apply $\sum F_y = 0,$ with $+y$ upward. Apply $\sum \tau_z = 0$ with the axis at the end where the first person applies a force and with counterclockwise torques positive.

EXECUTE: $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}.$ $\sum \tau_z = 0$ gives $F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}.$ The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.

EVALUATE: By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

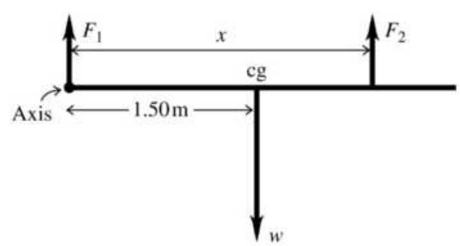


Figure 11.6

11.7. **IDENTIFY:** Apply $\sum F_y = 0$ and $\sum \tau_z = 0$ to the board.

SET UP: Let $+y$ be upward. Let x be the distance of the center of gravity of the motor from the end of the board where the 400 N force is applied.

EXECUTE: **(a)** If the board is taken to be massless, the weight of the motor is the sum of the applied forces, 1000 N. The motor is a distance $\dfrac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$ from the end where the 400 N force is applied, and so is 0.800 m from the end where the 600 N force is applied.

(b) The weight of the motor is $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}.$ Applying $\sum \tau_z = 0$ with the axis at the end of the board where the 400 N acts gives $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$ and $x = 1.25\text{ m}.$ The center of gravity of the motor is 0.75 m from the end of the board where the 600 N force is applied.

EVALUATE: The motor is closest to the end of the board where the larger force is applied.

11.8. **IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.
SET UP: The free-body diagram for the shelf is given in Figure 11.8. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.
EXECUTE: **(a)** $\sum\tau_z = 0$ gives $-w_\text{t}(0.200\text{ m}) - w_\text{s}(0.300\text{ m}) + T_2(0.400\text{ m}) = 0.$

$$T_2 = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$\sum F_y = 0$ gives $T_1 + T_2 - w_\text{t} - w_\text{s} = 0$ and $T_1 = 25.0\text{ N}$. The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.
EVALUATE: We can verify that $\sum\tau_z = 0$ is zero for any axis, for example for an axis at the right-hand end of the shelf.

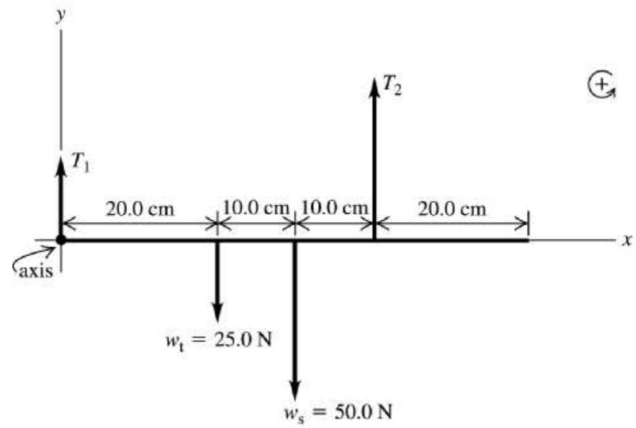


Figure 11.8

11.9. **IDENTIFY:** Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.
SET UP: Let cable A be at the left-hand end. Take the axis to be at the left-hand end of the bar and x be the distance of the weight w from this end. The free-body diagram for the bar is given in Figure 11.9.
EXECUTE: **(a)** $\sum F_y = 0$ gives $T_A + T_B - w - w_\text{bar} = 0$ and

$w = T_A + T_B - w_\text{bar} = 500.0\text{ N} + 400.0\text{ N} - 350.0\text{ N} = 550\text{ N}.$

(b) $\sum\tau_z = 0$ gives $T_B(1.50\text{ m}) - wx - w_\text{bar}(0.750\text{ m}) = 0.$

$x = \dfrac{T_B(1.50\text{ m}) - w_\text{bar}(0.750\text{ m})}{w} = \dfrac{(400.0\text{ N})(1.50\text{ m}) - (350\text{ N})(0.750\text{ m})}{550\text{ N}} = 0.614\text{ m}.$ The weight should be placed 0.614 m from the left-hand end of the bar (cable A).
EVALUATE: If the weight is moved to the left, T_A exceeds 500.0 N and if it is moved to the right T_B exceeds 400.0 N.

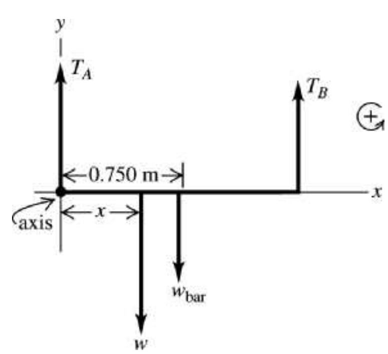


Figure 11.9

11.10. **IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.

SET UP: Let n_2 be the upward normal force exerted by the ground and let n_1 be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is $\mu_s n_2$.

EXECUTE: **(a)** Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force n_2. For the vertical forces to balance, $n_2 = w_1 + w_m = 160 \text{ N} + 740 \text{ N} = 900 \text{ N}$, and the maximum frictional force is $\mu_s n_2 = (0.40)(900\text{N}) = 360\text{N}$.

(b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

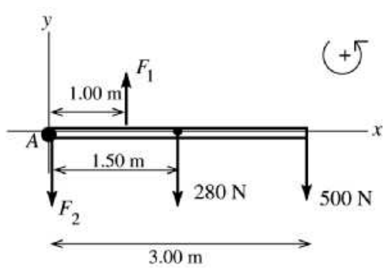
$(4.0 \text{ m})n_1 = (1.5 \text{ m})(160 \text{ N}) + (1.0 \text{ m})(3/5)(740 \text{ N}) = 684 \text{ N} \cdot \text{m}$, so $n_1 = 171.0 \text{ N}$. This horizontal force must be balanced by the friction force, which must then be 170 N to two figures.

(c) Setting the friction force, and hence n_1, equal to the maximum of 360 N and solving for the distance x along the ladder, $(4.0 \text{ m})(360 \text{ N}) = (1.50 \text{ m})(160 \text{ N}) + x(3/5)(740 \text{ N})$, so $x = 2.7 \text{ m}$.

EVALUATE: The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

11.11. **IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

(a) SET UP: The free-body diagram for the diving board is given in Figure 11.11. Take the origin of coordinates at the left-hand end of the board (point A).



$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

Figure 11.11

EXECUTE: $\sum \tau_A = 0$ gives $+F_1(1.0 \text{ m}) - (500 \text{ N})(3.00 \text{ m}) - (280 \text{ N})(1.50 \text{ m}) = 0$

$$F_1 = \frac{(500 \text{ N})(3.00 \text{ m}) + (280 \text{ N})(1.50 \text{ m})}{1.00 \text{ m}} = 1920 \text{ N}$$

(b) $\sum F_y = ma_y$

$F_1 - F_2 - 280 \text{ N} - 500 \text{ N} = 0$

$F_2 = F_1 - 280 \text{ N} - 500 \text{ N} = 1920 \text{ N} - 280 \text{ N} - 500 \text{ N} = 1140 \text{ N}$

EVALUATE: We can check our answers by calculating the net torque about some point and checking that $\sum \tau_z = 0$ for that point also. Net torque about the right-hand end of the board:

$(1140 \text{ N})(3.00 \text{ m}) + (280 \text{ N})(1.50 \text{ m}) - (1920 \text{ N})(2.00 \text{ m}) = 3420 \text{ N} \cdot \text{m} + 420 \text{ N} \cdot \text{m} - 3840 \text{ N} \cdot \text{m} = 0$, which checks.

11.12. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The boy exerts a downward force on the beam that is equal to his weight.

EXECUTE: **(a)** The graphs are given in Figure 11.12.

(b) $x = 6.25 \text{ m}$ when $F_A = 0$, which is 1.25 m beyond point B.

(c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900 \text{ N}$.

The distance that point B must be from the right end is then $\dfrac{(300 \text{ N})(4.50 \text{ m})}{(900 \text{ N})} = 1.50 \text{ m}$.

EVALUATE: When the beam is on the verge of tipping it starts to lift off the support A and the normal force F_A exerted by the support goes to zero.

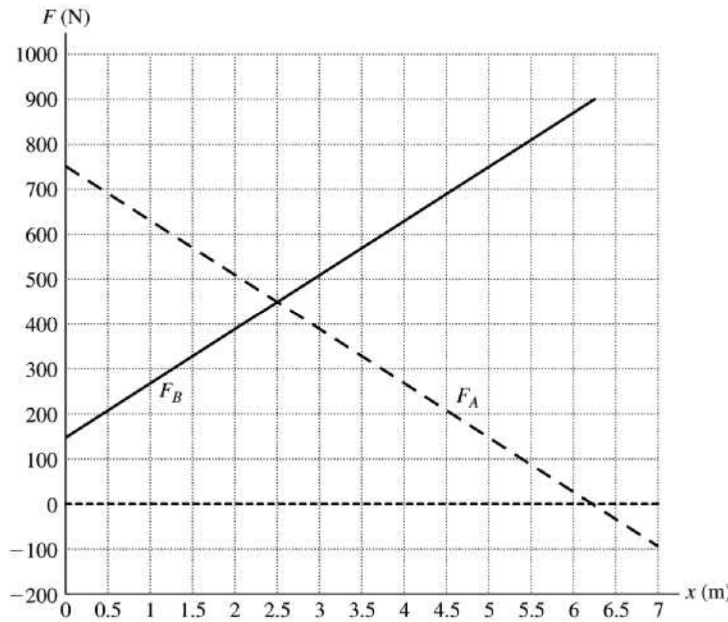


Figure 11.12

11.13. **IDENTIFY:** Apply the first and second conditions of equilibrium to the strut.
(a) SET UP: The free-body diagram for the strut is given in Figure 11.13a. Take the origin of coordinates at the hinge (point A) and $+y$ upward. Let F_h and F_v be the horizontal and vertical components of the force $\vec{F}$ exerted on the strut by the pivot. The tension in the vertical cable is the weight w of the suspended object. The weight w of the strut can be taken to act at the center of the strut. Let L be the length of the strut.

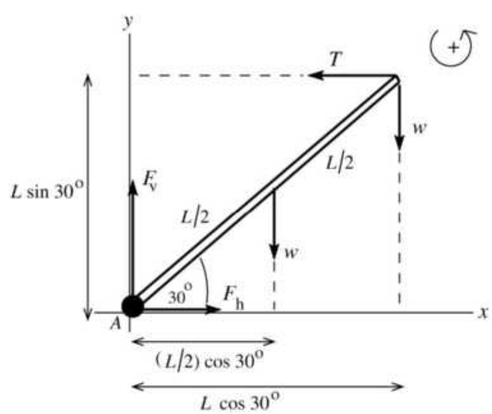


Figure 11.13a

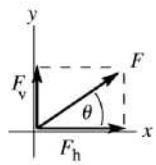
EXECUTE:
$\sum F_y = ma_y$
$F_v - w - w = 0$
$F_v = 2w$

Sum torques about point A. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.
$\tau_A = 0$
$TL\sin 30.0° - w((L/2)\cos 30.0°) - w(L\cos 30.0°) = 0$
$T\sin 30.0° - (3w/2)\cos 30.0° = 0$

$$T = \frac{3w\cos 30.0°}{2\sin 30.0°} = 2.60w$$

Then $\sum F_x = ma_x$ implies $T - F_\text{h} = 0$ and $F_\text{h} = 2.60w.$

We now have the components of $\vec{F}$ so can find its magnitude and direction (Figure 11.13b).



$$F = \sqrt{F_\text{h}^2 + F_\text{v}^2}$$

$$F = \sqrt{(2.60w)^2 + (2.00w)^2}$$

$$F = 3.28w$$

$$\tan\theta = \frac{F_\text{v}}{F_\text{h}} = \frac{2.00w}{2.60w}$$

$$\theta = 37.6°$$

Figure 11.13b

(b) SET UP: The free-body diagram for the strut is given in Figure 11.13c.

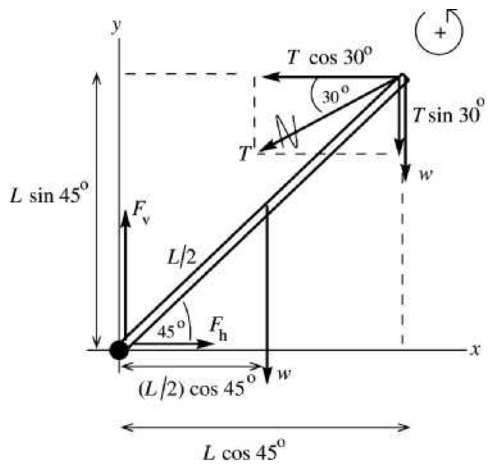


Figure 11.13c

The tension T has been replaced by its x and y components. The torque due to T equals the sum of the torques of its components, and the latter are easier to calculate.

EXECUTE: $\sum \tau_A = 0 + (T\cos 30.0°)(L\sin 45.0°) - (T\sin 30.0°)(L\cos 45.0°) -$
$w((L/2)\cos 45.0°) - w(L\cos 45.0°) = 0$

The length L divides out of the equation. The equation can also be simplified by noting that $\sin 45.0° = \cos 45.0°.$

Then $T(\cos 30.0° - \sin 30.0°) = 3w/2.$

$$T = \frac{3w}{2(\cos 30.0° - \sin 30.0°)} = 4.10w$$

$$\sum F_x = ma_x$$

$$F_\text{h} - T\cos 30.0° = 0$$

$$F_\text{h} = T\cos 30.0° = (4.10w)(\cos 30.0°) = 3.55w$$

$$\sum F_y = ma_y$$

$$F_\text{v} - w - w - T\sin 30.0° = 0$$

$$F_\text{v} = 2w + (4.10w)\sin 30.0° = 4.05w$$

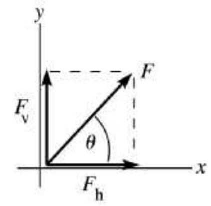


Figure 11.13d

From Figure 11.13d,

$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$$

$$\tan\theta = \frac{F_v}{F_h} = \frac{4.05w}{3.55w}$$

$$\theta = 48.8°$$

EVALUATE: In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

11.14. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.14. H_v and H_h are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle θ has $\cos\theta = 0.800$ and $\sin\theta = 0.600$. The tension T has been replaced by its x and y components.

EXECUTE: **(a)** H_v, H_h and $T_x = T\cos\theta$ all produce zero torque. $\sum\tau_z = 0$ gives

$$-w(2.00\text{ m}) - w_{load}(4.00\text{ m}) + T\sin\theta(4.00\text{ m}) = 0 \text{ and } T = \frac{(150\text{ N})(2.00\text{ m}) + (300\text{ N})(4.00\text{ m})}{(4.00\text{ m})(0.600)} = 625\text{ N.}$$

(b) $\sum F_x = 0$ gives $H_h - T\cos\theta = 0$ and $H_h = (625\text{ N})(0.800) = 500\text{ N}$. $\sum F_y = 0$ gives

$H_v - w - w_{load} + T\sin\theta = 0$ and $H_v = w + w_{load} - T\sin\theta = 150\text{ N} + 300\text{ N} - (625\text{ N})(0.600) = 75\text{ N}$.

EVALUATE: For an axis at the right-hand end of the beam, only w and H_v produce torque. The torque due to w is counterclockwise so the torque due to H_v must be clockwise. To produce a clockwise torque, H_v must be upward, in agreement with our result from $\sum F_y = 0$.

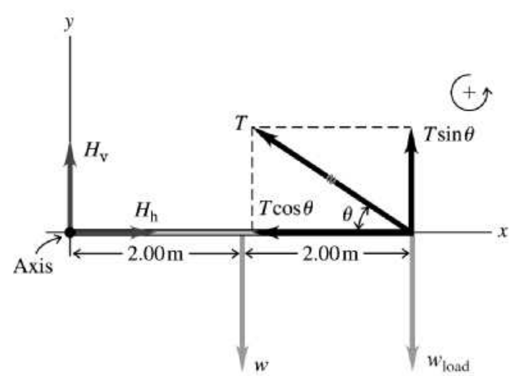


Figure 11.14

11.15. **IDENTIFY:** The athlete is in equilibrium, so the forces and torques on him must balance. The target variables are the forces on his hands and feet due to the floor.

SET UP: The free-body diagram is given in Figure 11.15. F_f is the force on each foot and F_h is the force on each hand. Use coordinates as shown. Take the pivot at his feet and let counterclockwise torques be positive. $\sum\tau_z = 0$ and $\sum F_y = 0$.

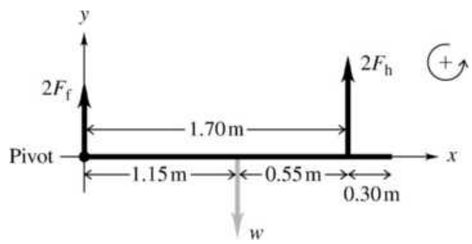


Figure 11.15

EXECUTE: $\sum\tau_z = 0$ gives $(2F_h)(1.70\text{ m}) - w(1.15\text{ m}) = 0$. Solving for F_h gives

$F_h = w\dfrac{1.15\text{ m}}{2(1.70\text{ m})} = 0.338w = 272\text{ N}$. Applying $\sum F_y = 0$, we get $2F_f + 2F_h - w = 0$ which gives

$F_f = \frac{1}{2}w - F_h = 402\text{ N} - 272\text{ N} = 130\text{ N}$.

EVALUATE: His center of mass is closer to his hands than to his feet, so his hands exert a greater force.

