12

FLUID MECHANICS

12.1. **IDENTIFY:** Use Eq. (12.1) to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 12.1, $\rho = 7.8\times10^3\ \text{kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R,
$V = (\pi R^2)L = \pi(0.01425\ \text{m})^2(0.858\ \text{m}) = 5.474\times10^{-4}\ \text{m}^3$.

Then $m = \rho V = (7.8\times10^3\ \text{kg/m}^3)(5.474\times10^{-4}\ \text{m}^3) = 4.27\ \text{kg}$, and

$w = mg = (4.27\ \text{kg})(9.80\ \text{m/s}^2) = 41.8\ \text{N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

12.2. **IDENTIFY:** The volume of the remaining object is the volume of a cube minus the volume of a cylinder, and it is this object for which we know the mass. The target variables are the density of the metal of the cube and the original weight of the cube.

SET UP: The volume of a cube with side length L is L^3, the volume of a cylinder of radius r and length L is $\pi r^2 L$, and density is $\rho = m/V$.

EXECUTE: **(a)** The volume of the metal left after the hole is drilled is the volume of the solid cube minus the volume of the cylindrical hole:
$V = L^3 - \pi r^2 L = (5.0\ \text{cm})^3 - \pi(1.0\ \text{cm})^2(5.0\ \text{cm}) = 109\ \text{cm}^3 = 1.09\times10^{-4}\ \text{m}^3$. The cube with the hole has mass $m = \dfrac{w}{g} = \dfrac{7.50\ \text{N}}{9.80\ \text{m/s}^2} = 0.765\ \text{kg}$ and density $\rho = \dfrac{m}{V} = \dfrac{0.765\ \text{kg}}{1.09\times10^{-4}\ \text{m}^3} = 7.02\times10^3\ \text{kg/m}^3$.

(b) The solid cube has volume $V = L^3 = 125\ \text{cm}^3 = 1.25\times10^{-4}\ \text{m}^3$ and mass
$m = \rho V = (7.02\times10^3\ \text{kg/m}^3)(1.25\times10^{-4}\ \text{m}^3) = 0.878\ \text{kg}$. The original weight of the cube was
$w = mg = 8.60\ \text{N}$.

EVALUATE: As Table 12.1 shows, the density of this metal is close to that of iron or steel, so it is reasonable.

12.3. **IDENTIFY:** $\rho = m/V$

SET UP: The density of gold is $19.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V = (5.0\times10^{-3}\ \text{m})(15.0\times10^{-3}\ \text{m})(30.0\times10^{-3}\ \text{m}) = 2.25\times10^{-6}\ \text{m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158\ \text{kg}}{2.25\times10^{-6}\ \text{m}^3} = 7.02\times10^3\ \text{kg/m}^3$. The metal is not pure gold.

EVALUATE: The average density is only 36% that of gold, so at most 36% of the mass is gold.

12.4. **IDENTIFY:** Find the mass of gold that has a value of $\$1.00\times10^6$. Then use the density of gold to find the volume of this mass of gold.

SET UP: For gold, $\rho = 19.3\times10^3\ \text{kg/m}^3$. The volume V of a cube is related to the length L of one side by $V = L^3$.

EXECUTE: $m = (\$1.00\times10^6)\left(\dfrac{1\text{ troy ounce}}{\$426.60}\right)\left(\dfrac{31.1035\times10^{-3}\text{ kg}}{1\text{ troy ounce}}\right) = 72.9\text{ kg}.$ $\rho = \dfrac{m}{V}$ so

$V = \dfrac{m}{\rho} = \dfrac{72.9\text{ kg}}{19.3\times10^3\text{ kg/m}^3} = 3.78\times10^{-3}\text{ m}^3.$ $L = V^{1/3} = 0.156\text{ m} = 15.6\text{ cm}.$

EVALUATE: The cube of gold would weigh about 160 lbs.

12.5. **IDENTIFY:** Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

SET UP: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^3\text{ kg/m}^3$ and for aluminum, $\rho_a = 2.7\times10^3\text{ kg/m}^3$.

EXECUTE: $m = \rho V = \frac{4}{3}\pi r^3\rho$. Same mass means $r_a^3\rho_a = r_1^3\rho_1$. $\dfrac{r_a}{r_1} = \left(\dfrac{\rho_1}{\rho_a}\right)^{1/3} = \left(\dfrac{11.3\times10^3}{2.7\times10^3}\right)^{1/3} = 1.6.$

EVALUATE: The aluminum sphere is larger, since its density is less.

12.6. **IDENTIFY:** Average density is $\rho = m/V$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{\text{sun}} = 1.99\times10^{30}\text{ kg}$ and radius $6.96\times10^8\text{ m}$.

EXECUTE: **(a)** $\rho = \dfrac{M_{\text{sun}}}{V_{\text{sun}}} = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(6.96\times10^8\text{m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{1.412\times10^{27}\text{ m}^3} = 1.409\times10^3\text{ kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30}\text{ kg}}{\frac{4}{3}\pi(2.00\times10^4\text{ m})^3} = \dfrac{1.99\times10^{30}\text{ kg}}{3.351\times10^{13}\text{ m}^3} = 5.94\times10^{16}\text{ kg/m}^3$

EVALUATE: For comparison, the average density of the earth is $5.5\times10^3\text{kg/m}^3$. A neutron star is extremely dense.

12.7. **IDENTIFY:** $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

SET UP: For a hollow cylinder with inner radius R_1, outer radius R_2, and length L the volume is $V = \pi(R_2^2 - R_1^2)L$. $R_1 = 1.25\times10^{-2}\text{ m}$ and $R_2 = 1.75\times10^{-2}\text{ m}$.

EXECUTE: $V = \pi([0.0175\text{ m}]^2 - [0.0125\text{ m}]^2)(1.50\text{ m}) = 7.07\times10^{-4}\text{ m}^3.$

$m = \rho V = (8.9\times10^3\text{ kg/m}^3)(7.07\times10^{-4}\text{ m}^3) = 6.29\text{ kg}.$ $w = mg = 61.6\text{ N}.$

EVALUATE: The pipe weights about 14 pounds.

12.8. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh.$

SET UP: Ocean water is seawater and has a density of $1.03\times10^3\text{ kg/m}^3$.

EXECUTE: $p - p_0 = (1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(3200\text{ m}) = 3.23\times10^7\text{ Pa}.$

$p - p_0 = (3.23\times10^7\text{ Pa})\left(\dfrac{1\text{ atm}}{1.013\times10^5\text{ Pa}}\right) = 319\text{ atm}.$

EVALUATE: The gauge pressure is about 320 times the atmospheric pressure at the surface.

12.9. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh.$

SET UP: Freshwater has density $1.00\times10^3\text{ kg/m}^3$ and seawater has density $1.03\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^3\text{ kg/m}^3)(3.71\text{ m/s}^2)(500\text{ m}) = 1.86\times10^6\text{ Pa}.$

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\text{ Pa}}{(1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 184\text{ m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

12.10. **IDENTIFY:** The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^3\ \text{kg/m}^3$. $y_1 - y_2 = 1.65\ \text{m}$. The surface area of the segment is πDL, where $D = 1.50\times10^{-3}\ \text{m}$ and $L = 2.00\times10^{-2}\ \text{m}$.

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^4\ \text{Pa}$.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2$.

$\Delta F_\perp = (1.71\times10^4\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61\ \text{N}$.

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm.

12.11. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980\ \text{Pa}$ and

$$h = \frac{5980\ \text{Pa}}{\rho h} = \frac{5980\ \text{N/m}^2}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.581\ \text{m}.$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

12.12. **IDENTIFY:** $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth h below the surface.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.120\ \text{m}) = 706\ \text{Pa}$

(b) For the water layer, $p_{\text{surface}} = 706\ \text{Pa} + p_{\text{atm}}$.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706\ \text{Pa} + \rho gh = 706\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 3.16\times10^3\ \text{Pa}$

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer.

12.13. **IDENTIFY:** There will be a difference in blood pressure between your head and feet due to the depth of the blood.

SET UP: The added pressure is equal to ρgh.

EXECUTE: **(a)** $\rho gh = (1060\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.85\ \text{m}) = 1.92\times10^4\ \text{Pa}$.

(b) This additional pressure causes additional outward force on the walls of the blood vessels in your brain.

EVALUATE: The pressure difference is about 1/5 atm, so it would be noticeable.

12.14. **IDENTIFY** and **SET UP:** Use Eq. (12.8) to calculate the gauge pressure at this depth. Use Eq. (12.3) to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 12.1 the density of seawater is $1.03\times10^3\ \text{kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m}) = 2.52\times10^6\ \text{Pa}$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 12.14.

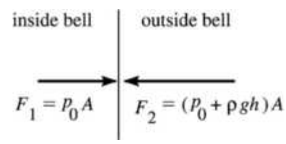


Figure 12.14

The net force is

$$F_2 - F_1 = (p_0 + \rho g h)A - p_0 A = (\rho g h)A$$
$$F_2 - F_1 = (2.52\times10^6\ \text{Pa})\pi(0.150\ \text{m})^2$$
$$F_2 - F_1 = 1.78\times10^5\ \text{N}$$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

12.15. **IDENTIFY:** The external pressure on the eardrum increases with depth in the ocean. This increased pressure could damage the eardrum.

SET UP: The density of seawater is $1.03\times10^3\ \text{kg/m}^3$. The area of the eardrum is $A = \pi r^2$, with $r = 4.1$ mm. The pressure increase with depth is $\Delta p = \rho g h$ and $F = pA$.

EXECUTE: $\Delta F = (\Delta p)A = \rho g h A$. Solving for h gives

$$h = \frac{\Delta F}{\rho g A} = \frac{1.5\ \text{N}}{(1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\pi(4.1\times10^{-3}\ \text{m})^2} = 2.8\ \text{m}.$$

EVALUATE: 2.8 m is less than 10 ft, so it is probably a good idea to wear ear plugs if you scuba dive.

12.16. **IDENTIFY** and **SET UP:** Use Eq. (12.6) to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_a = 980\ \text{millibar} = 9.80\times10^4\ \text{Pa}$

(a) Apply $p = p_0 + \rho g h$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_a$. The density of the liquid (mercury) is $13.6\times10^3\ \text{kg/m}^3$.

Thus $p = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0700\ \text{m}) = 1.07\times10^5\ \text{Pa}$.

(b) $p = p_0 + \rho g h = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 1.03\times10^5\ \text{Pa}$.

(c) Since $y_2 - y_1 = 4.00$ cm the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is 1.03×10^5 Pa.

(d) $p - p_0 = \rho g h = (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 5.33\times10^3\ \text{Pa}$.

EVALUATE: If Eq. (12.8) is evaluated with the density of mercury and $p - p_a = 1\ \text{atm} = 1.01\times10^5$ Pa, then $h = 76$cm. The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than 1.0×10^5 Pa.

12.17. **IDENTIFY:** Apply $p = p_0 + \rho g h$.

SET UP: For water, $\rho = 1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: $p - p_{\text{air}} = \rho g h = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(6.1\ \text{m}) = 6.0\times10^4\ \text{Pa}$.

EVALUATE: The pressure difference increases linearly with depth.

12.18. **IDENTIFY** and **SET UP:** Apply Eq. (12.6) to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_m g h_m$. With the water to a depth h_w, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_m g h_m + \rho_w g h_w$. If this is to be double the first value, then $\rho_w g h_w = \rho_m g h_m$.

$$h_w = h_m(\rho_m/\rho_w) = (0.0500\ \text{m})(13.6\times10^3/1.00\times10^3) = 0.680\ \text{m}$$

The volume of water is $V = hA = (0.680\ \text{m})(12.0\times10^{-4}\ \text{m}^2) = 8.16\times10^{-4}\ \text{m}^3 = 816\ \text{cm}^3$

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(5 \text{ cm}) = 68 \text{ cm}$, so the pressure increase from the top to the bottom of a 68-cm tall column of water is the same as the pressure increase from top to bottom for a 5-cm tall column of mercury.

12.19. **IDENTIFY:** $p = p_0 + \rho gh$. $F = pA$.

SET UP: For seawater, $\rho = 1.03\times10^3 \text{ kg/m}^3$

EXECUTE: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho gh)A - w = (1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(30 \text{ m})(0.75 \text{ m}^2) - 300 \text{ N} = 2.27\times10^5 \text{ N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

12.20. **IDENTIFY:** Apply $p = p_0 + \rho gh$, where p_0 is the pressure at the surface of the fluid. Gauge pressure is $p - p_{\text{air}}$.

SET UP: For water, $\rho = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: **(a)** The pressure difference between the surface of the water and the bottom is due to the weight of the water and is still 2500 Pa after the pressure increase above the surface. But the surface pressure increase is also transmitted to the fluid, making the total difference from atmospheric pressure $2500 \text{ Pa} + 1500 \text{ Pa} = 4000 \text{ Pa}$.

(b) Initially, the pressure due to the water alone is $2500 \text{ Pa} = \rho gh$. Thus

$h = \dfrac{2500 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.255 \text{ m}$. To keep the bottom gauge pressure at 2500 Pa after the 1500 Pa increase at the surface, the pressure due to the water's weight must be reduced to 1000 Pa:

$h = \dfrac{1000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.102 \text{ m}$. Thus the water must be lowered by $0.255 \text{ m} - 0.102 \text{ m} = 0.153 \text{ m}$.

EVALUATE: Note that ρgh, with $h = 0.153 \text{ m}$, is 1500 Pa.

12.21. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150 \text{ m})^2 = 0.0707 \text{ m}^2$.

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0 \text{ N}}{0.0707 \text{ m}^2} = 636 \text{ Pa}$.

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0 \text{ N}}{0.0707 \text{ m}^2} = 1170 \text{ Pa}$. (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

12.22. **IDENTIFY:** The force on an area A due to pressure p is $F_\perp = pA$. Use $p - p_0 = \rho gh$ to find the pressure inside the tank, at the bottom.

SET UP: $1 \text{ atm} = 1.013\times10^5 \text{ Pa}$. For benzene, $\rho = 0.90\times10^3 \text{ kg/m}^3$. The area of the bottom of the tank is $\pi D^2/4$, where $D = 1.72 \text{ m}$. The area of the vertical walls of the tank is πDL, where $L = 11.50 \text{ m}$.

EXECUTE: **(a)** At the bottom of the tank,

$p = p_0 + \rho gh = 92(1.013\times10^5 \text{ Pa}) + (0.90\times10^3 \text{ kg/m}^3)(0.894)(9.80 \text{ m/s}^2)(11.50 \text{ m})$.

$p = 9.32\times10^6 \text{ Pa} + 9.07\times10^4 \text{ Pa} = 9.41\times10^6 \text{ Pa}$. $F_\perp = pA = (9.41\times10^6 \text{ Pa})\pi(1.72 \text{ m})^2/4 = 2.19\times10^7 \text{ N}$.

(b) At the outside surface of the bottom of the tank, the air pressure is $p = (92)(1.013\times10^5\ \text{Pa}) = 9.32\times10^6\ \text{Pa}$. $F_\perp = pA = (9.32\times10^6\ \text{Pa})\pi(1.72\ \text{m})^2/4 = 2.17\times10^7\ \text{N}$.

(c) $F_\perp = pA = 92(1.013\times10^5\ \text{Pa})\pi(1.72\ \text{m})(11.5\ \text{m}) = 5.79\times10^8\ \text{N}$

EVALUATE: Most of the force in part (a) is due to the 92 atm of air pressure above the surface of the benzene and the net force on the bottom of the tank is much less than the inward and outward forces.

12.23. **IDENTIFY:** $F_2 = \frac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 of the diameter at the car.

EXECUTE: $mg = \frac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\frac{D_2}{D_1} = \sqrt{\frac{mg}{F_1}} = \sqrt{\frac{(1520\ \text{kg})(9.80\ \text{m/s}^2)}{125\ \text{N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

12.24. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the piston, with $+y$ upward. $F = pA$.

SET UP: $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$. The force diagram for the piston is given in Figure 12.24. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and

$$p - p_{\text{atm}} = p_{\text{gauge}} = \frac{w}{A} = \frac{mg}{\pi r^2} = \frac{(1200\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(0.15\ \text{m})^2} = 1.7\times10^5\ \text{Pa} = 1.7\ \text{atm}$$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

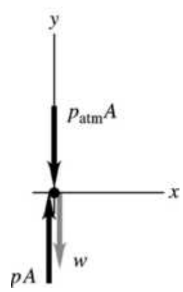


Figure 12.24

12.25. **IDENTIFY:** By Archimedes's principle, the additional buoyant force will be equal to the additional weight (the man).

SET UP: $V = \frac{m}{\rho}$ where $dA = V$ and d is the additional distance the buoy will sink.

EXECUTE: With man on buoy must displace additional 70.0 kg of water.

$$V = \frac{m}{\rho} = \frac{70.0\ \text{kg}}{1030\ \text{kg/m}^3} = 0.06796\ \text{m}^3.\quad dA = V \text{ so } d = \frac{V}{A} = \frac{0.06796\ \text{m}^3}{\pi(0.450\ \text{m})^2} = 0.107\ \text{m}.$$

EVALUATE: We do not need to use the mass of the buoy because it is already floating and hence in balance.

12.26. **IDENTIFY:** Apply Newton's second law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}}V_{\text{displ}}g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 12.26.

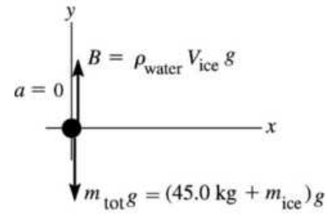


Figure 12.26

EXECUTE: $\Sigma F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{water}}V_{\text{ice}}g = (45.0 \text{ kg} + m_{\text{ice}})g$

But $\rho = m/V$ so $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}}$

$$V_{\text{ice}} = \frac{45.0 \text{ kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \frac{45.0 \text{ kg}}{1000 \text{ kg/m}^3 - 920 \text{ kg/m}^3} = 0.562 \text{ m}^3.$$

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}} = 517 \text{ kg}.$

12.27. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}}V_{\text{obj}}g.$

SET UP: $w = mg = 17.50 \text{ N}$ and $m = 1.79 \text{ kg}.$

EXECUTE: $T + B - mg = 0.$ $B = mg - T = 17.50 \text{ N} - 11.20 \text{ N} = 6.30 \text{ N}.$

$$V_{\text{obj}} = \frac{B}{\rho_{\text{water}}g} = \frac{6.30 \text{ N}}{(1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 6.43\times10^{-4} \text{ m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79 \text{ kg}}{6.43\times10^{-4} \text{ m}^3} = 2.78\times10^3 \text{ kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

12.28. **IDENTIFY:** The upward buoyant force *B* exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{\text{gly}} = 1.26\times10^3 \text{ kg/m}^3$ and seawater has density $\rho_{\text{sw}} = 1.03\times10^3 \text{ kg/m}^3.$ Let V_{obj} be the volume of the apparatus. $g_{\text{E}} = 9.80 \text{ m/s}^2;$ $g_{\text{C}} = 4.15 \text{ m/s}^2.$ Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{\text{sw}}(0.250V_{\text{obj}})g_{\text{E}} = mg_{\text{E}}.$ $m = (0.250)\rho_{\text{sw}}V_{\text{obj}}.$ On Caasi, $B = \rho_{\text{gly}}V_{\text{sub}}g_{\text{C}} = mg_{\text{C}}.$ $m = \rho_{\text{gyl}}V_{\text{sub}}.$ The two expressions for *m* must be equal, so

$$(0.250)V_{\text{obj}}\rho_{\text{sw}} = \rho_{\text{gly}}V_{\text{sub}} \text{ and } V_{\text{sub}} = \left(\frac{0.250\rho_{\text{sw}}}{\rho_{\text{gly}}}\right)V_{\text{obj}} = \left(\frac{[0.250][1.03\times10^3 \text{ kg/m}^3]}{1.26\times10^3 \text{ kg/m}^3}\right)V_{\text{obj}} = 0.204V_{\text{obj}}.$$

20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of *g* on each planet cancels out and has no effect on the answer. The value of *g* changes the weight of the apparatus and the buoyant force by the same factor.

12.29. **IDENTIFY:** For a floating object, the weight of the object equals the upward buoyancy force, *B*, exerted by the fluid.

SET UP: $B = \rho_{\text{fluid}}V_{\text{submerged}}g.$ The weight of the object can be written as $w = \rho_{\text{object}}V_{\text{object}}g.$ For seawater, $\rho = 1.03\times10^3 \text{ kg/m}^3.$

EXECUTE: **(a)** The displaced fluid has less volume than the object but must weigh the same, so $\rho < \rho_{\text{fluid}}.$

(b) If the ship does not leak, much of the water will be displaced by air or cargo, and the average density of the floating ship is less than that of water.

(c) Let the portion submerged have volume *V*, and the total volume be $V_0.$ Then $\rho V_0 = \rho_{\text{fluid}}V,$ so $\dfrac{V}{V_0} = \dfrac{\rho}{\rho_{\text{fluid}}}.$ The fraction above the fluid surface is then $1 - \dfrac{\rho}{\rho_{\text{fluid}}}.$ If $\rho \to 0,$ the entire object floats, and if $\rho \to \rho_{\text{fluid}},$ none of the object is above the surface.

(d) Using the result of part (c), $1-\dfrac{\rho}{\rho_{\text{fluid}}}=1-\dfrac{(0.042\text{ kg})/([5.0][4.0][3.0]\times10^{-6}\text{m}^3)}{1030\text{kg/m}^3}=0.32=32\%.$

EVALUATE: For a given object, the fraction of the object above the surface increases when the density of the fluid in which it floats increases.

12.30. **IDENTIFY:** $B=\rho_{\text{water}}V_{\text{obj}}g$. The net force on the sphere is zero.

SET UP: The density of water is $1.00\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $B=(1000\text{ kg/m}^3)(0.650\text{ m}^3)(9.80\text{ m/s}^2)=6.37\times10^3\text{ N}$

(b) $B=T+mg$ and $m=\dfrac{B-T}{g}=\dfrac{6.37\times10^3\text{ N}-900\text{ N}}{9.80\text{ m/s}^2}=558\text{ kg}.$

(c) Now $B=\rho_{\text{water}}V_{\text{sub}}g$, where V_{sub} is the volume of the sphere that is submerged. $B=mg$.

$\rho_{\text{water}}V_{\text{sub}}g=mg$ and $V_{\text{sub}}=\dfrac{m}{\rho_{\text{water}}}=\dfrac{558\text{ kg}}{1000\text{ kg/m}^3}=0.558\text{ m}^3$. $\dfrac{V_{\text{sub}}}{V_{\text{obj}}}=\dfrac{0.558\text{ m}^3}{0.650\text{ m}^3}=0.858=85.8\%.$

EVALUATE: The average density of the sphere is $\rho_{\text{sph}}=\dfrac{m}{V}=\dfrac{558\text{ kg}}{0.650\text{ m}^3}=858\text{ kg/m}^3$. $\rho_{\text{sph}}<\rho_{\text{water}}$, and that is why it floats with 85.8% of its volume submerged.

12.31. **IDENTIFY** and **SET UP:** Use Eq. (12.8) to calculate the gauge pressure at the two depths.
(a) The distances are shown in Figure 12.31a.