11.16. IDENTIFY: Apply the conditions of equilibrium to the wheelbarrow plus its contents. The upward force applied by the person is 650 N.

SET UP: The free-body diagram for the wheelbarrow is given in Figure 11.16. $F = 650\text{ N}$, $w_{wb} = 80.0\text{ N}$ and w is the weight of the load placed in the wheelbarrow.

EXECUTE: **(a)** $\sum\tau_z = 0$ with the axis at the center of gravity gives $n(0.50\text{ m}) - F(0.90\text{ m}) = 0$ and

$n = F\left(\dfrac{0.90\text{ m}}{0.50\text{ m}}\right) = 1170\text{ N}$. $\sum F_y = 0$ gives $F + n - w_{wb} - w = 0$ and

$w = F + n - w_{wb} = 650\text{ N} + 1170\text{ N} - 80.0\text{ N} = 1740\text{ N}$.

(b) The extra force is applied by the ground pushing up on the wheel.

EVALUATE: You can verify that $\sum\tau_z = 0$ for any axis, for example for an axis where the wheel contacts the ground.

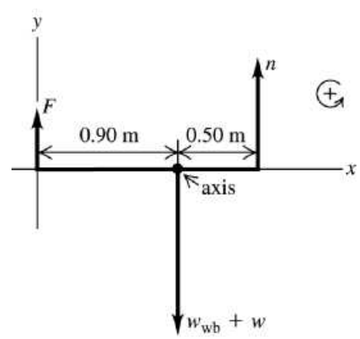
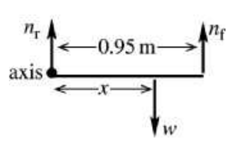


Figure 11.6

11.17. IDENTIFY: Apply the first and second conditions of equilibrium to Clea.

SET UP: Consider the forces on Clea. The free-body diagram is given in Figure 11.17



EXECUTE:

$n_r = 89\text{ N}$, $n_f = 157\text{ N}$

$n_r + n_f = w$ so $w = 246\text{ N}$

Figure 11.17

$\sum\tau_z = 0,$ axis at rear feet

Let x be the distance from the rear feet to the center of gravity.

$n_f(0.95\text{ m}) - xw = 0$

$x = 0.606\text{ m}$ from rear feet so 0.34 m from front feet.

EVALUATE: The normal force at her front feet is greater than at her rear feet, so her center of gravity is closer to her front feet.

11.18. **IDENTIFY:** Apply the conditions for equilibrium to the crane.

SET UP: The free-body diagram for the crane is sketched in Figure 11.18. F_h and F_v are the components of the force exerted by the axle. $\vec{T}$ pulls to the left so F_h is to the right. $\vec{T}$ also pulls downward and the two weights are downward, so F_v is upward.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T([13\text{ m}]\sin 25°) - w_c([7.0\text{ m}]\cos 55°) - w_b([16.0\text{ m}]\cos 55°) = 0.$

$$T = \frac{(11{,}000\text{ N})([16.0\text{ m}]\cos 55°) + (15{,}000\text{ N})([7.0\text{ m}]\cos 55°)}{(13.0\text{ m})\sin 25°} = 2.93\times 10^4\text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T\cos 30° = 0$ and $F_h = 2.54\times 10^4$ N.

$\sum F_y = 0$ gives $F_v - T\sin 30° - w_c - w_b = 0$ and $F_v = 4.06\times 10^4$ N.

EVALUATE: $\tan\theta = \dfrac{F_v}{F_h} = \dfrac{4.06\times 10^4\text{ N}}{2.54\times 10^4\text{ N}}$ and $\theta = 58°.$ The force exerted by the axle is not directed along the crane.

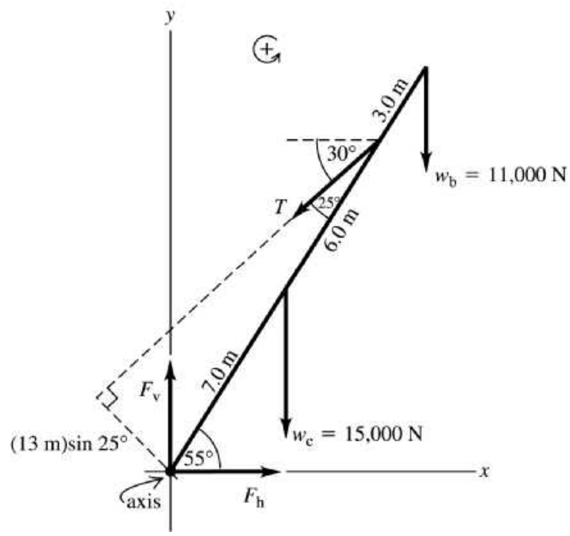


Figure 11.18

11.19. **IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.

SET UP: The force diagram for the rod is given in Figure 11.19.

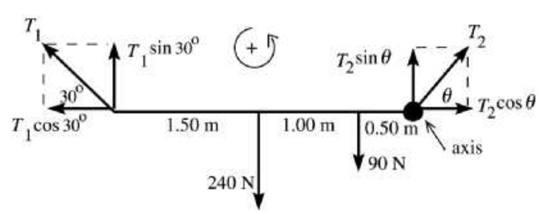


Figure 11.19

EXECUTE: $\sum\tau_z = 0,$ axis at right end of rod, counterclockwise torque is positive

$(240\text{ N})(1.50\text{ m}) + (90\text{ N})(0.50\text{ m}) - (T_1\sin 30.0°)(3.00\text{ m}) = 0$

$T_1 = \dfrac{360\text{ N}\cdot\text{m} + 45\text{ N}\cdot\text{m}}{1.50\text{ m}} = 270\text{ N}$

$\sum F_x = ma_x$

$T_2\cos\theta - T_1\cos 30° = 0$ and $T_2\cos\theta = 234\text{ N}$

$\sum F_y = ma_y$

$T_1\sin 30° + T_2\sin\theta - 240\text{ N} - 90\text{ N} = 0$

$T_2\sin\theta = 330\text{ N} - (270\text{ N})\sin 30° = 195\text{ N}$

Then $\dfrac{T_2\sin\theta}{T_2\cos\theta} = \dfrac{195\text{ N}}{234\text{ N}}$ gives $\tan\theta = 0.8333$ and $\theta = 40°$

And $T_2 = \dfrac{195\text{ N}}{\sin 40°} = 303\text{ N}.$

EVALUATE: The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1,$ the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

11.20. **IDENTIFY:** Apply the first and second conditions for equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.20.

EXECUTE: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point; $T(3.00\text{ m}) = (1.00\text{ kN})(2.00\text{ m})\cos 25.0° + (5.00\text{ kN})(4.50\text{ m})\cos 25.0°,$ and $T = 7.40\text{ kN}.$ The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of $T,$ $6.00\text{ kN} - T\cos 25.0° = -0.71\text{ kN}.$ The vertical component is downward. The horizontal force is $T\sin 25.0° = 3.13\text{ kN}.$

EVALUATE: The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

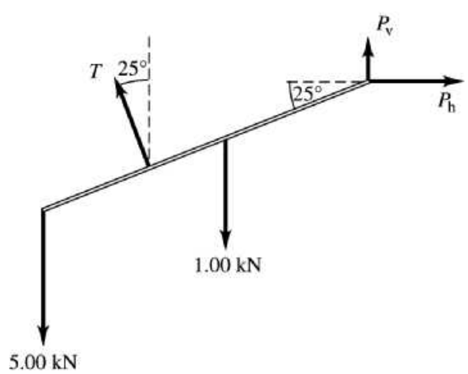


Figure 11.20

11.21. **(a) IDENTIFY** and **SET UP:** Use Eq. (10.3) to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.21a.

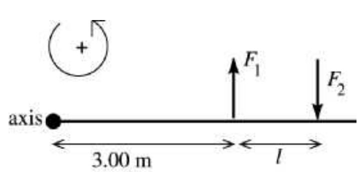


Figure 11.21a

EXECUTE:

$\tau_1 = F_1 l_1 = +(8.00\text{ N})(3.00\text{ m})$

$\tau_1 = +24.0\text{ N}\cdot\text{m}$

$\tau_2 = -F_2 l_2 = -(8.00\text{ N})(l + 3.00\text{ m})$

$\tau_2 = -24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l$

$\sum\tau_z = \tau_1 + \tau_2 = +24.0\ \text{N}\cdot\text{m} - 24.0\ \text{N}\cdot\text{m} - (8.00\ \text{N})l = -(8.00\ \text{N})l$

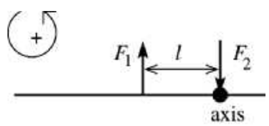
Want l that makes $\sum\tau_z = -6.40\ \text{N}\cdot\text{m}$ (net torque must be clockwise)

$-(8.00\ \text{N})l = -6.40\ \text{N}\cdot\text{m}$

$l = (6.40\ \text{N}\cdot\text{m})/8.00\ \text{N} = 0.800\ \text{m}$

(b) $|\tau_2| > |\tau_1|$ since F_2 has a larger moment arm; the net torque is clockwise.

(c) See Figure 11.21b.



$\tau_1 = -F_1 l_1 = -(8.00\ \text{N})l$

$\tau_2 = 0$ since $\vec{F}_2$ is at the axis

Figure 11.21b

$\sum\tau_z = -6.40\ \text{N}\cdot\text{m}$ gives $-(8.00\ \text{N})l = -6.40\ \text{N}\cdot\text{m}$

$l = 0.800\ \text{m}$, same as in part (a).

EVALUATE: The force couple gives the same magnitude of torque for the pivot at any point.

11.22. **IDENTIFY:** The person is in equilibrium, so the torques on him must balance. The target variable is the force exerted by the deltoid muscle.

SET UP: The free-body diagram for the arm is given in Figure 11.22. Take the pivot at the shoulder joint and let counterclockwise torques be positive. Use coordinates as shown. Let F be the force exerted by the deltoid muscle. There are also the weight of the arm and forces at the shoulder joint, but none of these forces produce any torque when the arm is in this position. The forces F and T have been replaced by their x and y components. $\sum\tau_z = 0$.

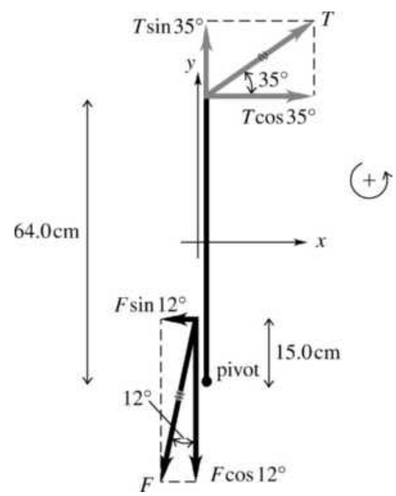


Figure 11.22

EXECUTE: $\sum\tau_z = 0$ gives $(F\sin 12.0°)(15.0\ \text{cm}) - (T\cos 35°)(64.0\ \text{cm}) = 0$.

$$F = \frac{(36.0\ \text{N})(\cos 35°)(64.0\ \text{cm})}{(\sin 12.0°)(15.0\ \text{cm})} = 605\ \text{N}.$$

EVALUATE: The force exerted by the deltoid muscle is much larger than the tension in the cable because the deltoid muscle makes a small angle (only 12.0°) with the humerus.

11.23. **IDENTIFY:** The student's head is at rest, so the torques on it must balance. The target variable is the tension in her neck muscles.

SET UP: Let the pivot be at point P and let counterclockwise torques be positive. $\sum\tau_z = 0$.

EXECUTE: **(a)** The free-body diagram is given in Figure 11.23.

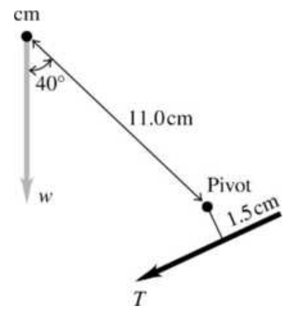


Figure 11.23

(b) $\sum \tau_z = 0$ gives $w(11.0\text{ cm})(\sin 40.0°) - T(1.50\text{ cm}) = 0.$

$$T = \frac{(4.50\text{ kg})(9.80\text{ m/s}^2)(11.0\text{ cm})\sin 40.0°}{1.50\text{ cm}} = 208\text{ N}.$$

EVALUATE: Her head weighs about 45 N but the tension in her neck muscles must be much larger because the tension has a small moment arm.

11.24. **IDENTIFY:** $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = 50.0\text{ cm}^2 = 50.0\times10^{-4}\text{ m}^2.$

EXECUTE: relaxed: $Y = \dfrac{(0.200\text{ m})(25.0\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 3.33\times10^4\text{ Pa}$

maximum tension: $Y = \dfrac{(0.200\text{ m})(500\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 6.67\times10^5\text{ Pa}$

EVALUATE: The muscle tissue is much more difficult to stretch when it is under maximum tension.

11.25. **IDENTIFY** and **SET UP:** Apply Eq. (11.10) and solve for A and then use $A = \pi r^2$ to get the radius and $d = 2r$ to calculate the diameter.

EXECUTE: $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ so $A = \dfrac{l_0 F_\perp}{Y\Delta l}$ (A is the cross-section area of the wire)

For steel, $Y = 2.0\times10^{11}\text{ Pa}$ (Table 11.1)

Thus $A = \dfrac{(2.00\text{ m})(400\text{ N})}{(2.0\times10^{11}\text{ Pa})(0.25\times10^{-2}\text{ m})} = 1.6\times10^{-6}\text{ m}^2.$

$A = \pi r^2$, so $r = \sqrt{A/\pi} = \sqrt{1.6\times10^{-6}\text{ m}^2/\pi} = 7.1\times10^{-4}\text{ m}$

$d = 2r = 1.4\times10^{-3}\text{ m} = 1.4\text{ mm}$

EVALUATE: Steel wire of this diameter doesn't stretch much; $\Delta l/l_0 = 0.12\%$.

11.26. **IDENTIFY:** Apply Eq. (11.10).

SET UP: From Table 11.1, for steel, $Y = 2.0\times10^{11}\text{ Pa}$ and for copper, $Y = 1.1\times10^{11}\text{ Pa}$.
$A = \pi(d^2/4) = 1.77\times10^{-4}\text{ m}^2$. $F_\perp = 4000\text{ N}$ for each rod.

EXECUTE: **(a)** The strain is $\dfrac{\Delta l}{l_0} = \dfrac{F}{YA}$. For steel $\dfrac{\Delta l}{l_0} = \dfrac{(4000\text{ N})}{(2.0\times10^{11}\text{ Pa})(1.77\times10^{-4}\text{ m}^2)} = 1.1\times10^{-4}.$

Similarly, the strain for copper is 2.1×10^{-4}.

(b) Steel: $(1.1\times10^{-4})(0.750\text{ m}) = 8.3\times10^{-5}\text{ m}$. Copper: $(2.1\times10^{-4})(0.750\text{ m}) = 1.6\times10^{-4}\text{ m}$.

EVALUATE: Copper has a smaller Y and therefore a greater elongation.

11.27. **IDENTIFY:** $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = 0.50\text{ cm}^2 = 0.50\times10^{-4}\text{ m}^2$

EXECUTE: $Y = \dfrac{(4.00\text{ m})(5000\text{ N})}{(0.50\times10^{-4}\text{ m}^2)(0.20\times10^{-2}\text{m})} = 2.0\times10^{11}\text{ Pa}$

EVALUATE: Our result is the same as that given for steel in Table 11.1.

11.28. **IDENTIFY:** $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = \pi r^2 = \pi(3.5\times10^{-3}\text{ m})^2 = 3.85\times10^{-5}\text{ m}^2$. The force applied to the end of the rope is the weight of the climber: $F_\perp = (65.0\text{ kg})(9.80\text{ m/s}^2) = 637\text{ N}$.

EXECUTE: $Y = \dfrac{(45.0\text{ m})(637\text{ N})}{(3.85\times10^{-5}\text{ m}^2)(1.10\text{ m})} = 6.77\times10^8\text{ Pa}$

EVALUATE: Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

11.29. **IDENTIFY:** Use the first condition of equilibrium to calculate the tensions T_1 and T_2 in the wires (Figure 11.29a). Then use Eq. (11.10) to calculate the strain and elongation of each wire.

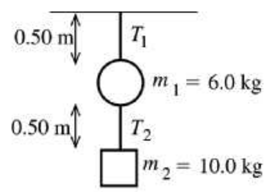
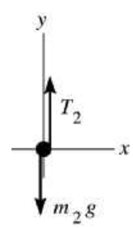


Figure 11.29a

SET UP: The free-body diagram for m_2 is given in Figure 11.27b.



EXECUTE:

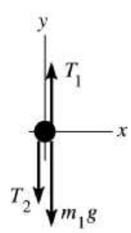
$\sum F_y = ma_y$

$T_2 - m_2 g = 0$

$T_2 = 98.0\text{ N}$

Figure 11.29b

SET UP: The free-body-diagram for m_1 is given in Figure 11.29c.



EXECUTE:

$\sum F_y = ma_y$

$T_1 - T_2 - m_1 g = 0$

$T_1 = T_2 + m_1 g$

$T_1 = 98.0\text{ N} + 58.8\text{ N} = 157\text{ N}$

Figure 11.29c

(a) $Y = \dfrac{\text{stress}}{\text{strain}}$ so strain $= \dfrac{\text{stress}}{Y} = \dfrac{F_\perp}{AY}$

upper wire: strain $= \dfrac{T_1}{AY} = \dfrac{157\text{ N}}{(2.5\times10^{-7}\text{ m}^2)(2.0\times10^{11}\text{ Pa})} = 3.1\times10^{-3}$

lower wire: strain $= \dfrac{T_2}{AY} = \dfrac{98\text{ N}}{(2.5\times10^{-7}\text{ m}^2)(2.0\times10^{11}\text{ Pa})} = 2.0\times10^{-3}$

(b) strain $= \Delta l/l_0$ so $\Delta l = l_0(\text{strain})$

upper wire: $\Delta l = (0.50 \text{ m})(3.1\times 10^{-3}) = 1.6\times 10^{-3} \text{ m} = 1.6 \text{ mm}$

lower wire: $\Delta l = (0.50 \text{ m})(2.0\times 10^{-3}) = 1.0\times 10^{-3} \text{ m} = 1.0 \text{ mm}$

EVALUATE: The tension is greater in the upper wire because it must support both objects. The wires have the same length and diameter, so the one with the greater tension has the greater strain and elongation.

11.30. **IDENTIFY:** Apply Eqs. (11.8), (11.9) and (11.10).

SET UP: The cross-sectional area of the post is $A = \pi r^2 = \pi(0.125 \text{ m})^2 = 0.0491 \text{ m}^2$. The force applied to the end of the post is $F_\perp = (8000 \text{ kg})(9.80 \text{ m/s}^2) = 7.84\times 10^4 \text{ N}$. The Young's modulus of steel is $Y = 2.0\times 10^{11} \text{ Pa}$.

EXECUTE: **(a)** stress $= \dfrac{F_\perp}{A} = -\dfrac{7.84\times 10^4 \text{ N}}{0.0491 \text{ m}^2} = -1.60\times 10^6 \text{ Pa}$. The minus sign indicates that the stress is compressive.

(b) strain $= \dfrac{\text{stress}}{Y} = -\dfrac{1.60\times 10^6 \text{ Pa}}{2.0\times 10^{11} \text{ Pa}} = -8.0\times 10^{-6}$. The minus sign indicates that the length decreases.

(c) $\Delta l = l_0(\text{strain}) = (2.50 \text{ m})(-8.0\times 10^{-6}) = -2.0\times 10^{-5} \text{ m}$

EVALUATE: The fractional change in length of the post is very small.

11.31. **IDENTIFY:** The amount of compression depends on the bulk modulus of the bone.

SET UP: $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $1 \text{ atm} = 1.01\times 10^5 \text{ Pa}$.

EXECUTE: **(a)** $\Delta p = -B\dfrac{\Delta V}{V_0} = -(15\times 10^9 \text{ Pa})(-0.0010) = 1.5\times 10^7 \text{ Pa} = 150 \text{ atm}$.

(b) The depth for a pressure increase of 1.5×10^7 Pa is 1.5 km.

EVALUATE: An extremely large pressure increase is needed for just a 0.10% bone compression, so pressure changes do not appreciably affect the bones. Unprotected dives do not approach a depth of 1.5 km, so bone compression is not a concern for divers.

11.32. **IDENTIFY:** Apply Eq. (11.13).

SET UP: $\Delta V = -\dfrac{V_0 \Delta p}{B}$. Δp is positive when the pressure increases.

EXECUTE: **(a)** The volume would increase slightly.
(b) The volume change would be twice as great.
(c) The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

EVALUATE: For lead, $B = 4.1\times 10^{10} \text{ Pa}$, so $\Delta p/B$ is very small and the fractional change in volume is very small.

11.33. **IDENTIFY:** Vigorous downhill hiking produces a shear force on the knee cartilage which could deform the cartilage. The target variable is the angle of deformation of the cartilage.

SET UP: $S = \dfrac{F_{\|}}{A\phi}$, where $\phi = x/h$. $F_{\|} = F\sin 12°$. ϕ is in radians. $F = 8mg$, with $m = 10$ kg. 1 rad $= 180°$.

EXECUTE: $\phi = \dfrac{F_{\|}}{AS} = \dfrac{8mg\sin 12°}{(10\times 10^{-4} \text{ m}^2)(12\times 10^6 \text{ Pa})} = 0.1494 \text{ rad} = 8.6°$.

EVALUATE: The shear modulus of cartilage is much less than the values for metals given in Table 11.1 in the text.

11.34. **IDENTIFY:** Apply Eq. (11.13). Density $= m/V$.

SET UP: At the surface the pressure is 1.0×10^5 Pa, so $\Delta p = 1.16\times 10^8$ Pa. $V_0 = 1.00 \text{ m}^3$. At the surface 1.00 m^3 of water has mass 1.03×10^3 kg.

EXECUTE: **(a)** $B = -\dfrac{(\Delta p)V_0}{\Delta V}$ gives $\Delta V = -\dfrac{(\Delta p)V_0}{B} = -\dfrac{(1.16\times 10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times 10^9\ \text{Pa}} = -0.0527\ \text{m}^3$

(b) At this depth 1.03×10^3 kg of seawater has volume $V_0 + \Delta V = 0.9473\ \text{m}^3$. The density is $\dfrac{1.03\times 10^3\ \text{kg}}{0.9473\ \text{m}^3} = 1.09\times 10^3\ \text{kg/m}^3$.

EVALUATE: The density is increased because the volume is compressed due to the increased pressure.

11.35. **IDENTIFY** and **SET UP:** Use Eqs. (11.13) and (11.14) to calculate B and k.

EXECUTE: $B = -\dfrac{\Delta p}{\Delta V/V_0} = -\dfrac{(3.6\times 10^6\ \text{Pa})(600\ \text{cm}^3)}{(-0.45\ \text{cm}^3)} = +4.8\times 10^9\ \text{Pa}$

$k = 1/B = 1/4.8\times 10^9\ \text{Pa} = 2.1\times 10^{-10}\ \text{Pa}^{-1}$

EVALUATE: k is the same as for glycerine (Table 11.2).

11.36. **IDENTIFY:** Apply Eq. (11.17).

SET UP: $F_{\parallel} = 9.0\times 10^5$ N. $A = (0.100\ \text{m})(0.500\times 10^{-2}\ \text{m})$. $h = 0.100$ m. From Table 11.1, $S = 7.5\times 10^{10}$ Pa for steel.

EXECUTE: **(a)** Shear strain $= \dfrac{F_{\parallel}}{AS} = \dfrac{(9\times 10^5\ \text{N})}{[(0.100\ \text{m})(0.500\times 10^{-2}\text{m})][7.5\times 10^{10}\ \text{Pa}]} = 2.4\times 10^{-2}$.

(b) Using Eq. (11.16), $x = (\text{Shear strain})\cdot h = (0.024)(0.100\ \text{m}) = 2.4\times 10^{-3}\text{m}$.

EVALUATE: This very large force produces a small displacement; $x/h = 2.4\%$.

11.37. **IDENTIFY:** The forces on the cube must balance. The deformation x is related to the force by $S = \dfrac{F_{\parallel}}{A}\dfrac{h}{x}$. $F_{\parallel} = F$ since F is applied parallel to the upper face.

SET UP: $A = (0.0600\ \text{m})^2$ and $h = 0.0600$ m. Table 11.1 gives $S = 4.4\times 10^{10}$ Pa for copper and 0.6×10^{10} Pa for lead.

EXECUTE: **(a)** Since the horizontal forces balance, the glue exerts a force F in the opposite direction.

(b) $F = \dfrac{AxS}{h} = \dfrac{(0.0600\ \text{m})^2(0.250\times 10^{-3}\ \text{m})(4.4\times 10^{10}\ \text{Pa})}{0.0600\ \text{m}} = 6.6\times 10^5\ \text{N}$

(c) $x = \dfrac{Fh}{AS} = \dfrac{(6.6\times 10^5\ \text{N})(0.0600\ \text{m})}{(0.0600\ \text{m})^2(0.6\times 10^{10}\ \text{Pa})} = 1.8\ \text{mm}$

EVALUATE: Lead has a smaller S than copper, so the lead cube has a greater deformation than the copper cube.

11.38. **IDENTIFY:** The force components parallel to the face of the cube produce a shear which can deform the cube.

SET UP: $S = \dfrac{F_{\parallel}}{A\phi}$, where $\phi = x/h$. $F_{\parallel}$ is the component of the force tangent to the surface, so $F_{\parallel} = (1375\ \text{N})\cos 8.50° = 1360$ N. ϕ must be in radians, $\phi = 1.24° = 0.0216$ rad.

EXECUTE: $S = \dfrac{1360\ \text{N}}{(0.0925\ \text{m})^2(0.0216\ \text{rad})} = 7.36\times 10^6\ \text{Pa}$.

EVALUATE: The shear modulus of this material is much less than the values for metals given in Table 11.1 in the text.

11.39. **IDENTIFY** and **SET UP:** Use Eq. (11.8).

EXECUTE: Tensile stress $= \dfrac{F_{\perp}}{A} = \dfrac{F_{\perp}}{\pi r^2} = \dfrac{90.8\ \text{N}}{\pi(0.92\times 10^{-3}\ \text{m})^2} = 3.41\times 10^7\ \text{Pa}$

EVALUATE: A modest force produces a very large stress because the cross-sectional area is small.

11.40. **IDENTIFY:** The proportional limit and breaking stress are values of the stress, $F_{\perp}/A$. Use Eq. (11.10) to calculate Δl.

SET UP: For steel, $Y = 20\times 10^{10}$ Pa. $F_{\perp} = w$.

EXECUTE: **(a)** $w = (1.6\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2) = 1.60\times10^3\ \text{N}.$

(b) $\Delta l = \left(\dfrac{F_\perp}{A}\right)\dfrac{l_0}{Y} = (1.6\times10^{-3})(4.0\ \text{m}) = 6.4\ \text{mm}$

(c) $(6.5\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2) = 6.5\times10^3\ \text{N}.$

EVALUATE: At the proportional limit, the fractional change in the length of the wire is 0.16%.

11.41. **IDENTIFY:** The elastic limit is a value of the stress, $F_\perp/A$. Apply $\sum\vec{F} = m\vec{a}$ to the elevator in order to find the tension in the cable.

SET UP: $\dfrac{F_\perp}{A} = \frac{1}{3}(2.40\times10^8\ \text{Pa}) = 0.80\times10^8\ \text{Pa}.$ The free-body diagram for the elevator is given in Figure 11.41. $F_\perp$ is the tension in the cable.

EXECUTE: $F_\perp = A(0.80\times10^8\ \text{Pa}) = (3.00\times10^{-4}\ \text{m}^2)(0.80\times10^8\ \text{Pa}) = 2.40\times10^4\ \text{N}.$ $\sum F_y = ma_y$ applied to the elevator gives $F_\perp - mg = ma$ and $a = \dfrac{F_\perp}{m} - g = \dfrac{2.40\times10^4\ \text{N}}{1200\ \text{kg}} - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2$

EVALUATE: The tension in the cable is about twice the weight of the elevator.

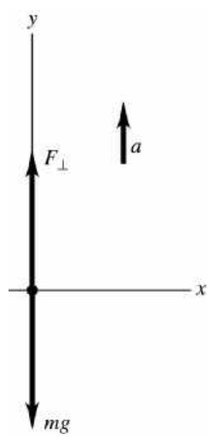


Figure 11.41

11.42. **IDENTIFY:** The breaking stress of the wire is the value of $F_\perp/A$ at which the wire breaks.

SET UP: From Table 11.3, the breaking stress of brass is 4.7×10^8 Pa. The area A of the wire is related to its diameter by $A = \pi d^2/4$.

EXECUTE: $A = \dfrac{350\ \text{N}}{4.7\times10^8\ \text{Pa}} = 7.45\times10^{-7}\ \text{m}^2$, so $d = \sqrt{4A/\pi} = 0.97$ mm.

EVALUATE: The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

11.43. **IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use Eq. (11.3) to calculate x_{cm}.

SET UP: The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and $+x$ to the right. Let $m_1 = 25.0$ kg, so $x_1 = 0.500$ m. Let $m_2 = m_{\text{sand}}$, so $x_2 = -0.625$ m. $x_{\text{cm}} = 0$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$ and $m_2 = -m_1\dfrac{x_1}{x_2} = -(25.0\ \text{kg})\left(\dfrac{0.500\ \text{m}}{-0.625\ \text{m}}\right) = 20.0$ kg.

EVALUATE: The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

11.44. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

SET UP: The free-body diagram for the door is given in Figure 11.44. Let $\vec{H}_1$ and $\vec{H}_2$ be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point *A*) and $+y$ upward.

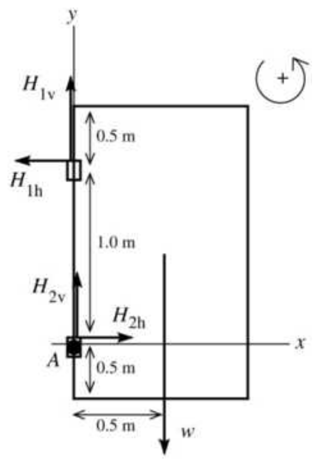


Figure 11.44

EXECUTE:
We are given that
$H_{1v} = H_{2v} = w/2 = 140$ N.
$\sum F_x = ma_x$
$H_{2h} - H_{1h} = 0$
$H_{1h} = H_{2h}$
The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Sum torques about point *A*. H_{1v}, H_{2v} and H_{2h} all have zero moment arm and hence zero torque about an axis at this point. Thus $\sum \tau_A = 0$ gives $H_{1h}(1.00\text{ m}) - w(0.50\text{ m}) = 0$

$$H_{1h} = w\left(\frac{0.50\text{ m}}{1.00\text{ m}}\right) = \tfrac{1}{2}(280\text{ N}) = 140\text{ N}.$$

The horizontal component of each hinge force is 140 N.

EVALUATE: The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at *A*, the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

11.45. **IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value $f_s = \mu_s n$.

SET UP: The free-body diagram for the climber is given in Figure 11.45. f_s and *n* are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force *T* is $(1.4\text{ m})\cos 10°$.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $T(1.4\text{ m})\cos 10° - w(1.1\text{ m})\cos 35.0° = 0.$

$$T = \frac{(1.1\text{ m})\cos 35.0°}{(1.4\text{ m})\cos 10°}(82.0\text{ kg})(9.80\text{ m/s}^2) = 525\text{ N}$$

(b) $\sum F_x = 0$ gives $n = T\sin 25.0° = 222$ N. $\sum F_y = 0$ gives $f_s + T\cos 25° - w = 0$ and $f_s = (82.0\text{ kg})(9.80\text{ m/s}^2) - (525\text{ N})\cos 25° = 328$ N.

(c) $\mu_s = \dfrac{f_s}{n} = \dfrac{328\text{ N}}{222\text{ N}} = 1.48$

EVALUATE: To achieve this large value of μ_s the climber must wear special rough-soled shoes.

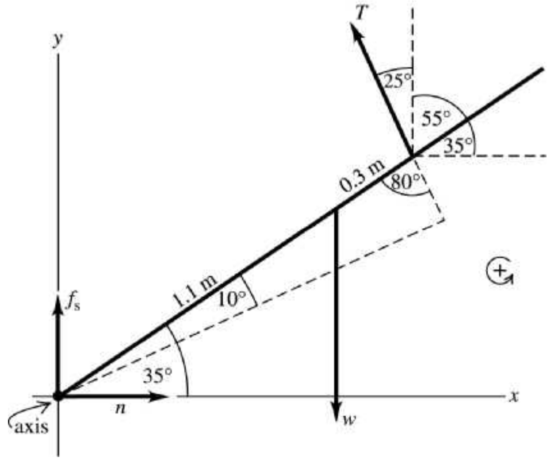


Figure 11.45

11.46. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the bridge.

SET UP: Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.
EXECUTE: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0 \text{ m}) = (600 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) + (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m})$, so $T = 6860$ N.

This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance *x* Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by *x* and the tension T by $T_{\text{max}} = 5.80 \times 10^3 \text{ N}$ and solve for *x*;

$$x = \frac{(5.80 \times 10^3 \text{ N})(12.0 \text{ m}) - (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m})}{(600 \text{ kg})(9.80 \text{ m/s}^2)} = 9.84 \text{ m}.$$

EVALUATE: Before Lancelot goes onto the bridge, the tension in the supporting cable is
$T = \dfrac{(6.0 \text{ m})(200 \text{ kg})(9.80 \text{ m/s}^2)}{12.0 \text{ m}} = 980$ N, well below the breaking strength of the cable. As he moves along the bridge, the increase in tension is proportional to *x*, the distance he has moved along the bridge.

11.47. **IDENTIFY:** For the airplane to remain in level flight, both $\sum F_y = 0$ and $\sum \tau_z = 0$.

SET UP: The free-body diagram for the airplane is given in Figure 11.47. Let $+y$ be upward.

EXECUTE: $-F_{\text{tail}} - W + F_{\text{wing}} = 0$. Taking the counterclockwise direction as positive, and taking torques about the point where the tail force acts, $-(3.66 \text{ m})(6700 \text{ N}) + (3.36 \text{ m})F_{\text{wing}} = 0$. This gives

$F_{\text{wing}} = 7300 \text{ N(up)}$ and $F_{\text{tail}} = 7300 \text{ N} - 6700 \text{ N} = 600 \text{ N(down)}$.

EVALUATE: We assumed that the wing force was upward and the tail force was downward. When we solved for these forces we obtained positive values for them, which confirms that they do have these directions. Note that the rear stabilizer provides a *downward* force. It does not hold up the tail of the aircraft, but serves to counter the torque produced by the wing. Thus balance, along with weight, is a crucial factor in airplane loading.

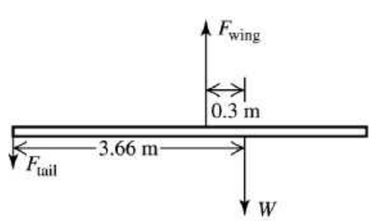


Figure 11.47

11.48. **IDENTIFY:** Apply the first and second conditions of equilibrium to the truck.