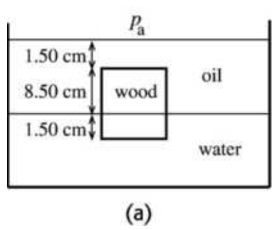


Figure 12.31a

EXECUTE: $p-p_0=\rho gh$

The upper face is 1.50 cm below the top of the oil, so

$p-p_0=(790\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.0150\text{ m})$

$p-p_0=116\text{ Pa}$

(b) The pressure at the interface is $p_{\text{interface}}=p_{\text{a}}+\rho_{\text{oil}}g(0.100\text{ m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p=p_{\text{interface}}+\rho_{\text{water}}g(0.0150\text{ m})$. Combining these two equations gives

$p-p_{\text{a}}=\rho_{\text{oil}}g(0.100\text{ m})+\rho_{\text{water}}g(0.0150\text{ m})$

$p-p_{\text{a}}=[(790\text{ kg/m}^3)(0.100\text{ m})+(1000\text{ kg/m}^3)(0.0150\text{ m})](9.80\text{ m/s}^2)$

$p-p_{\text{a}}=921\text{ Pa}$

(c) IDENTIFY and **SET UP:** Consider the forces on the block. The area of each face of the block is $A=(0.100\text{ m})^2=0.0100\text{ m}^2$. Let the absolute pressure at the top face be p_{t} and the pressure at the bottom face be p_{b}. In Eq. (12.3) use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 12.31b.

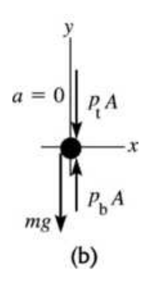


Figure 12.31b

EXECUTE: $\Sigma F_y=ma_y$

$p_{\text{b}}A-p_{\text{t}}A-mg=0$

$(p_{\text{b}}-p_{\text{t}})A=mg$

Note that $(p_b - p_t) = (p_b - p_a) - (p_t - p_a) = 921\ \text{Pa} - 116\ \text{Pa} = 805\ \text{Pa};$ the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_b - p_t)A}{g} = \frac{(805\ \text{Pa})(0.0100\ \text{m}^2)}{9.80\ \text{m/s}^2} = 0.821\ \text{kg}.$$

And then $\rho = m/V = 0.821\ \text{kg}/(0.100\ \text{m})^3 = 821\ \text{kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}})g$ where $V_{\text{oil}} = (0.0100\ \text{m}^2)(0.0850\ \text{m}) = 8.50 \times 10^{-4}\ \text{m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100\ \text{m}^2)(0.0150\ \text{m}) = 1.50 \times 10^{-4}\ \text{m}^3$ is the volume of water displaced by the block. This gives $B = (0.821\ \text{kg})g$. The mass of water displaced equals the mass of the block.

12.32. **IDENTIFY:** The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7 \times 10^3\ \text{kg/m}^3$. The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7 \times 10^3\ \text{kg/m}^3} = 3.36 \times 10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00 \times 10^3\ \text{kg/m}^3)(3.36 \times 10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$.

$T = mg - B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

12.33. **IDENTIFY:** The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\ \text{N} - 28.4\ \text{N} = 10.8\ \text{N}$. The mass of this much water is thus $10.8\ \text{N}/(9.80\ \text{m/s}^2) = 1.102\ \text{kg}$ and its volume, equal to the rock's volume, is $\dfrac{1.102\ \text{kg}}{1.00 \times 10^3\ \text{kg/m}^3} = 1.102 \times 10^{-3}\ \text{m}^3$. The weight of unknown liquid displaced is $39.2\ \text{N} - 18.6\ \text{N} = 20.6\ \text{N}$, and its mass is $20.6\ \text{N}/(9.80\ \text{m/s}^2) = 2.102\ \text{kg}$. The liquid's density is thus $2.102\ \text{kg}/(1.102 \times 10^{-3}\ \text{m}^3) = 1.91 \times 10^3\ \text{kg/m}^3$.

EVALUATE: The density of the unknown liquid is roughly twice the density of water.

12.34. **IDENTIFY:** The volume flow rate is Av.

SET UP: $Av = 0.750\ \text{m}^3/\text{s}$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\ \text{m}^3/\text{s}$. $v = \dfrac{4(0.750\ \text{m}^3/\text{s})}{\pi(4.50 \times 10^{-2}\ \text{m})^2} = 472\ \text{m/s}$.

(b) vD^2 must be constant, so $v_1 D_1^2 = v_2 D_2^2$. $v_2 = v_1 \left(\dfrac{D_1}{D_2}\right)^2 = (472\ \text{m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\ \text{m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

12.35. **IDENTIFY:** Apply the equation of continuity, $v_1 A_1 = v_2 A_2$.

SET UP: $A = \pi r^2$

EXECUTE: $v_2 = v_1(A_1/A_2)$. $A_1 = \pi(0.80\ \text{cm})^2$, $A_2 = 20\pi(0.10\ \text{cm})^2$. $v_2 = (3.0\ \text{m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\ \text{m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross-sectional area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

12.36. **IDENTIFY:** $v_1A_1 = v_2A_2$. The volume flow rate is vA.

SET UP: $1.00 \text{ h} = 3600 \text{ s}$.

EXECUTE: **(a)** $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (3.50 \text{ m/s})\left(\frac{0.070 \text{ m}^2}{0.105 \text{ m}^2}\right) = 2.33 \text{ m/s}$

(b) $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (3.50 \text{ m/s})\left(\frac{0.070 \text{ m}^2}{0.047 \text{ m}^2}\right) = 5.21 \text{ m/s}$

(c) $V = v_1A_1t = (3.50 \text{ m/s})(0.070 \text{ m}^2)(3600 \text{ s}) = 882 \text{ m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.37. **IDENTIFY** and **SET UP:** Apply Eq. (12.10). In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20 \text{ m}^3/\text{s}$

$$v = \frac{1.20 \text{ m}^3/\text{s}}{A} = \frac{1.20 \text{ m}^3/\text{s}}{\pi r^2} = \frac{1.20 \text{ m}^3/\text{s}}{\pi(0.150 \text{ m})^2} = 17.0 \text{ m/s}$$

(b) $vA = 1.20 \text{ m}^3/\text{s}$

$v\pi r^2 = 1.20 \text{ m}^3/\text{s}$

$$r = \sqrt{\frac{1.20 \text{ m}^3/\text{s}}{v\pi}} = \sqrt{\frac{1.20 \text{ m}^3/\text{s}}{(3.80 \text{ m/s})\pi}} = 0.317 \text{ m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

12.38. **IDENTIFY:** Narrowing the width of the pipe will increase the speed of flow of the fluid.

SET UP: The continuity equation is $A_1v_1 = A_2v_2$. $A = \frac{1}{2}\pi d^2$, where d is the pipe diameter.

EXECUTE: The continuity equation gives $\frac{1}{2}\pi d_1^2 v_1 = \frac{1}{2}\pi d_2^2 v_2$, so

$$v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{2.50 \text{ in.}}{1.00 \text{ in.}}\right)^2 (6.00 \text{ cm/s}) = 37.5 \text{ cm/s}$$

EVALUATE: To achieve the same volume flow rate the water flows faster in the smaller diameter pipe. Note that the pipe diameters entered in a ratio so there was no need to convert units.

12.39. **IDENTIFY:** A change in the speed of the water indicates that the cross-sectional area of the canal must have changed.

SET UP: The continuity equation is $A_1v_1 = A_2v_2$.

EXECUTE: If h is the depth of the canal, then $(18.5 \text{ m})(3.75 \text{ m})(2.50 \text{ cm/s}) = (16.5 \text{ m})h(11.0 \text{ cm/s})$ so $h = 0.956 \text{ m}$, the depth of the canal at the second point.

EVALUATE: The speed of the water has increased, so the cross-sectional area must have decreased, which is consistent with our result for h.

12.40. **IDENTIFY:** A change in the speed of the blood indicates that there is a difference in the cross-sectional area of the artery. Bernoulli's equation applies to the fluid.

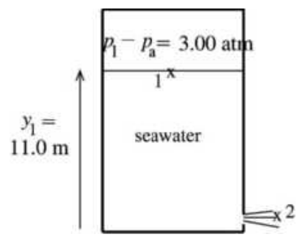
SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. The two points are close together so we can neglect $\rho g(y_1 - y_2)$. $\rho = 1.06 \times 10^3 \text{ kg/m}^3$. The continuity equation is $A_1v_1 = A_2v_2$.

EXECUTE: Solve $p_1 - p_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$ for v_2:

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho} + v_1^2} = \sqrt{\frac{2(1.20 \times 10^4 \text{ Pa} - 1.15 \times 10^4 \text{ Pa})}{1.06 \times 10^3 \text{ kg/m}^3} + (0.300 \text{ m/s})^2}.$$ $v_2 = 1.0 \text{ m/s} = 100 \text{ cm/s}$. The continuity equation gives $\frac{A_2}{A_1} = \frac{v_1}{v_2} = \frac{30 \text{ cm/s}}{100 \text{ cm/s}} = 0.30$. $A_2 = 0.30A_1$, so 70% of the artery is blocked.

EVALUATE: A 70% blockage reduces the blood speed from 100 cm/s to 30 cm/s, which should easily be detectable.

12.41. **IDENTIFY** and **SET UP:**



Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 12.41. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0$ m and $y_2 = 0$. The target variable is v_2.

Figure 12.41

$$p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

$A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-sectional area of the tank (A_1) is much larger than the cross-sectional area of the hole (A_2), so $v_1 << v_2$ and the $\tfrac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\tfrac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times10^5 \text{ Pa})}{1030 \text{ kg/m}^3} + 2(9.80 \text{ m/s}^2)(11.0 \text{ m})}$$

$v_2 = 28.4$ m/s

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example: 12.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7$ m/s. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

12.42. **IDENTIFY:** Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

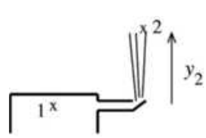
SET UP: $A = \pi D^2/4$, with $D = 6.00\times10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80 \text{ m/s}^2)(14.0 \text{ m})} = 16.6$ m/s

(b) $vA = (16.6 \text{ m/s})\pi(6.00\times10^{-3} \text{ m})^2/4 = 4.69\times10^{-4} \text{ m}^3\text{/s}$. A volume of $4.69\times10^{-4} \text{ m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

12.43. **IDENTIFY** and **SET UP:**



Apply Bernoulli's equation to points 1 and 2 as shown in Figure 12.43. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Figure 12.43

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(15.0 \text{ m}) = 1.47\times10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

12.44. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1A_1 = v_2A_2$. In Eq. (12.17) either absolute or gauge pressures can be used at both points.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00\times10^4 \text{ Pa} + (1.00\times10^3 \text{ kg/m}^3)\left(\frac{15}{32}(3.00 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(11.0 \text{ m})\right) = 1.62\times10^5 \text{ Pa.}$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

12.45. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: $y_1 = y_2$. $v_1A_1 = v_2A_2$. $A_2 = 2A_1$.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_2 = v_1\left(\frac{A_1}{A_2}\right) = (2.50 \text{ m/s})\left(\frac{A_1}{2A_1}\right) = 1.25 \text{ m/s.}$

$$p_2 = p_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) = 1.80\times10^4 \text{ Pa} + \tfrac{1}{2}(1000 \text{ kg/m}^3)([2.50 \text{ m/s}]^2 - [1.25 \text{ m/s}]^2) = 2.03\times10^4 \text{ Pa}$$

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

12.46. **IDENTIFY:** $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\dfrac{(220)(0.355 \text{ kg})}{60.0 \text{ s}} = 1.30 \text{ kg/s.}$