SET UP: The weight on the front wheels is n_f, the normal force exerted by the ground on the front wheels. The weight on the rear wheels is n_r, the normal force exerted by the ground on the rear wheels. When the front wheels come off the ground, $n_f \rightarrow 0$. The free-body diagram for the truck without the box is given in Figure 11.48a and with the box in Figure 11.48b. The center of gravity of the truck, without the box, is a distance x from the rear wheels.

EXECUTE: $\sum F_y = 0$ in Figure 11.48a gives $w = n_r + n_f = 8820\text{ N} + 10{,}780\text{ N} = 19{,}600\text{ N}$.

$\sum \tau_z = 0$ in Figure 11.48a, with the axis at the rear wheels and counterclockwise torques positive, gives

$n_f(3.00\text{ m}) - wx = 0$ and $x = \dfrac{n_f(3.00\text{ m})}{w} = \left(\dfrac{10{,}780\text{ N}}{19{,}600\text{ N}}\right)(3.00\text{ m}) = 1.65\text{ m}$.

(a) $\sum \tau_z = 0$ in Figure 11.48b, with the axis at the rear wheels and counterclockwise torques positive, gives $w_{box}(1.00\text{ m}) + n_f(3.00\text{ m}) - w(1.65\text{ m}) = 0$.

$$n_f = \frac{-(3600\text{ N})(1.00\text{ m}) + (19{,}600\text{ N})(1.65\text{ m})}{3.00\text{ m}} = 9580\text{ N}$$

$\sum F_y = 0$ gives $n_r + n_f = w_{box} + w$ and $n_r = 3600\text{ N} + 19{,}600\text{ N} - 9580\text{ N} = 13{,}620\text{ N}$. There is 9580 N on the front wheels and 13,620 N on the rear wheels.

(b) $n_f \rightarrow 0$. $\sum \tau_z = 0$ gives $w_{box}(1.00\text{ m}) - w(1.65\text{ m}) = 0$ and $w_{box} = 1.65w = 3.23\times10^4\text{ N}$.

EVALUATE: Placing the box on the tailgate in part (b) reduces the normal force exerted at the front wheels.

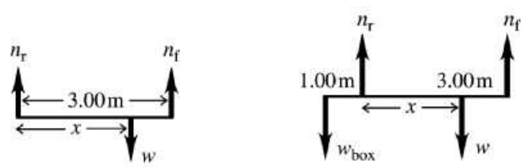


Figure 11.48a, b

11.49. **IDENTIFY:** In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use Eq. (11.3) to locate x_{cm}, with m_i replaced by w_i.

SET UP: Let the origin be at the left-hand end of the rod and take the $+x$ axis to lie along the rod. Let $w_1 = 255\text{ N}$ (the rod) so $x_1 = 1.00\text{ m}$, let $w_2 = 225\text{ N}$ so $x_2 = 2.00\text{ m}$ and let $w_3 = W$. In part (a) $x_3 = 0.500\text{ m}$ and in part (b) $x_3 = 0.750\text{ m}$.

EXECUTE: **(a)** $x_{cm} = 1.25\text{ m}$. $x_{cm} = \dfrac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$ gives $w_3 = \dfrac{(w_1 + w_2)x_{cm} - w_1x_1 - w_2x_2}{x_3 - x_{cm}}$ and

$W = \dfrac{(480\text{ N})(1.25\text{ m}) - (255\text{ N})(1.00\text{ m}) - (225\text{ N})(2.00\text{ m})}{0.500\text{ m} - 1.25\text{ m}} = 140\text{ N}$.

(b) Now $w_3 = W = 140\text{ N}$ and $x_3 = 0.750\text{ m}$.

$x_{cm} = \dfrac{(255\text{ N})(1.00\text{ m}) + (225\text{ N})(2.00\text{ m}) + (140\text{ N})(0.750\text{ m})}{255\text{ N} + 225\text{ N} + 140\text{ N}} = 1.31\text{ m}$. W must be moved $1.31\text{ m} - 1.25\text{ m} = 6\text{ cm}$ to the right.

EVALUATE: Moving W to the right means x_{cm} for the system moves to the right.

11.50. **IDENTIFY:** The beam is at rest, so the forces and torques on it must balance.

SET UP: The weight of the beam acts 4.0 m from each end. Take the pivot at the hinge and let counterclockwise torques be positive. Represent the force exerted by the hinge by its horizontal and vertical components, H_h and H_v. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum \tau_z = 0$.

EXECUTE: **(a)** The free-body diagram for the beam is given in Figure 11.50a.

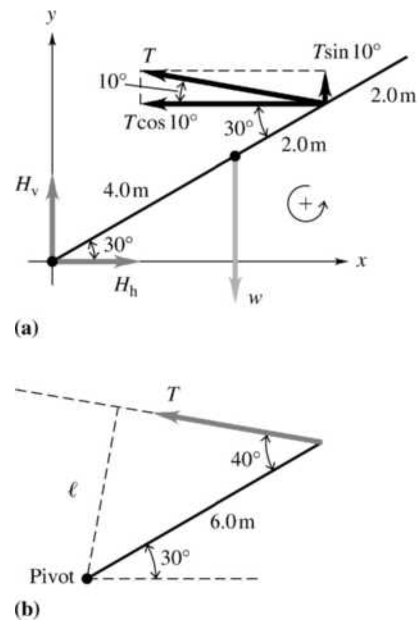


Figure 11.50

(b) The moment arm for T is sketched in Figure 11.50b and is equal to $(6.0\text{ m})\sin 40.0°$. $\sum\tau_z = 0$ gives $T(6.0\text{ m})(\sin 40.0°) - w(4.0\text{ m})(\cos 30.0°) = 0$. $T = \dfrac{(1500\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m})(\cos 30.0°)}{(6.0\text{ m})(\sin 40.0°)} = 1.32\times10^4\text{ N}$.

(c) $\sum F_x = 0$ gives $H_h - T\cos 10.0° = 0$ and $H_h = T\cos 10.0° = 1.30\times10^4\text{ N}$. $\sum F_y = 0$ gives $H_v + T\sin 10.0° - w = 0$ and $H_v = w - T\sin 10.0° = (1500\text{ kg})(9.80\text{ m/s}^2) - 2.29\times10^3\text{ N} = 1.24\times10^4\text{ N}$. $H = \sqrt{H_h^2 + H_v^2} = 1.80\times10^4\text{ N}$. This is the force the hinge exerts on the beam. By Newton's third law, the force the beam exerts on the wall has the same magnitude, so is $1.80\times10^4\text{ N}$.

EVALUATE: The tension is less than the weight of the beam because it has a larger moment arm than the weight force has.

11.51. **IDENTIFY:** Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to $T_w = mg/2 = 137\text{ N}$.

SET UP: The free-body diagram for the beam is given in Figure 11.51. F_v and F_h are the horizontal and vertical forces exerted by the hinge on the sign. Since the cable is 2.00 m long and the beam is 1.50 m long, $\cos\theta = \dfrac{1.50\text{ m}}{2.00\text{ m}}$ and $\theta = 41.4°$. The tension T_c in the cable has been replaced by its horizontal and vertical components.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T_c(\sin 41.4°)(1.50\text{ m}) - w_{\text{beam}}(0.750\text{ m}) - T_w(1.50\text{ m}) - T_w(0.60\text{ m}) = 0$.

$$T_c = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m}) + (137\text{ N})(1.50\text{ m} + 0.60\text{ m})}{(1.50\text{ m})(\sin 41.4°)} = 379\text{ N}.$$

(b) $\sum F_y = 0$ gives $F_v + T_c\sin 41.4° - w_{\text{beam}} - 2T_w = 0$ and $F_v = 2T_w + w_{\text{beam}} - T_c\sin 41.4° = 2(137\text{ N}) + (12.0\text{ kg})(9.80\text{ m/s}^2) - (379\text{ N})(\sin 41.4°) = 141\text{ N}$. The hinge must be able to supply a vertical force of 141 N.

EVALUATE: The force from the two wires could be replaced by the weight of the sign acting at a point 0.60 m to the left of the right-hand edge of the sign.

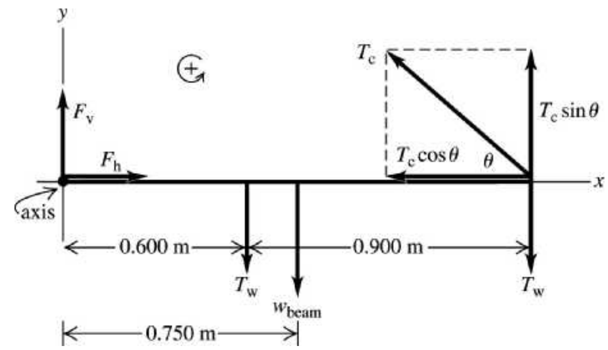


Figure 11.51

11.52. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the hammer.

SET UP: Take the axis of rotation to be at point A.

EXECUTE: The force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of $(0.080\text{ m})\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1 \frac{(0.0800\text{ m})\sin 60°}{(0.300\text{ m})} = (400\text{ N})(0.231) = 92.4\text{ N}.$$

EVALUATE: The force F_2 that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

11.53. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

SET UP: The free-body diagram for the bar is given in Figure 11.53. n is the normal force exerted on the bar by the surface. There is no friction force at this surface. H_h and H_v are the components of the force exerted on the bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

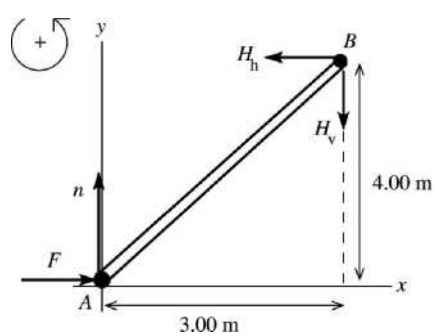


Figure 11.53

EXECUTE:

$\sum F_x = ma_x$

$F = H_h = 160\text{ N}$

$\sum F_y = ma_y$

$n - H_v = 0$

$H_v = n$, but we don't know either of these forces.

$\sum \tau_B = 0$ gives $F(4.00\text{ m}) - n(3.00\text{ m}) = 0$.

$n = (4.00\text{ m}/3.00\text{ m})F = \frac{4}{3}(160\text{ N}) = 213\text{ N}$ and then $H_v = 213\text{ N}$.

Force of bar on hinge:
horizontal component 160 N, to right
vertical component 213 N, upward

EVALUATE: $H_h/H_v = 160/213 = 0.750 = 3.00/4.00$, so the force the hinge exerts on the bar is directed along the bar. $\vec{n}$ and $\vec{F}$ have zero torque about point A, so the line of action of the hinge force $\vec{H}$ must pass through this point also if the net torque is to be zero.

11.54. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the piece of art.

SET UP: The free-body diagram for the piece of art is given in Figure 11.54.

EXECUTE: $\sum \tau_z = 0$ gives $T_B(1.25\text{ m}) - w(1.02\text{ m}) = 0.$ $T_B = (426\text{ N})\left(\dfrac{1.02\text{ m}}{1.25\text{ m}}\right) = 348\text{ N}.$

$\sum F_y = 0$ gives $T_A + T_B - w = 0$ and $T_A = w - T_B = 426\text{ N} - 348\text{ N} = 78\text{ N}.$

EVALUATE: If we consider the sum of torques about the center of gravity of the piece of art, T_A has a larger moment arm than T_B, and this is why $T_A < T_B$.

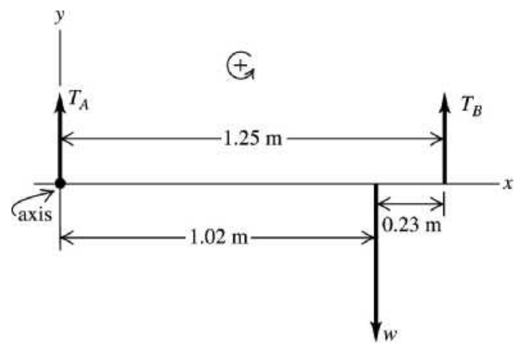


Figure 11.54

11.55. **IDENTIFY:** We want to locate the center of mass of the leg-cast system. We can treat each segment of the leg and cast as a point-mass located at its center of mass.

SET UP: The force diagram for the leg is given in Figure 11.55. The weight of each piece acts at the center of mass of that piece. The mass of the upper leg is $m_{\text{ul}} = (0.215)(37\text{ kg}) = 7.955\text{ kg}.$ The mass of the lower leg is $m_{\text{ll}} = (0.140)(37\text{ kg}) = 5.18\text{ kg}.$ Use the coordinates shown, with the origin at the hip and the x-axis along the leg, and use $x_{\text{cm}} = \dfrac{x_{\text{ul}}m_{\text{ul}} + x_{\text{ll}}m_{\text{ll}} + x_{\text{cast}}m_{\text{cast}}}{m_{\text{ul}} + m_{\text{ll}} + m_{\text{cast}}}.$

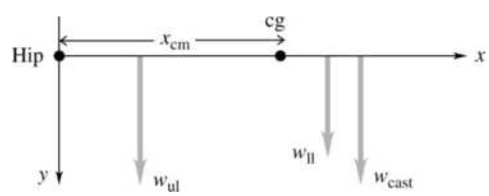


Figure 11.55

EXECUTE: Using $x_{\text{cm}} = \dfrac{x_{\text{ul}}m_{\text{ul}} + x_{\text{ll}}m_{\text{ll}} + x_{\text{cast}}m_{\text{cast}}}{m_{\text{ul}} + m_{\text{ll}} + m_{\text{cast}}}$, we have

$$x_{\text{cm}} = \frac{(18.0\text{ cm})(7.955\text{ kg}) + (69.0\text{ cm})(5.18\text{ kg}) + (78.0\text{ cm})(5.50\text{ kg})}{7.955\text{ kg} + 5.18\text{ kg} + 5.50\text{ kg}} = 49.9\text{ cm}$$

EVALUATE: The strap is attached to the left of the center of mass of the cast, but it is still supported by the rigid cast since the cast extends beyond its center of mass.

11.56. **IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.

SET UP: Find torques about the hinge. Use L as the length of the bridge and w_{T} and w_{B} for the weights of the truck and the raised section of the bridge. Take $+y$ to be upward and $+x$ to be to the right.

EXECUTE: **(a)** $TL\sin 70° = w_{\text{T}}(\tfrac{3}{4}L)\cos 30° + w_{\text{B}}(\tfrac{1}{2}L)\cos 30°,$ so

$$T = \frac{(\tfrac{3}{4}m_{\text{T}} + \tfrac{1}{2}m_{\text{B}})(9.80\text{ m/s}^2)\cos 30°}{\sin 70°} = 2.84\times 10^5\text{ N}.$$

(b) Horizontal: $T\cos(70° - 30°) = 2.18\times 10^5\text{ N}$ (to the right).

Vertical: $w_{\text{T}} + w_{\text{B}} - T\sin 40° = 2.88\times 10^5\text{ N}$ (upward).

EVALUATE: If ϕ is the angle of the hinge force above the horizontal,

$$\tan\phi = \frac{2.88\times10^5 \text{ N}}{2.18\times10^5 \text{ N}}$$ and $\phi = 52.9°$. The hinge force is not directed along the bridge.

11.57. **IDENTIFY:** The leg is not rotating, so the external torques on it must balance.
SET UP: The free-body diagram for the leg is given in Figure 11.57. Take the pivot at the hip joint and let counterclockwise torque be positive. There are also forces on the leg exerted by the hip joint but these forces produce no torque and aren't shown. $\sum\tau_z = 0$ for no rotation.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T(10 \text{ cm})(\sin\theta) - w(44 \text{ cm})(\cos\theta) = 0.$

$$T = \frac{4.4w\cos\theta}{\sin\theta} = \frac{4.4w}{\tan\theta} \text{ and for } \theta = 60°,\ T = \frac{4.4(15 \text{ kg})(9.80 \text{ m/s}^2)}{\tan 60°} = 370 \text{ N}.$$

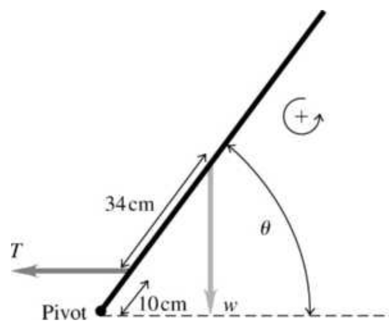


Figure 11.57

(b) For $\theta = 5°$, $T = 7400$ N. The tension is much greater when he just starts to raise his leg off the ground.
(c) $T \to \infty$ as $\theta \to 0$. The person could not raise his leg. If the leg is horizontal so θ is zero, the moment arm for *T* is zero and *T* produces no torque to rotate the leg against the torque due to its weight.
EVALUATE: Most of the exercise benefit of leg-raises occurs when the person just starts to raise his legs off the ground.

11.58. **IDENTIFY:** Apply the first and second conditions of equilibrium to the ladder.
SET UP: Take torques about the pivot. Let $+y$ be upward.

EXECUTE: **(a)** The force F_V that the ground exerts on the ladder is given to be vertical, so $\sum\tau_z = 0$ gives $F_V(6.0 \text{ m})\sin\theta = (250 \text{ N})(4.0 \text{ m})\sin\theta + (750 \text{ N})(1.50 \text{ m})\sin\theta$, so $F_V = 354$ N.

(b) There are no other horizontal forces on the ladder, so the horizontal pivot force is zero. The vertical force that the pivot exerts on the ladder must be (750 N) + (250 N) – (354 N) = 646 N, up, so the ladder exerts a downward force of 646 N on the pivot.
(c) The results in parts (a) and (b) are independent of θ.
EVALUATE: All the forces on the ladder are vertical, so all the moment arms are vertical and are proportional to $\sin\theta$. Therefore, $\sin\theta$ divides out of the torque equations and the results are independent of θ.