(b) The density of the liquid is $\dfrac{0.355 \text{ kg}}{0.355\times10^{-3} \text{ m}^3} = 1000 \text{ kg/m}^3$, and so the volume flow rate is

$\dfrac{1.30 \text{ kg/s}}{1000 \text{ kg/m}^3} = 1.30\times10^{-3} \text{ m}^3\text{/s} = 1.30 \text{ L/s.}$ This result may also be obtained from

$\dfrac{(220)(0.355 \text{ L})}{60.0 \text{ s}} = 1.30 \text{ L/s.}$

(c) $v_1 = \dfrac{1.30\times10^{-3} \text{ m}^3\text{/s}}{2.00\times10^{-4} \text{m}^2} = 6.50 \text{ m/s.}$ $v_2 = v_1/4 = 1.63 \text{ m/s.}$

(d) $p_1 = p_2 + \dfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1).$

$p_1 = 152 \text{ kPa} + (1000 \text{ kg/m}^3)\left(\frac{1}{2}[(1.63 \text{ m/s})^2 - (6.50 \text{ m/s})^2] + (9.80 \text{ m/s}^2)(-1.35 \text{ m})\right)$. $p_1 = 119 \text{ kPa.}$

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

12.47. **IDENTIFY** and **SET UP:** Let point 1 be where $r_1 = 4.00 \text{ cm}$ and point 2 be where $r_2 = 2.00 \text{ cm}$. The volume flow rate vA has the value $7200 \text{ cm}^3\text{/s}$ at all points in the pipe. Apply Eq. (12.10) to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1A_1 = v_1\pi r_1^2 = 7200 \text{ cm}^3$, so $v_1 = 1.43 \text{ m/s}$

$v_2A_2 = v_2\pi r_2^2 = 7200 \text{ cm}^3\text{/s}$, so $v_2 = 5.73 \text{ m/s}$

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$y_1 = y_2$ and $p_1 = 2.40\times10^5 \text{ Pa}$, so $p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.25\times10^5 \text{ Pa}$

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

12.48. **IDENTIFY:** Since a pressure difference is needed to keep the fluid flowing, there must be viscosity in the fluid.

SET UP: From Section 12.6, the pressure difference Δp over a length L of cylindrical pipe of radius R is proportional to L/R^4. In this problem, the length L is the same in both cases, so $R^4\,\Delta p$ must be constant. The target variable is the pressure difference.

EXECUTE: Since $R^4\,\Delta p$ is constant, we have $\Delta p_1 R_1^4 = \Delta p_2 R_2^4$.

$$\Delta p_2 = \Delta p_1\left(\frac{R_1}{R_2}\right)^4 = (6.00\times 10^4\ \text{Pa})\left(\frac{0.21\ \text{m}}{0.0700\ \text{m}}\right)^4 = 4.86\times 10^6\ \text{Pa}.$$

EVALUATE: The pipe is narrower, so the pressure difference must be greater.

12.49. **IDENTIFY:** Increasing the cross-sectional area of the artery will increase the amount of blood that flows through it per second.

SET UP: The flow rate, $\dfrac{\Delta V}{\Delta t}$, is related to the radius R or diameter D of the artery by Poiseuille's law: $\dfrac{\Delta V}{\Delta t} = \dfrac{\pi R^4}{8\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \dfrac{\pi D^4}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right)$. Assume the pressure gradient $(p_1 - p_2)/L$ in the artery remains the same.

EXECUTE: $(\Delta V/\Delta t)/D^4 = \dfrac{\pi}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \text{constant}$, so $(\Delta V/\Delta t)_{\text{old}}/D_{\text{old}}{}^4 = (\Delta V/\Delta t)_{\text{new}}/D_{\text{new}}{}^4$.

$(\Delta V/\Delta t)_{\text{new}} = 2(\Delta V/\Delta t)_{\text{old}}$ and $D_{\text{old}} = D$. This gives $D_{\text{new}} = D_{\text{old}}\left[\dfrac{(\Delta V/\Delta t)_{\text{new}}}{(\Delta V/\Delta t)_{\text{old}}}\right]^{1/4} = 2^{1/4}D = 1.19D$.

EVALUATE: Since the flow rate is proportional to D^4, a 19% increase in D doubles the flow rate.

12.50. **IDENTIFY:** Apply $p = p_0 + \rho g h$ and $\Delta V = -\dfrac{(\Delta p)V_0}{B}$, where B is the bulk modulus.

SET UP: Seawater has density $\rho = 1.03\times 10^3\ \text{kg/m}^3$. The bulk modulus of water is $B = 2.2\times 10^9$ Pa. $p_{\text{air}} = 1.01\times 10^5$ Pa.

EXECUTE:

(a) $p_0 = p_{\text{air}} + \rho g h = 1.01\times 10^5\ \text{Pa} + (1.03\times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10.92\times 10^3\ \text{m}) = 1.10\times 10^8$ Pa

(b) At the surface $1.00\ \text{m}^3$ of seawater has mass 1.03×10^3 kg. At a depth of 10.92 km the change in volume is $\Delta V = -\dfrac{(\Delta p)V_0}{B} - \dfrac{(1.10\times 10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times 10^9\ \text{Pa}} = -0.050\ \text{m}^3$. The volume of this mass of water at this depth therefore is $V = V_0 + \Delta V = 0.950\ \text{m}^3$. $\rho = \dfrac{m}{V} = \dfrac{1.03\times 10^3\ \text{kg}}{0.950\ \text{m}^3} = 1.08\times 10^3\ \text{kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

12.51. **IDENTIFY:** $F = pA$, where A is the cross-sectional area presented by a hemisphere. The force F_{bb} that the body builder must apply must equal in magnitude the net force on each hemisphere due to the air inside and outside the sphere.

SET UP: $A = \pi\frac{D^2}{4}$.

EXECUTE: **(a)** $F_{\text{bb}} = (p_0 - p)\pi\frac{D^2}{4}$.

(b) The force on each hemisphere due to the atmosphere is $\pi(5.00\times 10^{-2}\ \text{m})^2(1.013\times 10^5\ \text{Pa/atm})(0.975\ \text{atm}) = 776$ N. The bodybuilder must exert this force on each hemisphere to pull them apart.

EVALUATE: The force is about 170 lbs, feasible only for a very strong person. The force required is proportional to the square of the diameter of the hemispheres.

12.52. **IDENTIFY:** As the fish inflates its swim bladder, it changes its volume and hence the volume of water it displaces. This in turn changes the buoyant force on it, by Archimedes's principle.

SET UP: The buoyant force exerted by the water is $F_B = \rho_w g V_{fish}$. When the fish is fully submerged the buoyant force on it must equal its weight.

EXECUTE: **(a)** The average density of the fish is very close to the density of water.

(b) Before inflation, $F_B = w = (2.75 \text{ kg})(9.80 \text{ m/s}^2) = 27.0 \text{ N}$. When the volume increases by a factor of 1.10, the buoyant force also increases by a factor of 1.10 and becomes $(1.10)(27.0 \text{ N}) = 29.7 \text{ N}$.

(c) The water exerts an upward force 29.7 N and gravity exerts a downward force of 27.0 N so there is a net upward force of 2.7 N; the fish moves upward.

EVALUATE: Normally the buoyant force on the fish is equal to its weight, but if the fish inflates itself, the buoyant force increases and the fish rises.

12.53. **IDENTIFY:** In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth h is ρgh. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00 \times 10^3 \text{ kg/m}^3$.

EXECUTE: **(a)** The weight of the water is

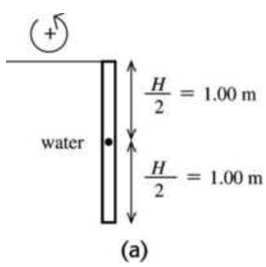
$\rho gV = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)((5.00 \text{ m})(4.0 \text{ m})(3.0 \text{ m})) = 5.9 \times 10^5 \text{ N}$.

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If d is the depth of the pool and A is the area of one end of the pool, then $F = \rho gA\dfrac{d}{2} = (1.00 \times 10^3 \text{kg/m}^3)(9.80 \text{ m/s}^2)((4.0 \text{ m})(3.0 \text{ m}))(1.50 \text{ m}) = 1.76 \times 10^5 \text{ N}$.

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho gd$ is the gauge pressure at the bottom of the pool and $A = (5.0 \text{ m})(4.0 \text{ m})$ is the area of the bottom of the pool.

12.54. **IDENTIFY:** Use Eq. (12.8) to find the gauge pressure versus depth, use Eq. (12.3) to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 12.54a.



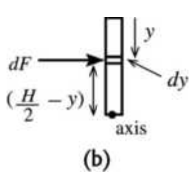
Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_l be the torque due to the force on the lower half.

Figure 12.54a

With the indicated sign convention, τ_l is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_l - \tau_u$. Let H be the height of the gate.

Upper-half of gate:

Calculate the torque due to the force on a narrow strip of height dy located a distance y below the top of the gate, as shown in Figure 12.54b. Then integrate to get the total torque.



The net force on the strip is $dF = p(y)\, dA$, where $p(y) = \rho gy$ is the pressure at this depth and $dA = W\, dy$ with $W = 4.00$ m.

$dF = \rho gyW\, dy$

Figure 12.54b

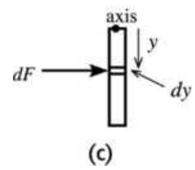
The moment arm is $(H/2-y)$, so $d\tau=\rho gW(H/2-y)y\,dy$.

$$\tau_{\text{u}}=\int_0^{H/2}d\tau=\rho gW\int_0^{H/2}(H/2-y)y\,dy=\rho gW((H/4)y^2-y^3/3)\Big|_0^{H/2}$$

$$\tau_{\text{u}}=\rho gW(H^3/16-H^3/24)=\rho gW(H^3/48)$$

$$\tau_{\text{u}}=(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})(2.00\text{ m})^3/48=6.533\times10^3\text{ N}\cdot\text{m}$$

<u>Lower-half of gate</u>:



Consider the narrow strip shown in Figure 12.54c. The depth of the strip is $(H/2+y)$ so the force dF is $dF=p(y)\,dA=\rho g(H/2+y)W\,dy$.

Figure 12.54c

The moment arm is y, so $d\tau=\rho gW(H/2+y)y\,dy$.

$$\tau_{\text{l}}=\int_0^{H/2}d\tau=\rho gW\int_0^{H/2}(H/2+y)y\,dy=\rho gW((H/4)y^2+y^3/3)\Big|_0^{H/2}$$

$$\tau_{\text{l}}=\rho gW(H^3/16+H^3/24)=\rho gW(5H^3/48)$$

$$\tau_{\text{l}}=(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})5(2.00\text{ m})^3/48=3.267\times10^4\text{ N}\cdot\text{m}$$

Then $\tau=\tau_{\text{l}}-\tau_{\text{u}}=3.267\times10^4\text{ N}\cdot\text{m}-6.533\times10^3\text{ N}\cdot\text{m}=2.61\times10^4\text{ N}\cdot\text{m}$.

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

12.55. **IDENTIFY:** Compute the force and the torque on a thin, horizontal strip at a depth h and integrate to find the total force and torque.

SET UP: The strip has an area $dA=(dh)L$, where dh is the height of the strip and L is its length. $A=HL$. The height of the strip about the bottom of the dam is $H-h$.

EXECUTE: **(a)** $dF=pdA=\rho ghLdh$. $F=\int_0^H dF=\rho gL\int_0^H hdh=\rho gLH^2/2=\rho gAH/2$.

(b) The torque about the bottom on a strip of vertical thickness dh is $d\tau=dF(H-h)=\rho gLh(H-h)dh$, and integrating from $h=0$ to $h=H$ gives $\tau=\rho gLH^3/6=\rho gAH^2/6$.

(c) The force depends on the width and on the square of the depth, and the torque about the bottom depends on the width and the cube of the depth; the surface area of the lake does not affect either result (for a given width).

EVALUATE: The force is equal to the average pressure, at depth $H/2$, times the area A of the vertical side of the dam that faces the lake. But the torque is not equal to $F(H/2)$, where $H/2$ is the moment arm for a force acting at the center of the dam.

12.56. **IDENTIFY:** The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B=\rho_{\text{air}}Vg$. Let g_{M} be the value of g for Mars. For a sphere $V=\frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A=4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B=mg_{\text{M}}$. $\rho_{\text{air}}Vg_{\text{M}}=mg_{\text{M}}$. $\rho_{\text{air}}\frac{4}{3}\pi R^3=(5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

$R=\dfrac{3(5.00\times10^{-3}\text{ kg/m}^2)}{\rho_{\text{air}}}=0.974\text{ m}$. $m=(5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)=0.0596\text{ kg}$.

(b) $F_{\text{net}}=B-mg=ma$. $B=\rho_{\text{air}}Vg=\rho_{\text{air}}\frac{4}{3}\pi R^3g=(1.20\text{ kg/m}^3)\left(\dfrac{4\pi}{3}\right)(0.974\text{ m})^3(9.80\text{ m/s}^2)=45.5\text{ N}$.

$$a = \frac{B - mg}{m} = \frac{45.5\ \text{N} - (0.0596\ \text{kg})(9.80\ \text{m/s}^2)}{0.0596\ \text{m}} = 754\ \text{m/s}^2,\ \text{upward.}$$

(c) $B = m_{\text{tot}}g.$ $\rho_{\text{air}}Vg = (m_{\text{balloon}} + m_{\text{load}})g.$ $m_{\text{load}} = \rho_{\text{air}}\frac{4}{3}\pi R^3 - (5.00\times10^{-3}\ \text{kg/m}^2)4\pi R^2.$

$$m_{\text{load}} = (0.0154\ \text{kg/m}^3)\left(\frac{4\pi}{3}\right)(5[0.974\ \text{m}])^3 - (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi)(5[0.974\ \text{m}])^2$$

$$m_{\text{load}} = 7.45\ \text{kg} - 1.49\ \text{kg} = 5.96\ \text{kg}$$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of *g* for Mars.

12.57. **IDENTIFY:** The buoyant force on an object in a liquid is equal to the weight of the liquid it displaces.

SET UP: $V = \dfrac{m}{\rho}$.

EXECUTE: When it is floating, the ice displaces an amount of glycerin equal to its weight. From Table 12.1, the density of glycerin is $1260\ \text{kg/m}^3$. The volume of this amount of glycerin is $V = \dfrac{m}{\rho} = \dfrac{0.180\ \text{kg}}{1260\ \text{kg/m}^3} = 1.429\times10^{-4}\ \text{m}^3$. The ice cube produces 0.180 kg of water. The volume of this mass of water is $V = \dfrac{m}{\rho} = \dfrac{0.180\ \text{kg}}{1000\ \text{kg/m}^3} = 1.80\times10^{-4}\ \text{m}^3$. The volume of water from the melted ice is greater than the volume of glycerin displaced by the floating cube and the level of liquid in the cylinder rises. The distance the level rises is $\dfrac{1.80\times10^{-4}\ \text{m}^3 - 1.429\times10^{-4}\ \text{m}^3}{\pi(0.0350\ \text{m})^2} = 9.64\times10^{-3}\ \text{m} = 0.964\ \text{cm}.$

EVALUATE: The melted ice has the same mass as the solid ice, but a different density.

12.58. **IDENTIFY:** The pressure must be the same at the bottom of the tube. Therefore since the liquids have different densities, they must have difference heights.

SET UP: After the barrier is removed the top of the water moves downward a distance *x* and the top of the oil moves up a distance *x*, as shown in Figure 12.58. After the heights have changed, the gauge pressure at the bottom of each of the tubes is the same. The gauge pressure *p* at a depth *h* is $p - p_{\text{atm}} = \rho gh$.

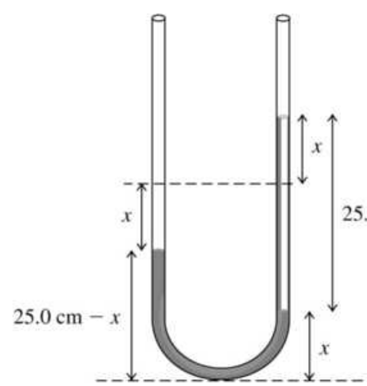


Figure 12.58

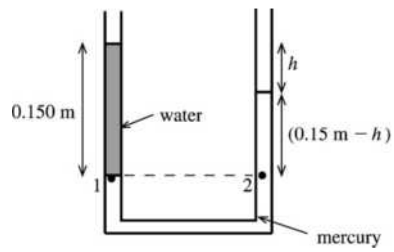
EXECUTE: The gauge pressure at the bottom of arm *A* of the tube is $p - p_{\text{atm}} = \rho_{\text{w}}g(25.0\ \text{cm} - x)$. The gauge pressure at the bottom of arm *B* of the tube is $p - p_{\text{atm}} = \rho_{\text{oil}}g(25.0\ \text{cm}) + \rho_{\text{w}}gx$. The gauge pressures must be equal, so $\rho_{\text{w}}g(25.0\ \text{cm} - x) = \rho_{\text{oil}}g(25.0\ \text{cm}) + \rho_{\text{w}}gx$. Dividing out *g* and using $\rho_{\text{oil}} = 0.80\rho_{\text{w}}$, we have $\rho_{\text{w}}(25.0\ \text{cm} - x) = 0.80\rho_{\text{w}}(25.0\ \text{cm}) + \rho_{\text{w}}x$. ρ_{w} divides out and leaves

$25.0 \text{ cm} - x = 20.0 \text{ cm} + x,$ so $x = 2.5$ cm. The height of fluid in arm A is $25.0 \text{ cm} - x = 22.5$ cm and the height in arm B is $25.0 \text{ cm} + x = 27.5$ cm.

(b) (i) If the densities were the same there would be no reason for a difference in height and the height would be 25.0 cm on each side. (ii) The pressure exerted by the column of oil would be very small and the water would divide equally on both sides. The height in arm A would be 12.5 cm and the height in arm B would be $25.0 \text{ cm} + 12.5 \text{ cm} = 37.5$ cm.

EVALUATE: The less dense fluid rises to a higher height, which is physically reasonable.

12.59. **(a) IDENTIFY** and **SET UP:**



Apply $p = p_0 + \rho g h$ to the water in the left-hand arm of the tube.
See Figure 12.59.

Figure 12.59

EXECUTE: $p_0 = p_a$, so the gauge pressure at the interface (point 1) is

$p - p_a = \rho g h = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 1470$ Pa.

(b) IDENTIFY and **SET UP:** The pressure at point 1 equals the pressure at point 2. Apply Eq. (12.6) to the right-hand arm of the tube and solve for h.

EXECUTE: $p_1 = p_a + \rho_w g(0.150 \text{ m})$ and $p_2 = p_a + \rho_{Hg} g(0.150 \text{ m} - h)$

$p_1 = p_2$ implies $\rho_w g(0.150 \text{ m}) = \rho_{Hg} g(0.150 \text{ m} - h)$

$$0.150 \text{ m} - h = \frac{\rho_w (0.150 \text{ m})}{\rho_{Hg}} = \frac{(1000 \text{ kg/m}^3)(0.150 \text{ m})}{13.6\times 10^3 \text{ kg/m}^3} = 0.011 \text{ m}$$

$h = 0.150 \text{ m} - 0.011 \text{ m} = 0.139 \text{ m} = 13.9$ cm

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

12.60. **IDENTIFY:** Follow the procedure outlined in the hint. $F = pA$.

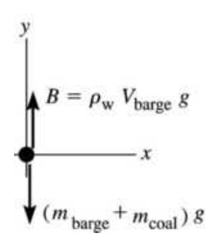
SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is $\rho g y$.

EXECUTE: $F = \int_0^h (\rho g y)(2\pi R)dy = \rho g \pi R h^2$ where R and h are the radius and height of the tank. Using the given numerical values gives $F = 2.11\times 10^8$ N.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi R h$, times the average pressure, the pressure $\rho g h/2$ at depth $h/2$.

12.61. **IDENTIFY:** Apply Newton's second law to the barge plus its contents. Apply Archimedes's principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 12.61.



EXECUTE: $\sum F_y = m a_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{barge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

Figure 12.61

$V_{\text{barge}} = (22\text{ m})(12\text{ m})(40\text{ m}) = 1.056\times10^4\text{ m}^3$

The mass of the barge is $m_{\text{barge}} = \rho_s V_s$, where s refers to steel.

From Table 12.1, $\rho_s = 7800\text{ kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge

$$V_s = (0.040\text{ m})[2(22\text{ m})(12\text{ m}) + 2(40\text{ m})(12\text{ m}) + (22\text{ m})(40\text{ m})] = 94.7\text{ m}^3.$$

Therefore, $m_{\text{barge}} = \rho_s V_s = (7800\text{ kg/m}^3)(94.7\text{ m}^3) = 7.39\times10^5\text{ kg}$.

Then $m_{\text{coal}} = \rho_w V_{\text{barge}} - m_{\text{barge}} = (1000\text{ kg/m}^3)(1.056\times10^4\text{ m}^3) - 7.39\times10^5\text{ kg} = 9.8\times10^6\text{ kg}$.

The volume of this mass of coal is $V_{\text{coal}} = m_{\text{coal}}/\rho_{\text{coal}} = 9.8\times10^6\text{ kg}/1500\text{ kg/m}^3 = 6500\text{ m}^3$; this is less than V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force *B* must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

12.62. **IDENTIFY:** The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{\text{gas}} = \rho_{\text{gas}} V$. $B = \rho_{\text{air}} V g$.

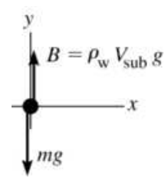
SET UP: The total weight, exclusive of the gas inside the balloon, is $900\text{ N} + 1700\text{ N} + 3200\text{ N} = 5800\text{ N}$.

EXECUTE: $5800\text{ N} + \rho_{\text{gas}} V g = \rho_{\text{air}} V g$ and $\rho_{\text{gas}} = 1.23\text{ kg/m}^3 - \dfrac{(5800\text{ N})}{(9.80\text{ m/s}^2)(2200\text{ m}^3)} = 0.96\text{ kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

12.63. **IDENTIFY:** Apply Newton's second law to the car. The buoyancy force is given by Archimedes's principle.

(a) SET UP: The free-body diagram for the floating car is given in Figure 12.63. (V_{sub} is the volume that is submerged.)



EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_w V_{\text{sub}} g - mg = 0$

Figure 12.63

$V_{\text{sub}} = m/\rho_w = (900\text{ kg})/(1000\text{ kg/m}^3) = 0.900\text{ m}^3$

$V_{\text{sub}}/V_{\text{obj}} = (0.900\text{ m}^3)/(3.0\text{ m}^3) = 0.30 = 30\%$

EVALUATE: The average density of the car is $(900\text{ kg})/(3.0\text{ m}^3) = 300\text{ kg/m}^3$. $\rho_{\text{car}}/\rho_{\text{water}} = 0.30$; this equals $V_{\text{sub}}/V_{\text{obj}}$.

(b) SET UP: When the car starts to sink it is fully submerged and the buoyant force is equal to the weight of the car plus the water that is inside it.