11.59. **IDENTIFY:** Apply the first and second conditions for equilibrium to the strut.
SET UP: Denote the length of the strut by L.
EXECUTE: **(a)** $V = mg + w$ and $H = T$. To find the tension, take torques about the pivot point.

$$T\left(\frac{2}{3}L\right)\sin\theta = w\left(\frac{2}{3}L\right)\cos\theta + mg\left(\frac{L}{6}\right)\cos\theta \text{ and } T = \left(w + \frac{mg}{4}\right)\cot\theta.$$

(b) Solving the above for w, and using the maximum tension for T,

$$w = T\ \tan\theta - \frac{mg}{4} = (700 \text{ N})\tan 55.0° - (7.50 \text{ kg})(9.80 \text{ m/s}^2) = 926 \text{ N}.$$

(c) Solving the expression obtained in part (a) for $\tan\theta$ and letting

$$w \to 0,\ \tan\theta = \frac{mg}{4T} = 0.105, \text{ so } \theta = 6.00°.$$

EVALUATE: As the strut becomes closer to the horizontal, the moment arm for the horizontal tension force approaches zero and the tension approaches infinity.

11.60. **IDENTIFY:** Apply the first and second conditions of equilibrium to each rod.

SET UP: Apply $\sum F_y = 0$ with $+y$ upward and apply $\sum \tau_z = 0$ with the pivot at the point of suspension for each rod.

EXECUTE: **(a)** The free-body diagram for each rod is given in Figure 11.60.

(b) $\sum \tau_z = 0$ for the lower rod: $(6.0\text{ N})(4.0\text{ cm}) = w_A(8.0\text{ cm})$ and $w_A = 3.0\text{ N}$.

$\sum F_y = 0$ for the lower rod: $S_3 = 6.0\text{ N} + w_A = 9.0\text{ N}$

$\sum \tau_z = 0$ for the middle rod: $w_B(3.0\text{ cm}) = (5.0\text{ cm})S_3$ and $w_B = \left(\dfrac{5.0}{3.0}\right)(9.0\text{ N}) = 15.0\text{ N}$.

$\sum F_y = 0$ for the middle rod: $S_2 = 9.0\text{ N} + S_3 = 24.0\text{ N}$

$\sum \tau_z = 0$ for the upper rod: $S_2(2.0\text{ cm}) = w_C(6.0\text{ cm})$ and $w_C = \left(\dfrac{2.0}{6.0}\right)(24.0\text{ N}) = 8.0\text{ N}$.

$\sum F_y = 0$ for the upper rod: $S_1 = S_2 + w_C = 32.0\text{ N}$.

In summary, $w_A = 3.0\text{ N}$, $w_B = 15.0\text{ N}$, $w_C = 8.0\text{ N}$. $S_1 = 32.0\text{ N}$, $S_2 = 24.0\text{ N}$, $S_3 = 9.0\text{ N}$.

(c) The center of gravity of the entire mobile must lie along a vertical line that passes through the point where S_1 is located.

EVALUATE: For the mobile as a whole the vertical forces must balance, so $S_1 = w_A + w_B + w_C + 6.0\text{ N}$.

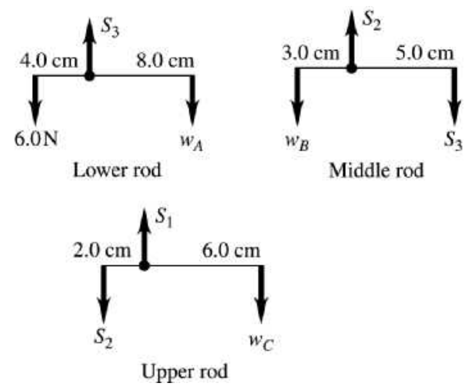


Figure 11.60

11.61. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.61.

EXECUTE: $\sum \tau_z = 0$, axis at hinge, gives $T(6.0\text{ m})(\sin 40°) - w(3.75\text{ m})(\cos 30°) = 0$ and $T = 4900\text{ N}$.

EVALUATE: The tension in the cable is less than the weight of the beam. $T\sin 40°$ is the component of T that is perpendicular to the beam.

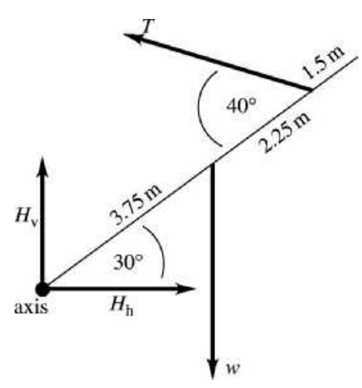


Figure 11.61

11.62. IDENTIFY: Apply the first and second conditions of equilibrium to the drawbridge.

SET UP: The free-body diagram for the drawbridge is given in Figure 11.62. H_{v} and H_{h} are the components of the force the hinge exerts on the bridge. In part (c), apply $\sum \tau_z = I\alpha$ to the rotating bridge and in part (d) apply energy conservation to the bridge.

EXECUTE: **(a)** $\sum \tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and $T = 2w\dfrac{\cos 37°}{\sin 37°} = 2\dfrac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times10^5\text{ N}.$

(b) $\sum F_x = 0$ gives $H_{\text{h}} = T = 1.19\times10^5\text{ N}$. $\sum F_y = 0$ gives $H_{\text{v}} = w = 4.50\times10^4\text{ N}$.

$H = \sqrt{H_{\text{h}}^2 + H_{\text{v}}^2} = 1.27\times10^5\text{ N}$. $\tan\theta = \dfrac{H_{\text{v}}}{H_{\text{h}}}$ and $\theta = 20.7°$. The hinge force has magnitude $1.27\times10^5\text{ N}$ and is directed at 20.7° above the horizontal.

(c) We can treat the bridge as a uniform bar rotating around one end, so $I = 1/3\, mL^2$. $\sum \tau_z = I\alpha_z$ gives $mg(L/2)\cos 37° = 1/3\, mL^2\alpha$. Solving for α gives $\alpha = \dfrac{3g\cos 37°}{2L} = \dfrac{3(9.80\text{ m/s}^2)\cos 37°}{2(14.0\text{ m})} = 0.839\text{ rad/s}^2$.

(d) Energy conservation gives $U_1 = K_2$, giving $mgh = 1/2\, I\omega^2 = (1/2)(1/3\, mL^2)\omega^2$. Trigonometry gives $h = L/2\,\sin 37°$. Canceling m, the energy conservation equation gives $g(L/2)\sin 37° = (1/6)L^2\omega^2$. Solving for ω gives $\omega = \sqrt{\dfrac{3g\sin 37°}{L}} = \sqrt{\dfrac{3(9.80\text{ m/s}^2)\sin 37°}{14.0\text{ m}}} = 1.12\text{ rad/s}.$

EVALUATE: The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

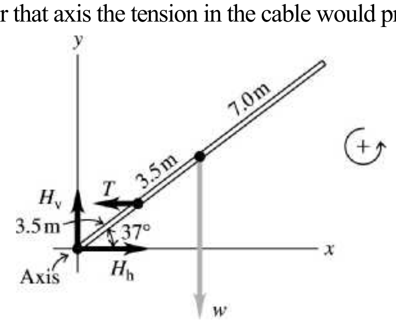


Figure 11.62

11.63. IDENTIFY: The amount the tendon stretches depends on Young's modulus for the tendon material. The foot is in rotational equilibrium, so the torques on it balance.

SET UP: $Y = \dfrac{F_{\text{T}}/A}{\Delta l / l_0}$. The foot is in rotational equilibrium, so $\sum \tau_z = 0$.

EXECUTE: **(a)** The free-body diagram for the foot is given in Figure 11.63. T is the tension in the tendon and A is the force exerted on the foot by the ankle. $n = (75\text{ kg})g$, the weight of the person.

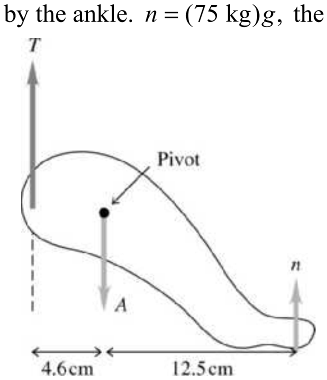


Figure 11.63

(b) Apply $\sum \tau_z = 0,$ letting counterclockwise torques be positive and with the pivot at the ankle:

$T(4.6\text{ cm}) - n(12.5\text{ cm}) = 0.$ $T = \left(\dfrac{12.5\text{ cm}}{4.6\text{ cm}}\right)(75\text{ kg})(9.80\text{ m/s}^2) = 2000\text{ N},$ which is 2.72 times his weight.

(c) The foot pulls downward on the tendon with a force of 2000 N.

$$\Delta l = \left(\frac{F_\text{T}}{YA}\right)l_0 = \frac{2000\text{ N}}{(1470\times10^6\text{ Pa})(78\times10^{-6}\text{ m}^2)}(25\text{ cm}) = 4.4\text{ mm}.$$

EVALUATE: The tension is quite large, but the Achilles tendon stretches about 4.4 mm, which is only about 1/6 of an inch, so it must be a strong tendon.

11.64. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The center of mass of the beam is 1.0 m from the suspension point.

EXECUTE: **(a)** Taking torques about the suspension point,
$w(4.00\text{ m})\sin 30° + (140.0\text{ N})(1.00\text{ m})\sin 30° = (100\text{ N})(2.00\text{ m})\sin 30°.$

The common factor of $\sin 30°$ divides out, from which $w = 15.0\text{ N}.$

(b) In this case, a common factor of $\sin 45°$ would be factored out, and the result would be the same.

EVALUATE: All the forces are vertical, so the moments are all horizontal and all contain the factor $\sin\theta$, where θ is the angle the beam makes with the horizontal.

11.65. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the flagpole.

SET UP: The free-body diagram for the flagpole is given in Figure 11.65. Let clockwise torques be positive. θ is the angle the cable makes with the horizontal pole.

EXECUTE: **(a)** Taking torques about the hinged end of the pole
$(200\text{ N})(2.50\text{ m}) + (600\text{ N})(5.00\text{ m}) - T_y(5.00\text{ m}) = 0.$ $T_y = 700\text{ N}.$ The x-component of the tension is then

$T_x = \sqrt{(1000\text{ N})^2 - (700\text{ N})^2} = 714\text{ N}.$ $\tan\theta = \dfrac{h}{5.00\text{ m}} = \dfrac{T_y}{T_x}.$ The height above the pole that the wire must

be attached is $(5.00\text{ m})\dfrac{700}{714} = 4.90\text{ m}.$

(b) The y-component of the tension remains 700 N. Now $\tan\theta = \dfrac{4.40\text{ m}}{5.00\text{ m}}$ and $\theta = 41.35°,$ so

$T = \dfrac{T_y}{\sin\theta} = \dfrac{700\text{ N}}{\sin 41.35°} = 1060\text{ N},$ an increase of 60 N.

EVALUATE: As the wire is fastened closer to the hinged end of the pole, the moment arm for T decreases and T must increase to produce the same torque about that end.

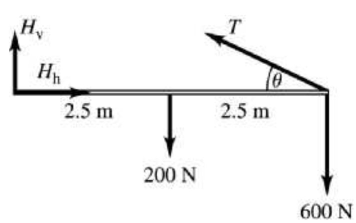


Figure 11.65

11.66. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where D, C and B are joined. Apply $\sum \tau_z = 0$ to the rod.

SET UP: To find T_C and T_D, use a coordinate system with axes parallel to the cords.

EXECUTE: A and B are straightforward, the tensions being the weights suspended:
$T_A = (0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.353\text{ N}$ and $T_B = (0.0240\text{ kg} + 0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.588\text{ N}.$ Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B\cos 36.9° = 0.470\text{ N}$ and $T_D = T_B\cos 53.1° = 0.353\text{ N}.$ To find T_E, take torques about the point where string F is attached.

$T_E(1.00\text{ m}) = T_D \sin 36.9°(0.800\text{ m}) + T_C \sin 53.1°(0.200\text{ m}) + (0.120\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m})$ and $T_E = 0.833$ N.

T_F may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\text{kg})(9.80\text{m/s}^2) = 1.76$ N, from which $T_F = 0.931$ N.

EVALUATE: The vertical line through the spheres is closer to *F* than to *E*, so we expect $T_F > T_E$, and this is indeed the case.

11.67. **IDENTIFY:** The torques must balance since the person is not rotating.

SET UP: Figure 11.67a shows the distances and angles. $\theta + \phi = 90°$. $\theta = 56.3°$ and $\phi = 33.7°$. The distances x_1 and x_2 are $x_1 = (90\text{ cm})\cos\theta = 50.0$ cm and $x_2 = (135\text{ cm})\cos\phi = 112$ cm. The free-body diagram for the person is given in Figure 11.67b. $w_l = 277$ N is the weight of his feet and legs, and $w_t = 473$ N is the weight of his trunk. n_f and f_f are the total normal and friction forces exerted on his feet and n_h and f_h are those forces on his hands. The free-body diagram for his legs is given in Figure 11.67c. *F* is the force exerted on his legs by his hip joints. For balance, $\sum \tau_z = 0$.

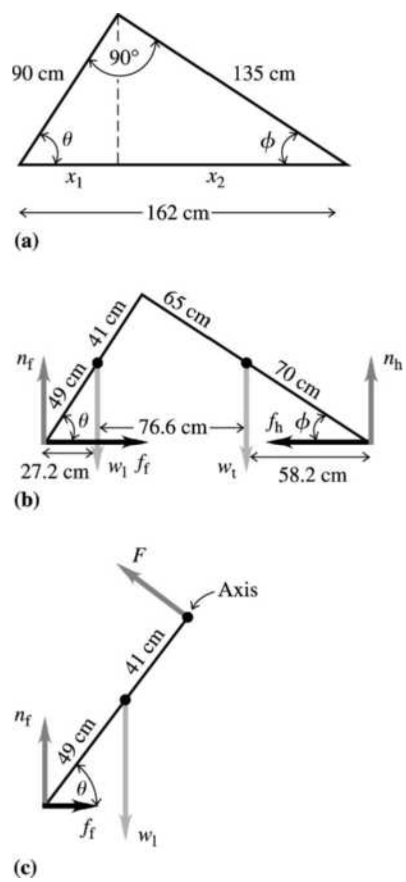


Figure 11.67

EXECUTE: **(a)** Consider the force diagram of Figure 11.67b. $\sum \tau_z = 0$ with the pivot at his feet and counterclockwise torques positive gives $n_h(162\text{ cm}) - (277\text{ N})(27.2\text{ cm}) - (473\text{ N})(103.8\text{ cm}) = 0$. $n_h = 350$ N, so there is a normal force of 175 N at each hand. $n_f + n_h - w_l - w_t = 0$ so $n_f = w_l + w_t - n_h = 750\text{ N} - 350\text{ N} = 400$ N, so there is a normal force of 200 N at each foot.

(b) Consider the force diagram of Figure 11.67c. $\sum \tau_z = 0$ with the pivot at his hips and counterclockwise torques positive gives $f_f(74.9 \text{ cm}) + w_1(22.8 \text{ cm}) - n_f(50.0 \text{ cm}) = 0.$

$f_f = \dfrac{(400 \text{ N})(50.0 \text{ cm}) - (277 \text{ N})(22.8 \text{ cm})}{74.9 \text{ cm}} = 182.7 \text{ N}.$ There is a friction force of 91 N at each foot.

$\sum F_x = 0$ in Figure 11.67b gives $f_h = f_f$, so there is a friction force of 91 N at each hand.

EVALUATE: In this position the normal forces at his feet and at his hands don't differ very much.

11.68. IDENTIFY: Apply Eq. (11.10) and the relation $\Delta w/w_0 = -\sigma\Delta l/l_0$ that is given in the problem.

SET UP: The steel rod in Example 11.5 has $\Delta l/l_0 = 9.0\times10^{-4}$. For nickel, $Y = 2.1\times10^{11}$ Pa. The width w_0 is $w_0 = \sqrt{4A/\pi}$.

EXECUTE: **(a)** $\Delta w = -\sigma\,(\Delta l/l)w_0 = -(0.23)(9.0\times10^{-4})\sqrt{4(0.30\times10^{-4}\text{ m}^2)/\pi} = -1.3\ \mu\text{m}.$

(b) $F_\perp = AY\dfrac{\Delta l}{l} = AY\dfrac{1}{\sigma}\dfrac{\Delta w}{w}$ and $F_\perp = \dfrac{(2.1\times10^{11}\text{ Pa})\,(\pi\,(2.0\times10^{-2}\text{m})^2)}{0.42}\;\dfrac{0.10\times10^{-3}\text{m}}{2.0\times10^{-2}\text{ m}} = 3.1\times10^6 \text{ N}.$

EVALUATE: For nickel and steel, $\sigma < 1$ and the fractional change in width is less than the fractional change in length.

11.69. IDENTIFY: Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation $f_s = \mu_s n.$

SET UP: The free-body diagram for the crate when it is ready to tip is given in Figure 11.69.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $P(1.50 \text{ m})\sin 53.0° - w(1.10 \text{ m}) = 0.$

$$P = w\left(\frac{1.10 \text{ m}}{[1.50 \text{ m}][\sin 53.0°]}\right) = 1.15\times10^3 \text{ N}$$

(b) $\sum F_y = 0$ gives $n - w - P\cos 53.0° = 0.$

$$n = w + P\cos 53.0° = 1250 \text{ N} + (1.15\times10^3 \text{ N})\cos 53° = 1.94\times10^3 \text{ N}$$

(c) $\sum F_y = 0$ gives $f_s = P\sin 53.0° = (1.15\times10^3 \text{ N})\sin 53.0° = 918 \text{ N}.$

(d) $\mu_s = \dfrac{f_s}{n} = \dfrac{918 \text{ N}}{1.94\times10^3 \text{ N}} = 0.473$

EVALUATE: The normal force is greater than the weight because P has a downward component.