EXECUTE: When the car is fully submerged $V_{\text{sub}} = V$, the volume of the car, and

$B = \rho_{\text{water}} V g = (1000\text{ kg/m}^3)(3.0\text{ m}^3)(9.80\text{ m/s}^2) = 2.94\times10^4\text{ N}$.

The weight of the car is $mg = (900\text{ kg})(9.80\text{ m/s}^2) = 8820\text{ N}$.

Thus the weight of the water in the car when it sinks is the buoyant force minus the weight of the car itself:

$m_{\text{water}} = (2.94\times10^4\text{ N} - 8820\text{ N})/(9.80\text{ m/s}^2) = 2.10\times10^3\text{ kg}$

And $V_{\text{water}} = m_{\text{water}}/\rho_{\text{water}} = (2.10\times10^3\text{ kg})/(1000\text{ kg/m}^3) = 2.10\text{ m}^3$

The fraction this is of the total interior volume is $(2.10\text{ m}^3)/(3.00\text{ m}^3) = 0.70 = 70\%$.

EVALUATE: The average density of the car plus the water inside it is $(900\text{ kg}+2100\text{ kg})/(3.0\text{ m}^3)=1000\text{ kg/m}^3$, so $\rho_{\text{car}}=\rho_{\text{water}}$ when the car starts to sink.

12.64. **IDENTIFY:** For a floating object, the buoyant force equals the weight of the object. $B=\rho_{\text{fluid}}V_{\text{submerged}}g$.

SET UP: Water has density $\rho=1.00\text{ g/cm}^3$.

EXECUTE: **(a)** The volume displaced must be that which has the same weight and mass as the ice, $\dfrac{9.70\text{ gm}}{1.00\text{ gm/cm}^3}=9.70\text{ cm}^3$.

(b) No; when melted, the cube produces the same volume of water as was displaced by the floating cube, and the water level does not change.

(c) $\dfrac{9.70\text{ gm}}{1.05\text{ gm/cm}^3}=9.24\text{ cm}^3$

(d) The melted water takes up more volume than the salt water displaced, and so 0.46 cm^3 flows over.

EVALUATE: The volume of water from the melted cube is less than the volume of the ice cube, but the cube floats with only part of its volume submerged.

12.65. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B=\rho_{\text{water}}V_{\text{tot}}g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho=m/V$.

SET UP: The density of lead is $11.3\times10^3\text{ kg/m}^3$.

EXECUTE: $V_{\text{wood}}=(0.600\text{ m})(0.250\text{ m})(0.080\text{ m})=0.0120\text{ m}^3$.

$m_{\text{wood}}=\rho_{\text{wood}}V_{\text{wood}}=(700\text{ kg/m}^3)(0.0120\text{ m}^3)=8.40\text{ kg}$.

$B=(m_{\text{wood}}+m_{\text{lead}})g$. Using $B=\rho_{\text{water}}V_{\text{tot}}g$ and $V_{\text{tot}}=V_{\text{wood}}+V_{\text{lead}}$ gives

$\rho_{\text{water}}(V_{\text{wood}}+V_{\text{lead}})g=(m_{\text{wood}}+m_{\text{lead}})g$. $m_{\text{lead}}=\rho_{\text{lead}}V_{\text{lead}}$ then gives

$\rho_{\text{water}}V_{\text{wood}}+\rho_{\text{water}}V_{\text{lead}}=m_{\text{wood}}+\rho_{\text{lead}}V_{\text{lead}}$.

$$V_{\text{lead}}=\frac{\rho_{\text{water}}V_{\text{wood}}-m_{\text{wood}}}{\rho_{\text{lead}}-\rho_{\text{water}}}=\frac{(1000\text{ kg/m}^3)(0.0120\text{ m}^3)-8.40\text{ kg}}{11.3\times10^3\text{ kg/m}^3-1000\text{ kg/m}^3}=3.50\times10^{-4}\text{ m}^3.$$

$m_{\text{lead}}=\rho_{\text{lead}}V_{\text{lead}}=3.95\text{ kg}$.

EVALUATE: The volume of the lead is only 2.9% of the volume of the wood. If the contribution of the volume of the lead to F_{B} is neglected, the calculation is simplified: $\rho_{\text{water}}V_{\text{wood}}g=(m_{\text{wood}}+m_{\text{lead}})g$ and $m_{\text{lead}}=3.6\text{ kg}$. The result of this calculation is in error by about 9%.

12.66. **IDENTIFY:** The fraction *f* of the volume that floats above the fluid is $f=1-\dfrac{\rho}{\rho_{\text{fluid}}}$, where ρ is the average density of the hydrometer (see Problem 12.29). This gives $\rho_{\text{fluid}}=\rho\dfrac{1}{1-f}$.

SET UP: The volume above the surface is *hA*, where *h* is the height of the stem above the surface and $A=0.400\text{ cm}^2$.

EXECUTE: If two fluids are observed to have floating fraction f_1 and f_2, $\rho_2=\rho_1\dfrac{1-f_1}{1-f_2}$. Using

$$f_1=\frac{(8.00\text{ cm})(0.400\text{ cm}^2)}{(13.2\text{ cm}^3)}=0.242,\quad f_2=\frac{(3.20\text{ cm})(0.400\text{ cm}^2)}{(13.2\text{ cm}^3)}=0.097$$ gives

$\rho_{\text{alcohol}}=(0.839)\rho_{\text{water}}=839\text{ kg/m}^3$.

EVALUATE: $\rho_{\text{alcohol}}<\rho_{\text{water}}$. When ρ_{fluid} increases, the fraction *f* of the object's volume that is above the surface increases.

12.67. **(a) IDENTIFY:** Apply Newton's second law to the airship. The buoyancy force is given by Archimedes's principle; the fluid that exerts this force is the air.

SET UP: The free-body diagram for the dirigible is given in Figure 12.67. The lift corresponds to a mass $m_{\text{lift}} = (90\times10^3\ \text{N})/(9.80\ \text{m/s}^2) = 9.184\times10^3\ \text{kg}$. The mass m_{tot} is $9.184\times10^3\ \text{kg}$ plus the mass m_{gas} of the gas that fills the dirigible. *B* is the buoyant force exerted by the air.

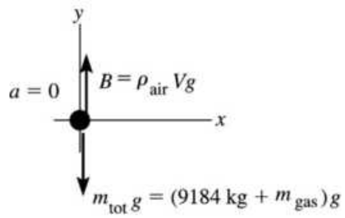


Figure 12.67

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{air}}Vg = (9.184\times10^3\ \text{kg} + m_{\text{gas}})g$

Write m_{gas} in terms of *V*: $m_{\text{gas}} = \rho_{\text{gas}}V$ and let *g* divide out; the equation becomes $\rho_{\text{air}}V = 9.184\times10^3\ \text{kg} + \rho_{\text{gas}}V$.

$$V = \frac{9.184\times10^3\ \text{kg}}{1.20\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3} = 8.27\times10^3\ \text{m}^3$$

EVALUATE: The density of the airship is less than the density of air and the airship is totally submerged in the air, so the buoyancy force exceeds the weight of the airship.

(b) SET UP: Let m_{lift} be the mass that could be lifted.

EXECUTE: From part (a), $m_{\text{lift}} = (\rho_{\text{air}} - \rho_{\text{gas}})V = (1.20\ \text{kg/m}^3 - 0.166\ \text{kg/m}^3)(8.27\times10^3\ \text{m}^3) = 8550\ \text{kg}$.

The lift force is $m_{\text{lift}} = (8550\ \text{kg})(9.80\ \text{m/s}^2) = 83.8\ \text{kN}$.

EVALUATE: The density of helium is less than that of air but greater than that of hydrogen. Helium provides lift, but less lift than hydrogen. Hydrogen is not used because it is highly explosive in air.

12.68. **IDENTIFY:** The buoyant force on the boat is equal to the weight of the water it displaces, by Archimedes's principle.

SET UP: $F_{\text{B}} = \rho_{\text{fluid}}gV_{\text{sub}}$, where V_{sub} is the volume of the object that is below the fluid's surface.

EXECUTE: **(a)** The boat floats, so the buoyant force on it equals the weight of the object: $F_{\text{B}} = mg$. Using Archimedes's principle gives $\rho_{\text{w}}gV = mg$ and $V = \dfrac{m}{\rho_{\text{w}}} = \dfrac{5750\ \text{kg}}{1.00\times10^3\ \text{kg/m}^3} = 5.75\ \text{m}^3$.

(b) $F_{\text{B}} = mg$ and $\rho_{\text{w}}gV_{\text{sub}} = mg$. $V_{\text{sub}} = 0.80V = 4.60\ \text{m}^3$, so the mass of the floating object is $m = \rho_{\text{w}}V_{\text{sub}} = (1.00\times10^3\ \text{kg/m}^3)(4.60\ \text{m}^3) = 4600\ \text{kg}$. He must throw out $5750\ \text{kg} - 4600\ \text{kg} = 1150\ \text{kg}$.

EVALUATE: He must throw out 20% of the boat's mass.

12.69. **IDENTIFY:** Bernoulli's principle will give us the speed with which the acid leaves the hole in the tank, and two-dimensional projectile motion will give us how far the acid travels horizontally after it leaves the tank.

SET UP: Apply Bernoulli's principle, $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, with point 1 at the surface of the acid in the tank and point 2 in the stream as it emerges from the hole. $p_1 = p_2 = p_{\text{air}}$. Since the hole is small the level in the tank drops slowly and $v_1 \approx 0$. After a drop of acid exits the hole the only force on it is gravity and it moves in projectile motion. For the projectile motion take $+y$ downward, so $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$.

EXECUTE: Bernoulli's equation with $p_1 = p_2$ and $v_1 = 0$ gives $v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\ \text{m/s}^2)(0.75\ \text{m})} = 3.83\ \text{m/s}$. Now apply projectile motion. Use the vertical motion to find the time in the air. Combining $v_{0y} = 0$, $a_y = +9.80\ \text{m/s}^2$, $y - y_0 = 1.4$ m.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(1.4\ \text{m})}{9.80\ \text{m/s}^2}} = 0.535$ s. The horizontal distance a drop travels in this time is $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (3.83\ \text{m/s})(0.535\ \text{s}) = 2.05$ m.

EVALUATE: If the depth of acid in the tank is increased, then the velocity of the stream as it emerges from the hole increases and the horizontal range of the stream increases.

12.70. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ gives $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(28.0\ \text{m})} = 23.4$ m/s.

$(\pi D^2/4)v = 0.500\ \text{m/s}^3$. $D = \sqrt{\dfrac{4(0.500\ \text{m/s}^3)}{\pi v}} = \sqrt{\dfrac{4(0.500\ \text{m/s}^3)}{\pi(23.4\ \text{m/s})}} = 0.165\ \text{m} = 16.5$ cm.

(b) $D^2 v$ is constant so if D is twice as great, then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \dfrac{28.0\ \text{m}}{16} = 1.75$ m.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

12.71. **IDENTIFY:** Find the horizontal range x as a function of the height y of the hole above the base of the cylinder. Then find the value of y for which x is a maximum. Once the water leaves the hole it moves in projectile motion.

SET UP: Apply Bernoulli's equation to points 1 and 2, where point 1 is at the surface of the water and point 2 is in the stream as the water leaves the hole. Since the hole is small the volume flow rate out the hole is small and $v_1 \approx 0$. $y_1 - y_2 = H - y$ and $p_1 = p_2 = \rho_{\text{air}}$. For the projectile motion, take $+y$ to be upward; $a_x = 0$ and $a_y = -9.80\ \text{m/s}^2$.