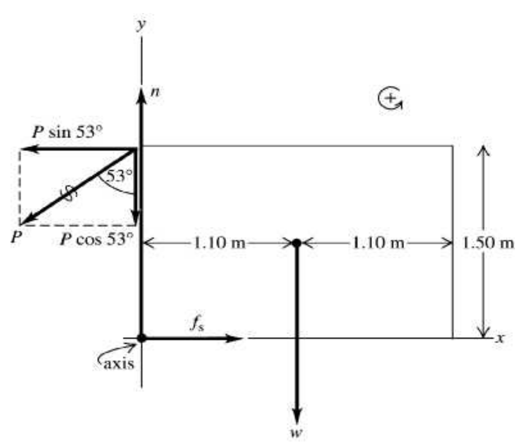


Figure 11.69

11.70. IDENTIFY: Apply $\sum \tau_z = 0$ to the meterstick.

SET UP: The wall exerts an upward static friction force f and a horizontal normal force n on the stick. Denote the length of the stick by l. $f = \mu_s n.$

EXECUTE: **(a)** Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = w/2$. Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is $l\tan\theta$, $n\tan\theta = w/2$. Then, $(f/n) = \tan\theta < 0.40$, so $\theta < \arctan(0.40) = 22°$.

(b) Taking torques as in part (a), $fl = w\frac{l}{2} + w(l-x)$ and $nl\tan\theta = w\frac{l}{2} + wx$. In terms of the coefficient of friction μ_s, $\mu_s > \frac{f}{n} = \frac{l/2 + (l-x)}{l/2 + x}\tan\theta = \frac{3l - 2x}{l + 2x}\tan\theta$. Solving for x, $x > \frac{l}{2}\frac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2$ cm.

(c) In the above expression, setting $x = 10$ cm and $l = 100$ cm and solving for μ_s gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

EVALUATE: For $\theta = 15°$ and without the block suspended from the stick, a value of $\mu_s \geq 0.268$ is required to prevent slipping. Hanging the block from the stick increases the value of μ_s that is required.

11.71. **IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

SET UP: The free-body diagram for the crate is given in Figure 11.71.

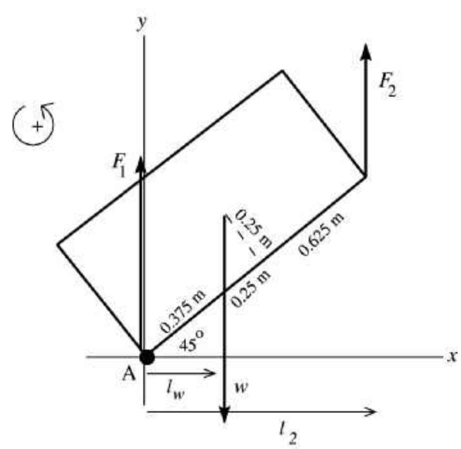


Figure 11.71

$l_w = (0.375\text{ m})\cos 45°$

$l_2 = (1.25\text{ m})\cos 45°$

Let $\vec{F}_1$ and $\vec{F}_2$ be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

EXECUTE: $\sum F_y = ma_y$ gives $F_1 + F_2 - w = 0$

$F_1 + F_2 = w = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

$\sum \tau_A = 0$ gives $F_2 l_2 - w l_w = 0$

$$F_2 = w\left(\frac{l_w}{l_2}\right) = 1960\text{ N}\left(\frac{0.375\text{ m}\cos 45°}{1.25\text{ m}\cos 45°}\right) = 590\text{ N}$$

Then $F_1 = w - F_2 = 1960\text{ N} - 590\text{ N} = 1370\text{ N}$.

EVALUATE: The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for $\vec{F}_1$ is less than for $\vec{F}_2$, so must have $F_1 > F_2$ to compensate.

11.72. **IDENTIFY:** Apply the first and second conditions for equilibrium to the forearm.

SET UP: The free-body diagram is given in Figure 11.72a, and when holding the weight in Figure 11.72b. Let $+y$ be upward.

EXECUTE: **(a)** $\sum \tau_{\text{Elbow}} = 0$ gives $F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm})$ and $F_B = 59.2$ N.

(b) $\sum \tau_{\text{Elbow}} = 0$ gives $F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm}) + (80.0\text{ N})(33.0\text{ cm})$ and $F_B = 754$ N. The biceps force has a short lever arm, so it must be large to balance the torques.

(c) $\sum F_y = 0$ gives $-F_E + F_B - 15.0\text{ N} - 80.0\text{ N} = 0$ and $F_E = 754\text{N} - 15.0\text{ N} - 80.0\text{ N} = 659\text{ N}.$

EVALUATE: (d) The biceps muscle acts perpendicular to the forearm, so its lever arm stays the same, but those of the other two forces decrease as the arm is raised. Therefore the tension in the biceps muscle *decreases*.

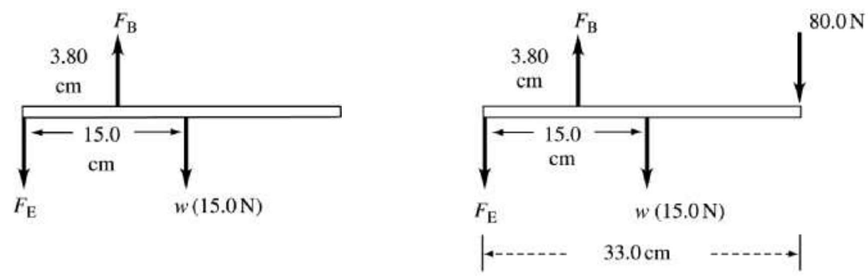


Figure 11.72a, b

11.73. IDENTIFY: Apply $\sum \tau_z = 0$ to the forearm.

SET UP: The free-body diagram for the forearm is given in Figure 11.10 in the textbook.

EXECUTE: (a) $\sum \tau_z = 0$, axis at elbow gives $wL - (T \sin\theta)D = 0.$ $\sin\theta = \dfrac{h}{\sqrt{h^2 + D^2}}$ so $w = T\dfrac{hD}{L\sqrt{h^2 + D^2}}.$

$$w_{\max} = T_{\max}\frac{hD}{L\sqrt{h^2 + D^2}}.$$

(b) $\dfrac{dw_{\max}}{dD} = \dfrac{T_{\max}h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive.

EVALUATE: (c) The result of part (b) shows that $w_{\max}$ increases when D increases, since the derivative is positive. $w_{\max}$ is larger for a chimp since D is larger.

11.74. IDENTIFY: Apply the first and second conditions for equilibrium to the table.

SET UP: Label the legs as shown in Figure 11.74a. Legs A and C are 3.6 m apart. Let the weight be placed closest to legs C and D. By symmetry, $A = B$ and $C = D$. Redraw the table as viewed from the AC side. The free-body diagram in this view is given in Figure 11.74b.

EXECUTE: $\sum \tau_z$ (about right end) $= 0$ gives $2A(3.6\text{ m}) = (90.0\text{ N})(1.8\text{ m}) + (1500\text{ N})(0.50\text{ m})$ and $A = 130\text{ N} = B.$ $\sum F_y = 0$ gives $A + B + C + D = 1590\text{ N}.$ Using $A = B = 130\text{ N}$ and $C = D$ gives $C = D = 670\text{ N}.$ By Newton's third law of motion, the forces A, B, C and D on the table are the same magnitude as the forces the table exerts on the floor.

EVALUATE: As expected, the legs closest to the 1500 N weight exert a greater force on the floor.

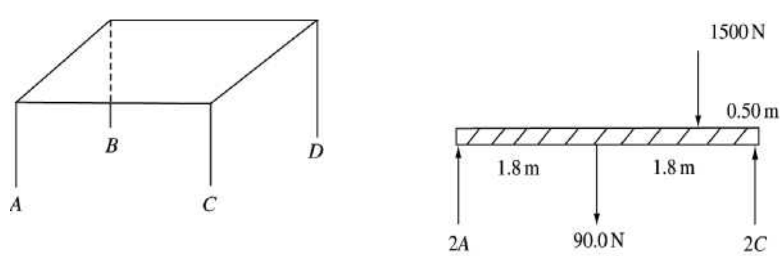
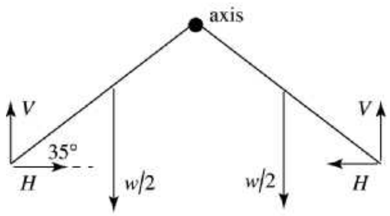


Figure 11.74a, b

11.75. IDENTIFY: Apply $\sum \tau_z = 0$ first to the roof and then to one wall.

(a) SET UP: Consider the forces on the roof; see Figure 11.75a.



V and H are the vertical and horizontal forces each wall exerts on the roof.
$w = 20{,}000\text{ N}$ is the total weight of the roof.
$2V = w$ so $V = w/2$

Figure 11.75a

Apply $\sum \tau_z = 0$ to one half of the roof, with the axis along the line where the two halves join. Let each half have length L.

EXECUTE: $(w/2)(L/2)(\cos 35.0°) + HL\sin 35.0° - VL\cos 35° = 0$

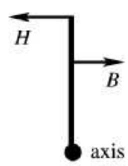
L divides out, and use $V = w/2$

$H\sin 35.0° = \frac{1}{4}w\cos 35.0°$

$$H = \frac{w}{4\tan 35.0°} = 7140\text{ N}$$

EVALUATE: By Newton's third law, the roof exerts a horizontal, outward force on the wall. For torque about an axis at the lower end of the wall, at the ground, this force has a larger moment arm and hence larger torque the taller the walls.

(b) SET UP: The force diagram for one wall is given in Figure 11.75b.



Consider the torques on this wall.

Figure 11.75b

H is the horizontal force exerted by the roof, as considered in part (a). B is the horizontal force exerted by the buttress. Now the angle is $40°$, so $H = \dfrac{w}{4\tan 40°} = 5959\text{ N}.$

EXECUTE: $\sum \tau_z = 0,$ axis at the ground

$H(40\text{ m}) - B(30\text{ m}) = 0$ and $B = 7900\text{ N}.$

EVALUATE: The horizontal force exerted by the roof is larger as the roof becomes more horizontal, since for torques applied to the roof the moment arm for H decreases. The force B required from the buttress is less the higher up on the wall this force is applied.

11.76. IDENTIFY: Apply $\sum \tau_z = 0$ to the wheel.

SET UP: Take torques about the upper corner of the curb.

EXECUTE: The force $\vec{F}$ acts at a perpendicular distance $R - h$ and the weight acts at a perpendicular distance $\sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$. Setting the torques equal for the minimum necessary force,

$$F = mg\frac{\sqrt{2Rh - h^2}}{R - h}.$$

(b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R - h$, so the minimum force is $(mg)\sqrt{2Rh - h^2}/(2R - h)$.

EVALUATE: **(c)** Less force is required when the force is applied at the top of the wheel, since in this case $\vec{F}$ has a larger moment arm.

11.77. IDENTIFY: Apply the first and second conditions of equilibrium to the gate.

SET UP: The free-body diagram for the gate is given in Figure 11.77.

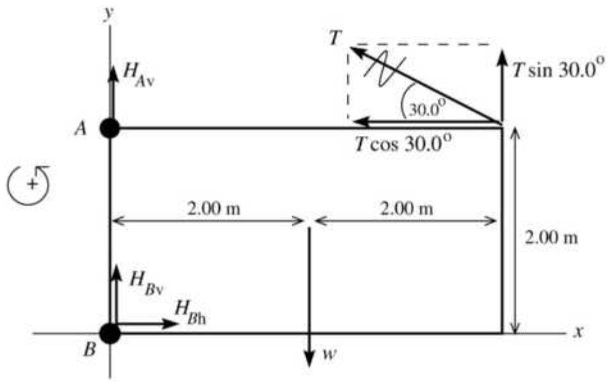


Figure 11.77

Use coordinates with the origin at B. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at A and B. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

EXECUTE: (a) $\sum \tau_B = 0$ gives $+(T\sin 30.0°)(4.00\text{ m}) + (T\cos 30.0°)(2.00\text{ m}) - w(2.00\text{ m}) = 0$

$T(2\sin 30.0° + \cos 30.0°) = w$

$$T = \frac{w}{2\sin 30.0° + \cos 30.0°} = \frac{500\text{ N}}{2\sin 30.0° + \cos 30.0°} = 268\text{ N}$$

(b) $\sum F_x = ma_x$ says $H_{Bh} - T\cos 30.0° = 0$

$H_{Bh} = T\cos 30.0° = (268\text{ N})\cos 30.0° = 232\text{ N}$

(c) $\sum F_y = ma_y$ says $H_{Av} + H_{Bv} + T\sin 30.0° - w = 0$

$H_{Av} + H_{Bv} = w - T\sin 30.0° = 500\text{ N} - (268\text{ N})\sin 30.0° = 366\text{ N}$

EVALUATE: T has a horizontal component to the left so H_{Bh} must be to the right, as these are the only two horizontal forces. Note that we cannot determine H_{Av} and H_{Bv} separately, only their sum.

11.78. IDENTIFY: Use Eq. (11.3) to locate the x-coordinate of the center of gravity of the block combinations.

SET UP: The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

EXECUTE: (a) The center of gravity of the top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ to the left of the right edge of the upper block, so the overhang is $3L/4$.

(b) Take the two-block combination from part (a), and place it on top of the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4) + (L/6) = 11L/12$. Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$.

(c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

EVALUATE: The sequence of maximum overhangs is $\frac{18L}{24}, \frac{22L}{24}, \frac{25L}{24}, \ldots$. The increase of overhang when one more block is added is decreasing.

11.79. IDENTIFY: Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

(a) SET UP: The forces on each marble are shown in Figure 11.79.

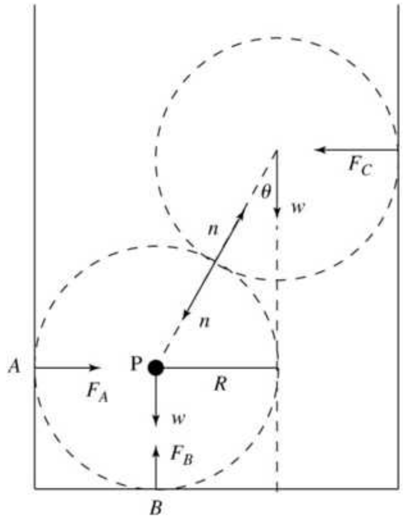


Figure 11.79

EXECUTE:
$F_B = 2w = 1.47\text{ N}$
$\sin\theta = R/2R$ so $\theta = 30°$
$\sum\tau_z = 0$, axis at P
$F_C(2R\cos\theta) - wR = 0$
$$F_C = \frac{mg}{2\cos 30°} = 0.424\text{ N}$$
$F_A = F_C = 0.424\text{ N}$

(b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so $F_A = n\sin\theta.$
$$n = \frac{F_A}{\sin 30°} = 0.848\text{ N}$$
Could use instead that the vertical forces sum to zero
$F_B - mg - n\cos\theta = 0$
$$n = \frac{F_B - mg}{\cos 30°} = 0.848\text{ N, which checks.}$$
EVALUATE: If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that $\sum F_x = 0$ and $\sum F_y = 0$ for the top marble.

11.80. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the right-hand beam.

SET UP: Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If F_{wire} is the tension in each wire and $w = 200\text{ N}$ is the weight of each beam, $2F_{\text{wire}} - 2w = 0$ and $F_{\text{wire}} = w.$ Let L be the length of each beam.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $F_{\text{wire}}L\sin\frac{\theta}{2} - F_{\text{c}}\frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0,$ where θ is the angle between the beams and F_{c} is the force exerted by the cross bar. The length drops out, and all other quantities except F_{c} are known, so $F_{\text{c}} = \dfrac{F_{\text{wire}}\sin(\theta/2)) - \frac{1}{2}w\sin(\theta/2)}{\frac{1}{2}\cos(\theta/2)} = (2F_{\text{wire}} - w)\tan\dfrac{\theta}{2}.$ Therefore $F = (260\text{ N})\tan\dfrac{53°}{2} = 130\text{ N}.$

(b) The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

(c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

EVALUATE: The force exerted on each beam increases as θ increases and exceeds the weight of the beam for $\theta \geq 90°.$

11.81. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

(a) SET UP: Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force n passes through the left-hand edge of the bale. Consider $\Sigma\tau_z = 0$ with point A at the lower left-hand corner.

Then $\tau_n = 0$ and $\tau_f = 0$, so it must be that $\tau_{mg} = 0$ also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.81a.

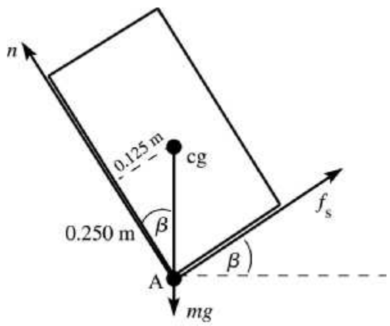


Figure 11.81a

EXECUTE: $\tan\beta = \dfrac{0.125\text{ m}}{0.250\text{ m}}$

$\beta = 27°$, angle where tips

SET UP: At the angle where the bale is ready to slip down the incline f_s has its maximum possible value, $f_s = \mu_s n$. The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.81b.

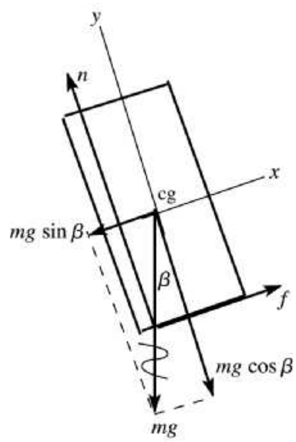


Figure 11.81b

EXECUTE:

$\sum F_y = ma_y$

$n - mg\cos\beta = 0$

$n = mg\cos\beta$

$f_s = \mu_s mg\cos\beta$

(f_s has maximum value when bale ready to slip)

$\sum F_x = ma_x$

$f_s - mg\sin\beta = 0$

$\mu_s mg\cos\beta - mg\sin\beta = 0$

$\tan\beta = \mu_s$

$\mu_s = 0.60$ gives that $\beta = 31°$

$\beta = 27°$ to tip; $\beta = 31°$ to slip, so tips first

(b) The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at $\beta = 27°$. For $\mu_s = 0.40$ slips at $\beta = \arctan(0.40) = 22°$.

Now the bale will start to slide down the incline before it tips.

EVALUATE: With a smaller μ_s the slope angle β where the bale slips is smaller.

11.82. **IDENTIFY:** Apply the equilibrium conditions to the pole. The horizontal component of the tension in the wire is 22.0 N.

SET UP: The free-body diagram for the pole is given in Figure 11.82. The tension in the cord equals the weight W. F_v and F_h are the components of the force exerted by the hinge. If either of these forces is actually in the opposite direction to what we have assumed, we will get a negative value when we solve for it.

EXECUTE: **(a)** $T\sin 37.0° = 22.0\text{ N}$ so $T = 36.6\text{ N}$. $\sum\tau_z = 0$ gives $(T\sin 37.0°)(1.75\text{ m}) - W(1.35\text{ m}) = 0$.

$$W = \frac{(22.0\text{ N})(1.75\text{ m})}{1.35\text{ m}} = 28.5\text{ N}.$$

(b) $\sum F_y = 0$ gives $F_v - T\cos 37.0° - w = 0$ and $F_v = (36.6\text{ N})\cos 37.0° + 55.0\text{ N} = 84.2\text{ N}$. $\sum F_x = 0$ gives $W - T\sin 37.0° - F_h = 0$ and $F_h = 28.5\text{ N} - 22.0\text{ N} = 6.5\text{ N}$. The magnitude of the hinge force is $F = \sqrt{F_h^2 + F_v^2} = 84.5\text{ N}$.

EVALUATE: If we consider torques about an axis at the top of the pole, we see that F_h must be to the left in order for its torque to oppose the torque produced by the force W.

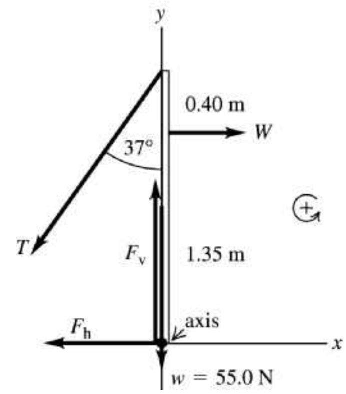


Figure 11.82

11.83. IDENTIFY: Apply the first and second conditions of equilibrium to the door.

(a) SET UP: The free-body diagram for the door is given in Figure 11.83.

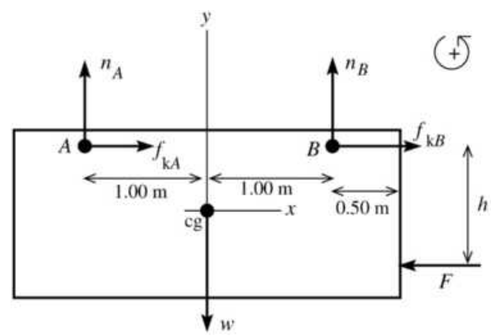


Figure 11.83

Take the origin of coordinates at the center of the door (at the cg). Let n_A, f_{kA}, n_B and f_{kB} be the normal and friction forces exerted on the door at each wheel.

EXECUTE: $\sum F_y = ma_y$

$n_A + n_B - w = 0$

$n_A + n_B = w = 950\text{ N}$

$\sum F_x = ma_x$

$f_{kA} + f_{kB} - F = 0$

$F = f_{kA} + f_{kB}$

$f_{kA} = \mu_k n_A$, $f_{kB} = \mu_k n_B$, so $F = \mu_k(n_A + n_B) = \mu_k w = (0.52)(950\text{ N}) = 494\text{ N}$

$\sum \tau_B = 0$

n_B, f_{kA} and f_{kB} all have zero moment arms and hence zero torque about this point.

Thus $+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$n_A = \frac{w(1.00\text{ m}) - F(h)}{2.00\text{ m}} = \frac{(950\text{ N})(1.00\text{ m}) - (494\text{ N})(1.60\text{ m})}{2.00\text{ m}} = 80\text{ N}$$

And then $n_B = 950\text{ N} - n_A = 950\text{ N} - 80\text{ N} = 870\text{ N}$.

(b) SET UP: If h is too large the torque of F will cause wheel A to leave the track. When wheel A just starts to lift off the track n_A and f_{kA} both go to zero.

EXECUTE: The equations in part (a) still apply.

$n_A + n_B - w = 0$ gives $n_B = w = 950\text{ N}$

Then $f_{kB} = \mu_k n_B = 0.52(950\text{ N}) = 494\text{ N}$

$F = f_{kA} + f_{kB} = 494\text{ N}$

$+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$h = \frac{w(1.00\text{ m})}{F} = \frac{(950\text{ N})(1.00\text{ m})}{494\text{ N}} = 1.92\text{ m}$$

EVALUATE: The result in part (b) is larger than the value of h in part (a). Increasing h increases the clockwise torque about B due to F and therefore decreases the clockwise torque that n_A must apply.

11.84. IDENTIFY: Apply the first and second conditions for equilibrium to the boom.

SET UP: Take the rotation axis at the left end of the boom.

EXECUTE: **(a)** The magnitude of the torque exerted by the cable must equal the magnitude of the torque due to the weight of the boom. The torque exerted by the cable about the left end is $TL\sin\theta$.
For any angle θ, $\sin(180° - \theta) = \sin\theta$, so the tension T will be the same for either angle. The horizontal component of the force that the pivot exerts on the boom will be $T\cos\theta$ or $T\cos(180° - \theta) = -T\cos\theta$.

(b) From the result of part (a), T is proportional to $\dfrac{1}{\sin\theta}$ and this becomes infinite as $\theta \to 0$ or $\theta \to 180°$.

(c) The tension is a minimum when $\sin\theta$ is a maximum, or $\theta = 90°$, a vertical cable.

(d) There are no other horizontal forces, so for the boom to be in equilibrium, the pivot exerts zero horizontal force on the boom.

EVALUATE: As the cable approaches the horizontal direction, its moment arm for the axis at the pivot approaches zero, so T must go to infinity in order for the torque due to the cable to continue to equal the gravity torque.

11.85. IDENTIFY: Apply the first and second conditions of equilibrium to the pole.

(a) SET UP: The free-body diagram for the pole is given in Figure 11.85.

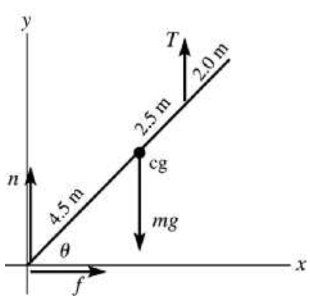


Figure 11.85

n and f are the vertical and horizontal components of the force the ground exerts on the pole.

$\sum F_x = ma_x$

$f = 0$

The force exerted by the ground has no horizontal component.

EXECUTE: $\sum \tau_A = 0$

$+T(7.0\text{ m})\cos\theta - mg(4.5\text{ m})\cos\theta = 0$

$T = mg(4.5\text{ m}/7.0\text{ m}) = (4.5/7.0)(5700\text{ N}) = 3700\text{ N}$

$\sum F_y = 0$

$n + T - mg = 0$

$n = mg - T = 5700\text{ N} - 3700\text{ N} = 2000\text{ N}$

The force exerted by the ground is vertical (upward) and has magnitude 2000 N.

EVALUATE: We can verify that $\sum \tau_z = 0$ for an axis at the cg of the pole. $T > n$ since T acts at a point closer to the cg and therefore has a smaller moment arm for this axis than n does.

(b) In the $\sum \tau_A = 0$ equation the angle θ divided out. All forces on the pole are vertical and their moment arms are all proportional to $\cos\theta$.

11.86. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the slab.

SET UP: The free-body diagram is given in Figure 11.86a. $\tan\beta = \dfrac{3.75\text{ m}}{1.75\text{ m}}$ so $\beta = 65.0°$. $20.0° + \beta + \alpha = 90°$ so $\alpha = 5.0°$. The distance from the axis to the center of the block is $\sqrt{\left(\dfrac{3.75\text{ m}}{2}\right)^2 + \left(\dfrac{1.75\text{ m}}{2}\right)^2} = 2.07\text{ m}.$

EXECUTE: **(a)** $w(2.07\text{ m})\sin 5.0° - T(3.75\text{ m})\sin 52.0° = 0$. $T = 0.061w$. Each worker must exert a force of $0.012w$, where w is the weight of the slab.

(b) As θ increases, the moment arm for w decreases and the moment arm for T increases, so the worker needs to exert less force.

(c) $T \to 0$ when w passes through the support point. This situation is sketched in Figure 11.86b. $\tan\theta = \dfrac{(1.75\text{ m})/2}{(3.75\text{ m})/2}$ and $\theta = 25.0°$. If θ exceeds this value the gravity torque causes the slab to tip over.

EVALUATE: The moment arm for T is much greater than the moment arm for w, so the force the workers apply is much less than the weight of the slab.

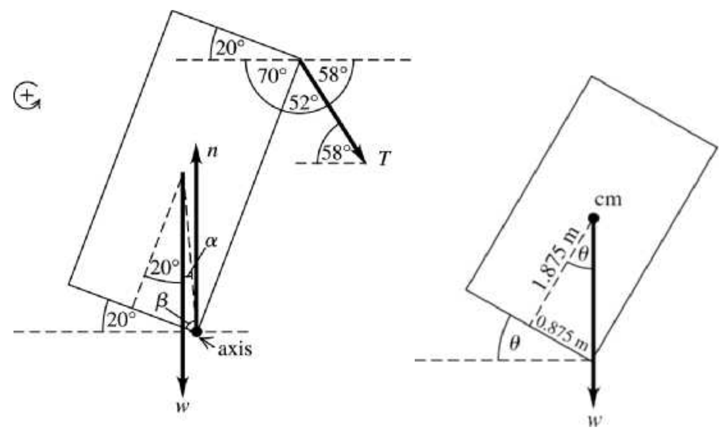


Figure 11.86 a, b

11.87. **IDENTIFY** and **SET UP:** $Y = F_\perp l_0 / A\,\Delta l$ (Eq. 11.10 holds since the problem states that the stress is proportional to the strain.) Thus $\Delta l = F_\perp l_0 / AY$. Use proportionality to see how changing the wire properties affects Δl.

EXECUTE: **(a)** Change l_0 but $F_\perp$ (same floodlamp), A (same diameter wire), and Y (same material) all stay the same.

$$\frac{\Delta l}{l_0} = \frac{F_\perp}{AY} = \text{constant, so } \frac{\Delta l_1}{l_{01}} = \frac{\Delta l_2}{l_{02}}$$

$$\Delta l_2 = \Delta l_1 (l_{02}/l_{01}) = 2\Delta l_1 = 2(0.18\text{ mm}) = 0.36\text{ mm}$$

(b) $A = \pi(d/2)^2 = \frac{1}{4}\pi d^2$, so $\Delta l = \dfrac{F_\perp l_0}{\frac{1}{4}\pi d^2 Y}$

$F_\perp$, l_0, Y all stay the same, so $\Delta l(d^2) = F_\perp l_0 / (\frac{1}{4}\pi Y) = \text{constant}$

$$\Delta l_1(d_1^2) = \Delta l_2(d_2^2)$$

$$\Delta l_2 = \Delta l_1 (d_1/d_2)^2 = (0.18\text{ mm})(1/2)^2 = 0.045\text{ mm}$$

(c) $F_\perp$, l_0, A all stay the same so $\Delta l Y = F_\perp l_0 / A = \text{constant}$

$$\Delta l_1 Y_1 = \Delta l_2 Y_2$$

$$\Delta l_2 = \Delta l_1 (Y_1/Y_2) = (0.18\text{ mm})(20\times10^{10}\text{ Pa}/11\times10^{10}\text{ Pa}) = 0.33\text{ mm}$$

EVALUATE: Greater l means greater Δl, greater diameter means less Δl, and smaller Y means greater Δl.

11.88. **IDENTIFY:** For a spring, $F = kx$. $Y = \dfrac{F_\perp l_0}{A\Delta l}$.

SET UP: $F_\perp = F = W$ and $\Delta l = x$. For copper, $Y = 11\times 10^{10}$ Pa.

EXECUTE: **(a)** $F = \left(\dfrac{YA}{l_0}\right)\Delta l = \left(\dfrac{YA}{l_0}\right)x$. This in the form of $F = kx$, with $k = \dfrac{YA}{l_0}$.

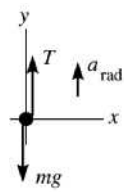
(b) $k = \dfrac{YA}{l_0} = \dfrac{(11\times 10^{10}\text{ Pa})\pi(6.455\times 10^{-4}\text{ m})^2}{0.750\text{ m}} = 1.9\times 10^5\text{ N/m}$

(c) $W = kx = (1.9\times 10^5\text{ N/m})(1.25\times 10^{-3}\text{ m}) = 240\text{ N}$

EVALUATE: For the wire the force constant is very large, much larger than for a typical spring.

11.89. **IDENTIFY:** Apply Newton's second law to the mass to find the tension in the wire. Then apply Eq. (11.10) to the wire to find the elongation this tensile force produces.

(a) SET UP: Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.89a.



The mass moves in an arc of a circle with radius $R = 0.50$ m. It has acceleration $\vec{a}_{\text{rad}}$ directed in toward the center of the circle, so at this point $\vec{a}_{\text{rad}}$ is upward.

Figure 11.89 a

EXECUTE: $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2)$.

But ω must be in rad/s:

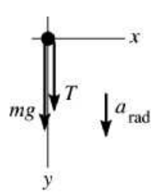
$\omega = (120\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 12.57\text{ rad/s}$.

Then $T = (12.0\text{ kg})(9.80\text{ m/s}^2 + (0.50\text{ m})(12.57\text{ rad/s})^2) = 1066\text{ N}$.

Now calculate the elongation Δl of the wire that this tensile force produces:

$Y = \dfrac{F_\perp l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_\perp l_0}{YA} = \dfrac{(1066\text{ N})(0.50\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 0.54\text{ cm}$.

(b) SET UP: The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.89b.



EXECUTE:

$\sum F_y = ma_y$

$mg + T = mR\omega^2$

$T = m(R\omega^2 - g)$

Figure 11.89 b

$T = (12.0\text{ kg})((0.50\text{ m})(12.57\text{ rad/s})^2 - 9.80\text{ m/s}^2) = 830\text{ N}$

$\Delta l = \dfrac{F_\perp l_0}{YA} = \dfrac{(830\text{ N})(0.50\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 0.42\text{ cm}$.

EVALUATE: At the lowest point T and w are in opposite directions and at the highest point they are in the same direction, so T is greater at the lowest point and the elongation is greatest there. The elongation is at most 1% of the length.

11.90. **IDENTIFY:** $F_{\perp} = \left(\dfrac{YA}{l_0}\right)\Delta l$ so the slope of the graph in part (a) depends on Young's modulus.

SET UP: $F_{\perp}$ is the total load, 20 N plus the added load.

EXECUTE: **(a)** The graph is given in Figure 11.90.

(b) The slope is $\dfrac{60\text{ N}}{(3.32-3.02)\times 10^{-2}\text{ m}} = 2.0\times 10^{4}\text{ N/m}.$

$$Y = \left(\frac{l_0}{\pi r^2}\right)(2.0\times 10^{4}\text{ N/m}) = \left(\frac{3.50\text{ m}}{\pi[0.35\times 10^{-3}\text{ m}]^2}\right)(2.0\times 10^{4}\text{ N/m}) = 1.8\times 10^{11}\text{ Pa}$$

(c) The stress is $F_{\perp}/A$. The total load at the proportional limit is $60\text{ N} + 20\text{ N} = 80\text{ N}.$

$$\text{stress} = \frac{80\text{ N}}{\pi(0.35\times 10^{-3}\text{ m})^2} = 2.1\times 10^{8}\text{ Pa}$$

EVALUATE: The value of Y we calculated is close to the value for iron, nickel and steel in Table 11.1.