EXECUTE: **(a)** $p_1 + \rho g y_1 + \dfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \dfrac{1}{2}\rho v_2^2$ gives $v_2 = \sqrt{2g(H-y)}$. In the projectile motion, $v_{0y} = 0$ and $y - y_0 = -y$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2y}{g}}$. The horizontal range is $x = v_{0x}t = v_2 t = 2\sqrt{y(H-y)}$. The y that gives maximum x satisfies $\dfrac{dx}{dy} = 0$. $(Hy - y^2)^{-1/2}(H - 2y) = 0$ and $y = H/2$.

(b) $x = 2\sqrt{y(H-y)} = 2\sqrt{(H/2)(H - H/2)} = H$.

EVALUATE: A smaller y gives a larger v_2, but a smaller time in the air after the water leaves the hole.

12.72. **IDENTIFY:** As water flows from the tank, the water level changes. This affects the speed with which the water flows out of the tank and the pressure at the bottom of the tank.

SET UP: Bernoulli's equation, $p_1 + \rho g y_1 + \dfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \dfrac{1}{2}\rho v_2^2$, and the continuity equation, $A_1 v_1 = A_2 v_2$, both apply.

EXECUTE: **(a)** Let point 1 be at the surface of the water in the tank and let point 2 be in the stream of water that is emerging from the tank. $p_1 + \rho g y_1 + \dfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \dfrac{1}{2}\rho v_2^2$. $v_1 = \dfrac{\pi d_2^2}{\pi d_1^2}v_2$, with $d_2 = 0.0200$ m and $d_1 = 2.00$ m. $v_1 \ll v_2$ so the $\dfrac{1}{2}\rho v_1^2$ term can be neglected. $v_2 = \sqrt{\dfrac{2p_0}{\rho} + 2gh}$, where $h = y_1 - y_2$ and $p_0 = p_1 - p_2 = 5.00 \times 10^3$ Pa. Initially $h = h_0 = 0.800$ m and when the tank has drained

$h = 0$. At $t = 0$, $v_2 = \sqrt{\dfrac{2(5.00\times10^3\ \text{Pa})}{1000\ \text{kg/m}^3} + 2(9.8\ \text{m/s}^2)(0.800\ \text{m})} = \sqrt{10 + 15.68}\ \text{m/s} = 5.07\ \text{m/s}$. If the tank is open to the air, $p_0 = 0$ and $v_2 = 3.96\ \text{m/s}$. The ratio is 1.28.

(b) $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1}v_2 = \left(\dfrac{d_2}{d_1}\right)^2\sqrt{\dfrac{2p_0}{\rho} + 2gh} = \left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\sqrt{\dfrac{p_0}{g\rho} + h}$. Separating variables gives

$\dfrac{dh}{\sqrt{\dfrac{p_0}{g\rho} + h}} = -\left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\,dt$. We now must integrate $\displaystyle\int_{h_0}^{0}\frac{dh'}{\sqrt{\dfrac{p_0}{g\rho} + h'}} = -\left(\frac{d_2}{d_1}\right)^2\sqrt{2g}\int_0^t dt'$. To do the left-hand side integral, make the substitution $u = \dfrac{p_0}{g\rho} + h'$, which makes $du = dh'$. The integral is then of the form $\displaystyle\int\frac{du}{u^{1/2}}$, which can be readily integrated using $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1}$. The result is

$2\left(\sqrt{\dfrac{p_0}{g\rho}} - \sqrt{\dfrac{p_0}{g\rho} + h_0}\right) = -\left(\dfrac{d_2}{d_1}\right)^2\sqrt{2g}\,t$. Solving for t gives $t = \left(\dfrac{d_1}{d_2}\right)^2\sqrt{\dfrac{2}{g}}\left(\sqrt{\dfrac{p_0}{g\rho} + h_0} - \sqrt{\dfrac{p_0}{g\rho}}\right)$. Since

$\dfrac{p_0}{g\rho} = \dfrac{5.00\times10^3\ \text{Pa}}{(9.8\ \text{m/s}^2)(1000\ \text{kg/m}^3)} = 0.5102\ \text{m}$, we get

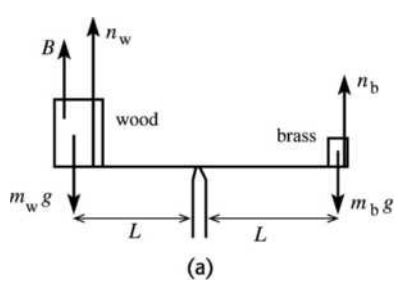
$t = \left(\dfrac{2.00}{0.0200}\right)^2\sqrt{\dfrac{2}{9.8\ \text{m/s}^2}}\left(\sqrt{0.5102\ \text{m} + 0.800\ \text{m}} - \sqrt{0.5102}\ \text{m}\right) = 1.944\times10^3\ \text{s} = 32.4\ \text{min}$. When $p_0 = 0$,

$t = \left(\dfrac{2.00}{0.0200}\right)^2\sqrt{\dfrac{2}{9.8\ \text{m/s}^2}}\left(\sqrt{0.800\ \text{m}}\right) = 4.04\times10^3\ \text{s} = 67.3\ \text{min}$. The ratio is 2.08.

EVALUATE: Both ratios are greater than one because a surface pressure greater than atmospheric pressure causes the water to drain with a greater speed and in a shorter time than if the surface were open to the atmosphere with a pressure of one atmosphere.

12.73. **IDENTIFY:** Apply the second condition of equilibrium to the balance arm and apply the first condition of equilibrium to the block and to the brass mass. The buoyancy force on the wood is given by Archimedes's principle and the buoyancy force on the brass mass is ignored.

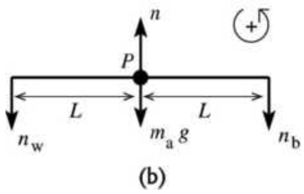
SET UP: The objects and forces are sketched in Figure 12.73a.



The buoyant force on the brass is neglected, but we include the buoyant force B on the block of wood. n_w and n_b are the normal forces exerted by the balance arm on which the objects sit.

Figure 12.73a

The free-body diagram for the balance arm is given in Figure 12.73b.



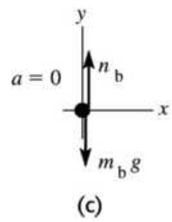
EXECUTE: $\tau_P = 0$

$n_w L - n_b L = 0$

$n_w = n_b$

Figure 12.73b

SET UP: The free-body diagram for the brass mass is given in Figure 12.73c.



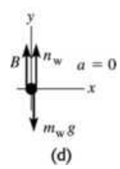
EXECUTE: $\sum F_y = ma_y$

$n_b - m_b g = 0$

$n_b = m_b g$

Figure 12.73c

The free-body diagram for the block of wood is given in Figure 12.73d.



$\sum F_y = ma_y$

$n_w + B - m_w g = 0$

$n_w = m_w g - B$

Figure 12.73d

But $n_b = n_w$ implies $m_b g = m_w g - B$.

And $B = \rho_{air} V_w g = \rho_{air}(m_w/\rho_w)g$, so $m_b g = m_w g - \rho_{air}(m_w/\rho_w)g$.

$$m_w = \frac{m_b}{1 - \rho_{air}/\rho_w} = \frac{0.115\text{ kg}}{1 - ((1.20\text{ kg/m}^3)/(150\text{ kg/m}^3))} = 0.116\text{ kg}.$$

EVALUATE: The mass of the wood is greater than the mass of the brass; the wood is partially supported by the buoyancy force exerted by the air. The buoyancy in air of the brass can be neglected because the density of brass is much more than the density of air; the buoyancy force exerted on the brass by the air is much less than the weight of the brass. The density of the balsa wood is much less than the density of the brass, so the buoyancy force on the balsa wood is not such a small fraction of its weight.

12.74. **IDENTIFY:** $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$. *D* and *E* read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale *D*; a similar statement applies to m_E.

$m_A = 3.50\text{ kg} + 7.50\text{ kg} - 1.00\text{ kg} - 1.80\text{ kg} = 8.20\text{ kg}$.

Forces on *A*: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

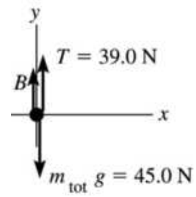
$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\text{ kg} - 3.50\text{ kg}}{3.80 \times 10^{-3}\text{ m}^3} = 1.24 \times 10^3\text{ kg/m}^3$$

(b) *D* reads the mass of *A*: 8.20 kg. *E* reads the total mass of *B* and *C*: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

12.75. **IDENTIFY:** Apply Newton's second law to the ingot. Use the expression for the buoyancy force given by Archimedes's principle to solve for the volume of the ingot. Then use the facts that the total mass is the mass of the gold plus the mass of the aluminum and that the volume of the ingot is the volume of the gold plus the volume of the aluminum.

SET UP: The free-body diagram for the piece of alloy is given in Figure 12.75.



EXECUTE: $\sum F_y = ma_y$

$B + T - m_{\text{tot}}g = 0$

$B = m_{\text{tot}}g - T$

$B = 45.0\text{ N} - 39.0\text{ N} = 6.0\text{ N}$

Figure 12.75

Also, $m_{\text{tot}}g = 45.0\text{ N}$ so $m_{\text{tot}} = 45.0\text{ N}/(9.80\text{ m/s}^2) = 4.59\text{ kg}$.

We can use the known value of the buoyant force to calculate the volume of the object:

$B = \rho_{\text{w}} V_{\text{obj}} g = 6.0\text{ N}$

$$V_{\text{obj}} = \frac{6.0\text{ N}}{\rho_{\text{w}} g} = \frac{6.0\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.122\times10^{-4}\text{ m}^3$$

We know two things:

(1) The mass m_{g} of the gold plus the mass m_{a} of the aluminum must add to m_{tot}: $m_{\text{g}} + m_{\text{a}} = m_{\text{tot}}$

We write this in terms of the volumes V_{g} and V_{a} of the gold and aluminum: $\rho_{\text{g}} V_{\text{g}} + \rho_{\text{a}} V_{\text{a}} = m_{\text{tot}}$

(2) The volumes V_{a} and V_{g} must add to give V_{obj}: $V_{\text{a}} + V_{\text{g}} = V_{\text{obj}}$ so that $V_{\text{a}} = V_{\text{obj}} - V_{\text{g}}$

Use this in the equation in (1) to eliminate V_{a}: $\rho_{\text{g}} V_{\text{g}} + \rho_{\text{a}}(V_{\text{obj}} - V_{\text{g}}) = m_{\text{tot}}$

$$V_{\text{g}} = \frac{m_{\text{tot}} - \rho_{\text{a}} V_{\text{obj}}}{\rho_{\text{g}} - \rho_{\text{a}}} = \frac{4.59\text{ kg} - (2.7\times10^3\text{ kg/m}^3)(6.122\times10^{-4}\text{ m}^3)}{19.3\times10^3\text{ kg/m}^3 - 2.7\times10^3\text{ kg/m}^3} = 1.769\times10^{-4}\text{ m}^3.$$

Then $m_{\text{g}} = \rho_{\text{g}} V_{\text{g}} = (19.3\times10^3\text{ kg/m}^3)(1.769\times10^{-4}\text{ m}^3) = 3.41\text{ kg}$ and the weight of gold is $w_{\text{g}} = m_{\text{g}} g = 33.4\text{ N}$.

EVALUATE: The gold is 29% of the volume but 74% of the mass, since the density of gold is much greater than the density of aluminum.