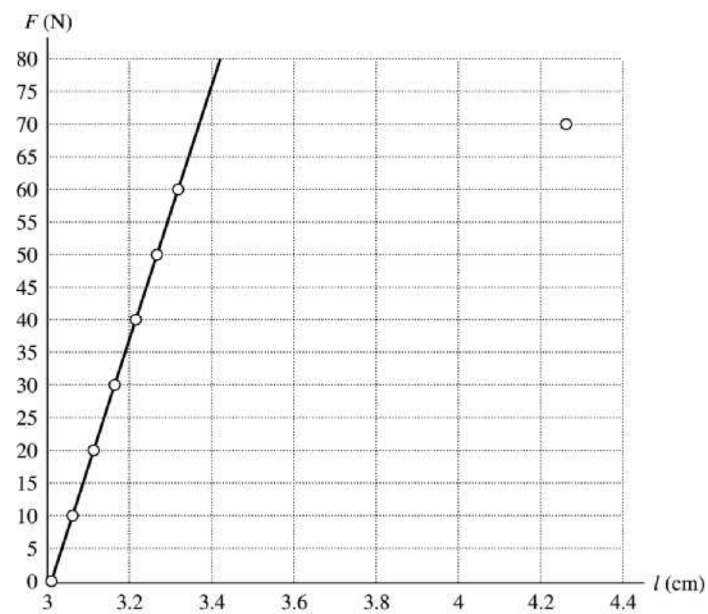


Figure 11.90

11.91. **IDENTIFY:** Use the second condition of equilibrium to relate the tension in the two wires to the distance w is from the left end. Use Eqs. (11.8) and (11.10) to relate the tension in each wire to its stress and strain.

(a) SET UP: stress $= F_{\perp}/A,$ so equal stress implies T/A same for each wire.

$T_A/2.00\text{ mm}^2 = T_B/4.00\text{ mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point C, a distance x to the right of wire A. The free-body diagram for the rod is given in Figure 11.91.

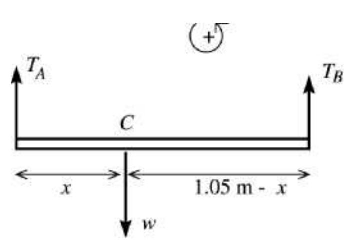


Figure 11.91

EXECUTE:

$\sum \tau_C = 0$

$+T_B(1.05\text{ m} - x) - T_A x = 0$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_A x = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70\text{ m}$ (measured from A).

(b) SET UP: $Y = \text{stress/strain}$ gives that strain $= \text{stress}/Y = F_\perp/AY$.

EXECUTE: Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times10^{11}\text{ Pa})} = \frac{T_B}{(4.00\text{ mm}^2)(1.20\times10^{11}\text{ Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A.$$

The $\sum\tau_C = 0$ equation still gives $T_B(1.05\text{ m} - x) - T_A x = 0$.

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\text{ m} - x) - T_A x = 0$.

$1.40\text{ m} = 2.33x$ and $x = 1.40\text{ m}/2.33 = 0.60\text{ m}$ (measured from A).

EVALUATE: Wire B has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller Y for B partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

11.92. **IDENTIFY:** Apply Eq. (11.10) and calculate Δl.

SET UP: When the ride is at rest the tension $F_\perp$ in the rod is the weight 1900 N of the car and occupants. When the ride is operating, the tension $F_\perp$ in the rod is obtained by applying $\sum\vec{F} = m\vec{a}$ to a car and its occupants. The free-body diagram is shown in Figure 11.92. The car travels in a circle of radius $r = l\sin\theta$, where l is the length of the rod and θ is the angle the rod makes with the vertical. For steel, $Y = 2.0\times10^{11}$ Pa. $\omega = 8.00\text{ rev/min} = 0.838\text{ rad/s}$.

EXECUTE: **(a)** $\Delta l = \dfrac{l_0 F_\perp}{YA} = \dfrac{(15.0\text{ m})(1900\text{ N})}{(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)} = 1.78\times10^{-4}\text{ m} = 0.18\text{ mm}$

(b) $\sum F_x = ma_x$ gives $F_\perp\sin\theta = mr\omega^2 = ml\sin\theta\omega^2$ and

$$F_\perp = ml\omega^2 = \left(\frac{1900\text{ N}}{9.80\text{ m/s}^2}\right)(15.0\text{ m})(0.838\text{ rad/s})^2 = 2.04\times10^3\text{ N}.$$

$$\Delta l = \left(\frac{2.04\times10^3\text{ N}}{1900\text{ N}}\right)(0.18\text{ mm}) = 0.19\text{ mm}$$

EVALUATE: $\sum F_y = ma_y$ gives $F_\perp\cos\theta = mg$ and $\cos\theta = mg/F_\perp$. As ω increases $F_\perp$ increases and $\cos\theta$ becomes small. Smaller $\cos\theta$ means θ increases, so the rods move toward the horizontal as ω increases.

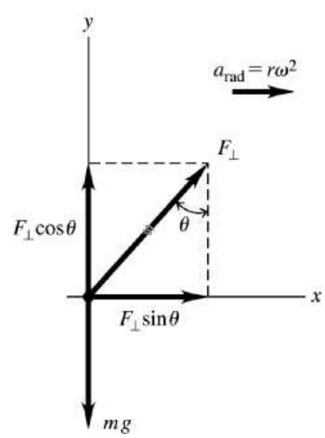


Figure 11.92

11.93. **IDENTIFY** and **SET UP:** The tension is the same at all points along the composite rod. Apply Eqs. (11.8) and (11.10) to relate the elongations, stresses and strains for each rod in the compound.

EXECUTE: Each piece of the composite rod is subjected to a tensile force of 4.00×10^4 N.

(a) $Y = \dfrac{F_\perp l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_\perp l_0}{YA}$

$\Delta l_\text{b} = \Delta l_\text{n}$ gives that $\dfrac{F_\perp l_{0,\text{b}}}{Y_\text{b} A_\text{b}} = \dfrac{F_\perp l_{0,\text{n}}}{Y_\text{n} A_\text{n}}$ (b for brass and n for nickel); $l_{0,\text{n}} = L$

But the $F_\perp$ is the same for both, so

$$l_{0,\text{n}} = \frac{Y_\text{n}}{Y_\text{b}}\frac{A_\text{n}}{A_\text{b}} l_{0,\text{b}}$$

$$L = \left(\frac{21\times10^{10}\ \text{Pa}}{9.0\times10^{10}\ \text{Pa}}\right)\left(\frac{1.00\ \text{cm}^2}{2.00\ \text{cm}^2}\right)(1.40\ \text{m}) = 1.63\ \text{m}$$

(b) stress $= F_\perp/A = T/A$

<u>brass</u>: stress $= T/A = (4.00\times10^4\ \text{N})/(2.00\times10^{-4}\ \text{m}^2) = 2.00\times10^8$ Pa

<u>nickel</u>: stress $= T/A = (4.00\times10^4\ \text{N})/(1.00\times10^{-4}\ \text{m}^2) = 4.00\times10^8$ Pa

(c) Y = stress/strain and strain = stress/Y

<u>brass</u>: strain $= (2.00\times10^8\ \text{Pa})/(9.0\times10^{10}\ \text{Pa}) = 2.22\times10^{-3}$

<u>nickel</u>: strain $= (4.00\times10^8\ \text{Pa})/(21\times10^{10}\ \text{Pa}) = 1.90\times10^{-3}$

EVALUATE: Larger Y means less Δl and smaller A means greater Δl, so the two effects largely cancel and the lengths don't differ greatly. Equal Δl and nearly equal l means the strains are nearly the same. But equal tensions and A differing by a factor of 2 means the stresses differ by a factor of 2.

11.94. **IDENTIFY:** Apply $\dfrac{F_\perp}{A} = Y\left(\dfrac{\Delta l}{l_0}\right)$. The height from which he jumps determines his speed at the ground. The acceleration as he stops depends on the force exerted on his legs by the ground.

SET UP: In considering his motion take $+y$ downward. Assume constant acceleration as he is stopped by the floor.

EXECUTE: **(a)** $F_\perp = YA\left(\dfrac{\Delta l}{l_0}\right) = (3.0\times10^{-4}\ \text{m}^2)(14\times10^9\ \text{Pa})(0.010) = 4.2\times10^4$ N

(b) As he is stopped by the ground, the net force on him is $F_\text{net} = F_\perp - mg$, where $F_\perp$ is the force exerted on him by the ground. From part (a), $F_\perp = 2(4.2\times10^4\ \text{N}) = 8.4\times10^4$ N and

$F = 8.4\times10^4\ \text{N} - (70\ \text{kg})(9.80\ \text{m/s}^2) = 8.33\times10^4$ N. $F_\text{net} = ma$ gives $a = 1.19\times10^3\ \text{m/s}^2$.

$a_y = -1.19\times10^3\ \text{m/s}^2$ since the acceleration is upward. $v_y = v_{0y} + a_y t$ gives

$v_{0y} = -a_y t = (-1.19\times10^3\ \text{m/s}^2)(0.030\ \text{s}) = 35.7$ m/s. His speed at the ground therefore is $v = 35.7$ m/s.

This speed is related to his initial height h above the floor by $\frac{1}{2}mv^2 = mgh$ and

$$h = \frac{v^2}{2g} = \frac{(35.7\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 65\ \text{m}.$$

EVALUATE: Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

11.95. **IDENTIFY:** Apply Eq. (11.13) and calculate ΔV.

SET UP: The pressure increase is w/A, where w is the weight of the bricks and A is the area πr^2 of the piston.

EXECUTE: $$\Delta p = \frac{(1420\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(0.150\ \text{m})^2} = 1.97\times10^5\ \text{Pa}$$

$$\Delta p = -B\frac{\Delta V}{V_0} \text{ gives } \Delta V = -\frac{(\Delta p)V_0}{B} = -\frac{(1.97\times 10^5 \text{ Pa})(250 \text{ L})}{9.09\times 10^8 \text{ Pa}} = -0.0542 \text{ L}$$

EVALUATE: The fractional change in volume is only 0.022%, so this attempt is not worth the effort.

11.96. IDENTIFY: Apply the equilibrium conditions to the ladder combination and also to each ladder.

SET UP: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

EXECUTE: **(a)** Taking torques about the right end, $F_L(5.00 \text{ m}) = (480 \text{ N})(3.40 \text{ m}) + (360 \text{ N})(0.90 \text{ m})$, so $F_L = 391 \text{ N}$. F_R may be found in a similar manner, or from $F_R = 840 \text{ N} - F_L = 449 \text{ N}$.

(b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50 \text{ m}) = F_L(3.20 \text{ m}) - (480 \text{ N})(1.60 \text{ m})$, so $T = 322.1 \text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50 \text{ m}) = F_R(1.80 \text{ m}) - (360 \text{ N})(0.90 \text{ m})$ gives the same result.

(c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, $480 \text{ N} - 391 \text{ N} = 449 \text{ N} - 360 \text{ N} = 89 \text{ N}$. The magnitude of the force at A is then $\sqrt{(322.1 \text{ N})^2 + (89 \text{ N})^2} = 334 \text{ N}$.

(d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L' = F_L + (0.36)(800 \text{ N}) = 679 \text{ N}$, and $F_R' = F_R + (0.64)(800 \text{ N}) = 961 \text{ N}$. Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20 \text{ m}) - (480 \text{ N})(1.60 \text{ m})}{(1.50 \text{ m})} = \frac{F'_R(1.80 \text{ m}) - (360 \text{ N})(0.90 \text{ m})}{(1.50 \text{ m})} = 937 \text{ N}.$$

EVALUATE: The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

11.97. IDENTIFY: Apply the first and second conditions for equilibrium to the bookcase.

SET UP: When the bookcase is on the verge of tipping, it contacts the floor only at its lower left-hand edge and the normal force acts at this point. When the bookcase is on the verge of slipping, the static friction force has its largest possible value, $\mu_s n$.

EXECUTE: **(a)** Taking torques about the left edge of the left leg, the bookcase would tip when $F = \frac{(1500 \text{ N})(0.90 \text{ m})}{(1.80 \text{ m})} = 750 \text{ N}$ and would slip when $F = (\mu_s)(1500 \text{ N}) = 600 \text{ N}$, so the bookcase slides before tipping.

(b) If F is vertical, there will be no net horizontal force and the bookcase could not slide. Again taking torques about the left edge of the left leg, the force necessary to tip the case is $\frac{(1500 \text{ N})(0.90 \text{ m})}{(0.10 \text{ m})} = 13.5 \text{ kN}$.

(c) To slide, the friction force is $f = \mu_s(w + F\cos\theta)$, and setting this equal to $F\sin\theta$ and solving for F gives $F = \frac{\mu_s w}{\sin\theta - \mu_s\cos\theta}$ (to slide). To tip, the condition is that the normal force exerted by the right leg is zero, and taking torques about the left edge of the left leg, $F\sin\theta(1.80 \text{ m}) + F\cos\theta(0.10 \text{ m}) = w(0.90 \text{ m})$, and solving for F gives $F = \frac{w}{(1/9)\cos\theta + 2\sin\theta}$ (to tip).

Setting the two expressions equal to each other gives $\mu_s((1/9)\cos\theta + 2\sin\theta) = \sin\theta - \mu_s\cos\theta$ and solving for θ gives $\theta = \arctan\left(\frac{(10/9)\mu_s}{(1-2\mu_s)}\right) = 66°$.

EVALUATE: The result in (c) depends not only on the numerical value of μ_s but also on the width and height of the bookcase.

11.98. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the post, for various choices of the location of the rotation axis.

SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s = \mu_s n$.

EXECUTE: **(a)** Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so

$$F\frac{h}{2} = (n - w)h\tan\theta.$$

Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}$. Using $f \le \mu_s n$ and solving for F, $F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\text{ N}.$

(b) The above relations between F, n and f become $F\frac{3}{5}h = (n - w)h\tan\theta, f = \frac{2}{5}F$, and eliminating f and n and solving for F gives $F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1}$, and substitution of numerical values gives 750 N to two figures.

(c) If the force is applied a distance y above the ground, the above relations become $Fy = (n - w)h\tan\theta$, $F(h - y) = fh$, which become, on eliminating n and f, $w \ge F\left[\frac{(1 - y/h)}{\mu_s} - \frac{(y/h)}{\tan\theta}\right]$.

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of y is found by setting the term in square brackets equal to zero. Solving for y gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

EVALUATE: For the post to slip, for an axis at the top of the post the torque due to F must balance the torque due to the friction force. As the point of application of F approaches the top of the post, its moment arm for this axis approaches zero.

11.99. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the girder.

SET UP: Assume that the center of gravity of the loaded girder is at $L/2$, and that the cable is attached a distance x to the right of the pivot. The sine of the angle between the lever arm and the cable is then $h/\sqrt{h^2 + ((L/2) - x)^2}$.

EXECUTE: The tension is obtained from balancing torques about the pivot;

$T\left[\frac{hx}{\sqrt{h^2 + ((L/2) - x)^2}}\right] = wL/2$, where w is the total load. The minimum tension will occur when the term in square brackets is a maximum; differentiating and setting the derivative equal to zero gives a maximum, and hence a minimum tension, at $x_{\min} = (h^2/L) + (L/2)$. However, if $x_{\min} > L$, which occurs if $h > L/\sqrt{2}$, the cable must be attached at L, the farthest point to the right.

EVALUATE: Note that $x_{\min}$ is greater than $L/2$ but approaches $L/2$ as $h \to 0$. The tension is a minimum when the cable is attached somewhere on the right-hand half of the girder.

11.100. **IDENTIFY:** Write $\Delta(pV)$ or $\Delta(pV^\gamma)$ in terms of Δp and ΔV and use the fact that pV or pV^γ is constant.

SET UP: B is given by Eq. (11.13).

EXECUTE: **(a)** For constant temperature $(\Delta T = 0)$, $\Delta(pV) = (\Delta p)V + p(\Delta V) = 0$ and $B = -\frac{(\Delta p)V}{(\Delta V)} = p$

(b) In this situation, $(\Delta p)V^\gamma + \gamma p(\Delta V)\ V^{\gamma-1} = 0$, $\quad (\Delta p) + \gamma p\frac{\Delta V}{V} = 0$, and $B = -\frac{(\Delta p)V}{\Delta V} = \gamma p.$

EVALUATE: We will see later that $\gamma > 1$, so B is larger in part (b).

11.101. **IDENTIFY:** Apply Eq. (11.10) to calculate Δl.

SET UP: For steel, $Y = 2.0\times 10^{11}$ Pa .

EXECUTE: **(a)** From Eq. (11.10), $\Delta l = \dfrac{(4.50\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{(20\times 10^{10}\text{ Pa})(5.00\times 10^{-7}\text{m}^2)} = 6.62\times 10^{-4}$ m, or 0.66 mm to two figures.

(b) $(4.50\text{ kg})(9.80\text{ m/s}^2)(0.0500\times 10^{-2}\text{ m}) = 0.022\text{ J}.$

(c) The magnitude F will vary with distance; the average force is $YA(0.0250\text{ cm}/l_0) = 16.7\text{ N},$ and so the work done by the applied force is $(16.7\text{ N})(0.0500\times 10^{-2}\text{ m}) = 8.35\times 10^{-3}\text{ J}.$

(d) The average force the wire exerts is $(4.50\text{ kg})g + 16.7\text{ N} = 60.8\text{ N}.$ The work done is negative, and equal to $-(60.8\text{ N})(0.0500\times 10^{-2}\text{ m}) = -3.04\times 10^{-2}\text{ J}.$

(e) Eq. (11.10) is in the form of Hooke's law, with $k = \dfrac{YA}{l_0}.$ $U_{\text{el}} = \frac{1}{2}kx^2,$ so $\Delta U_{\text{el}} = \frac{1}{2}k(x_2^2 - x_1^2).$

$x_1 = 6.62\times 10^{-4}$ m and $x_2 = 0.500\times 10^{-3}\text{ m} + x_1 = 11.62\times 10^{-4}$ m. The change in elastic potential energy is

$\dfrac{(20\times 10^{10}\text{ Pa})(5.00\times 10^{-7}\text{ m}^2)}{2(1.50\text{ m})}((11.62\times 10^{-4}\text{ m})^2 - (6.62\times 10^{-4}\text{ m})^2) = 3.04\times 10^{-2}\text{ J},$ the negative of the result of part (d).

EVALUATE: The tensile force in the wire is conservative and obeys the relation $W = -\Delta U$.