12.76. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.0\text{ cm})^3 = 7238\text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 76.0% of the ball's volume or $(0.760)(7238\text{ cm}^3) = 5501\text{ cm}^3$. This much water has a mass of $5501\text{ g} = 5.501\text{ kg}$ and weighs $(5.501\text{ kg})(9.80\text{ m/s}^2) = 53.9\text{ N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 53.9 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

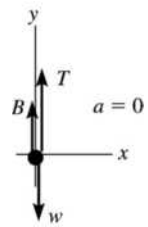
$$(0.240)(7238\text{ cm}^3)(1.00\text{ g/cm}^3) = 1737\text{ g} = 1.737\text{ kg},$$

and its acceleration upon release is thus $a = \frac{F_{\text{net}}}{m} = \frac{53.9\text{ N}}{1.737\text{ kg}} = 31.0\text{ m/s}^2$.

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

12.77. **(a) IDENTIFY:** Apply Newton's second law to the crown. The buoyancy force is given by Archimedes's principle. The target variable is the ratio ρ_c/ρ_w (c = crown, w = water).

SET UP: The free-body diagram for the crown is given in Figure 12.77.



EXECUTE: $\sum F_y = ma_y$

$T + B - w = 0$

$T = fw$

$B = \rho_w V_c g$, where ρ_w = density of water, V_c = volume of crown

Figure 12.77

Then $fw + \rho_w V_c g - w = 0$.

$(1 - f)w = \rho_w V_c g$

Use $w = \rho_c V_c g$, where ρ_c = density of crown.

$(1 - f)\rho_c V_c g = \rho_w V_c g$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$, as was to be shown.

$f \to 0$ gives $\rho_c/\rho_w = 1$ and $T = 0$. These values are consistent. If the density of the crown equals the density of the water, the crown just floats, fully submerged, and the tension should be zero.

When $f \to 1$, $\rho_c >> \rho_w$ and $T = w$. If $\rho_c >> \rho_w$ then B is negligible relative to the weight w of the crown and T should equal w.

(b) "apparent weight" equals T in the rope when the crown is immersed in water. $T = fw$, so need to compute f.

$\rho_c = 19.3\times10^3\ \text{kg/m}^3$; $\rho_w = 1.00\times10^3\ \text{kg/m}^3$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{19.3\times10^3\ \text{kg/m}^3}{1.00\times10^3\ \text{kg/m}^3} = \dfrac{1}{1-f}$

$19.3 = 1/(1-f)$ and $f = 0.9482$

Then $T = fw = (0.9482)(12.9\ \text{N}) = 12.2\ \text{N}$.

(c) Now the density of the crown is very nearly the density of lead;

$\rho_c = 11.3\times10^3\ \text{kg/m}^3$.

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{11.3\times10^3\ \text{kg/m}^3}{1.00\times10^3\ \text{kg/m}^3} = \dfrac{1}{1-f}$

$11.3 = 1/(1-f)$ and $f = 0.9115$

Then $T = fw = (0.9115)(12.9\ \text{N}) = 11.8\ \text{N}$.

EVALUATE: In part (c) the average density of the crown is less than in part (b), so the volume is greater. B is greater and T is less. These measurements can be used to determine if the crown is solid gold, without damaging the crown.

12.78. **IDENTIFY:** Problem 12.77 says $\dfrac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \dfrac{1}{1-f}$, where the apparent weight of the object when it is totally immersed in the fluid is fw.

SET UP: For the object in water, $f_{\text{water}} = w_{\text{water}}/w$ and for the object in the unknown fluid, $f_{\text{fluid}} = w_{\text{fluid}}/w$.

EXECUTE: (a) $\dfrac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \dfrac{w}{w - w_{\text{fluid}}}$, $\dfrac{\rho_{\text{steel}}}{\rho_{\text{water}}} = \dfrac{w}{w - w_{\text{water}}}$. Dividing the second of these by the first gives $\dfrac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \dfrac{w - w_{\text{fluid}}}{w - w_{\text{water}}}$.

(b) When w_{fluid} is greater than w_{water}, the term on the right in the above expression is less than one, indicating that the fluid is less dense than water, and this is consistent with the buoyant force when suspended in liquid being less than that when suspended in water. If the density of the fluid is the same as that of water $w_{\text{fluid}} = w_{\text{water}}$, as expected. Similarly, if w_{fluid} is less than w_{water}, the term on the right in the above expression is greater than one, indicating that the fluid is more dense than water.

(c) Writing the result of part (a) as $\dfrac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \dfrac{1 - f_{\text{fluid}}}{1 - f_{\text{water}}}$, and solving for f_{fluid},

$$f_{\text{fluid}} = 1 - \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}(1 - f_{\text{water}}) = 1 - (1.220)(0.128) = 0.844 = 84.4\%.$$

EVALUATE: Formic acid has density greater than the density of water. When the object is immersed in formic acid the buoyant force is greater and the apparent weight is less than when the object is immersed in water.

12.79. IDENTIFY and **SET UP:** Use Archimedes's principle for B.

(a) $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the total volume of the object.

$V_{\text{tot}} = V_{\text{m}} + V_0$, where V_{m} is the volume of the metal.

EXECUTE: $V_{\text{m}} = w/g\rho_{\text{m}}$ so $V_{\text{tot}} = w/g\rho_{\text{m}} + V_0$

This gives $B = \rho_{\text{water}} g (w/g\rho_{\text{m}} + V_0)$.

Solving for V_0 gives $V_0 = B/(\rho_{\text{water}} g) - w/(\rho_{\text{m}} g)$, as was to be shown.

(b) The expression derived in part (a) gives

$$V_0 = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} - \frac{156\text{ N}}{(8.9\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.52\times 10^{-4}\text{ m}^3$$

$$V_{\text{tot}} = \frac{B}{\rho_{\text{water}} g} = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.04\times 10^{-3}\text{ m}^3 \text{ and}$$

$V_0/V_{\text{tot}} = (2.52\times 10^{-4}\text{ m}^3)/(2.04\times 10^{-3}\text{ m}^3) = 0.124.$

EVALUATE: When $V_0 \to 0$, the object is solid and $V_{\text{obj}} = V_{\text{m}} = w/(\rho_{\text{m}} g)$. For $V_0 = 0$, the result in part (a) gives $B = (w/\rho_{\text{m}})\rho_{\text{water}} = V_{\text{m}}\rho_{\text{water}} g = V_{\text{obj}}\rho_{\text{water}} g$, which agrees with Archimedes's principle. As V_0 increases with the weight kept fixed, the total volume of the object increases and there is an increase in B.

12.80. IDENTIFY: For a floating object the buoyant force equals the weight of the object. Archimedes's principle says the buoyant force equals the weight of fluid displaced by the object. $m = \rho V$.

SET UP: Let d be the depth of the oil layer, h the depth that the cube is submerged in the water and L be the length of a side of the cube.

EXECUTE: (a) Setting the buoyant force equal to the weight and canceling the common factors of g and the cross-sectional area, $(1000)h + (750)d = (550)L$. d, h and L are related by $d + h + 0.35L = L$, so $h = 0.65L - d$. Substitution into the first relation gives $d = L\dfrac{(0.65)(1000) - (550)}{(1000) - (750)} = \dfrac{2L}{5.00} = 0.040\text{ m}$.

(b) The gauge pressure at the lower face must be sufficient to support the block (the oil exerts only sideways forces directly on the block), and $p = \rho_{\text{wood}} g L = (550\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.100\text{ m}) = 539\text{ Pa}$.

EVALUATE: As a check, the gauge pressure, found from the depths and densities of the fluids, is $[(0.040\text{ m})(750\text{ kg/m}^3) + (0.025\text{ m})(1000\text{ kg/m}^3)](9.80\text{ m/s}^2) = 539\text{ Pa}$.

12.81. **IDENTIFY** and **SET UP:** Apply the first condition of equilibrium to the barge plus the anchor. Use Archimedes's principle to relate the weight of the boat and anchor to the amount of water displaced. In both cases the total buoyant force must equal the weight of the barge plus the weight of the anchor. Thus the total amount of water displaced must be the same when the anchor is in the boat as when it is over the side. When the anchor is in the water the barge displaces less water, less by the amount the anchor displaces. Thus the barge rises in the water.

EXECUTE: The volume of the anchor is $V_{\text{anchor}} = m/\rho = (35.0\text{ kg})/(7860\text{ kg/m}^3) = 4.453\times10^{-3}\text{ m}^3$. The barge rises in the water a vertical distance h given by $hA = 4.453\times10^{-3}\text{ m}^3$, where A is the area of the bottom of the barge. $h = (4.453\times10^{-3}\text{ m}^3)/(8.00\text{ m}^2) = 5.57\times10^{-4}\text{ m}$.

EVALUATE: The barge rises a very small amount. The buoyancy force on the barge plus the buoyancy force on the anchor must equal the weight of the barge plus the weight of the anchor. When the anchor is in the water, the buoyancy force on it is less than its weight (the anchor doesn't float on its own), so part of the buoyancy force on the barge is used to help support the anchor. If the rope is cut, the buoyancy force on the barge must equal only the weight of the barge and the barge rises still farther.

12.82. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the barrel, with $+y$ upward. The buoyant force on the barrel is given by Archimedes's principle.

SET UP: $\rho_{\text{av}} = m_{\text{tot}}/V$. An object floats in a fluid if its average density is less than the density of the fluid. The density of seawater is 1030 kg/m^3.

EXECUTE: **(a)** The average density of a filled barrel is

$\dfrac{m_{\text{oil}} + m_{\text{steel}}}{V} = \rho_{\text{oil}} + \dfrac{m_{\text{steel}}}{V} = 750\text{ kg/m}^3 + \dfrac{15.0\text{ kg}}{0.120\text{ m}^3} = 875\text{ kg/m}^3$, which is less than the density of seawater, so the barrel floats.

(b) The fraction above the surface (see Problem 12.29) is

$$1 - \frac{\rho_{\text{av}}}{\rho_{\text{water}}} = 1 - \frac{875\text{ kg/m}^3}{1030\text{ kg/m}^3} = 0.150 = 15.0\%.$$

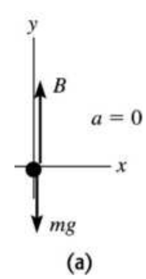
(c) The average density is $910\text{ kg/m}^3 + \dfrac{32.0\text{ kg}}{0.120\text{ m}^3} = 1172\text{ kg/m}^3$, which means the barrel sinks. In order to lift it, a tension

$T = w_{\text{tot}} - B = (1177\text{ kg/m}^3)(0.120\text{ m}^3)(9.80\text{ m/s}^2) - (1030\text{ kg/m}^3)(0.120\text{ m}^3)(9.80\text{ m/s}^2) = 173\text{ N}$ is required.

EVALUATE: When the barrel floats, the buoyant force B equals its weight, w. In part (c) the buoyant force is less than the weight and $T = w - B$.

12.83. **IDENTIFY:** Apply Newton's second law to the block. In part (a), use Archimedes's principle for the buoyancy force. In part (b), use Eq. (12.6) to find the pressure at the lower face of the block and then use Eq. (12.3) to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 12.83a.



EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_{\text{L}} V_{\text{sub}} g = \rho_{\text{B}} V_{\text{obj}} g$

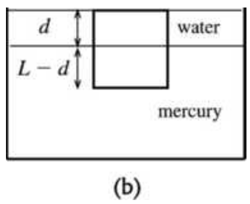
Figure 12.83a

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}} = \rho_{\text{B}}/\rho_{\text{L}}$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}} = 1-\rho_{\text{B}}/\rho_{\text{L}}$.

EVALUATE: If $\rho_{\text{B}} = \rho_{\text{L}}$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth d, as shown in Figure 12.83b.



EXECUTE: $p = p_0 + \rho_{\text{w}}gd + \rho_{\text{L}}g(L-d)$

Applying $\sum F_y = ma_y$ to the block gives

$(p-p_0)A - mg = 0.$

Figure 12.83b

$[\rho_{\text{w}}gd + \rho_{\text{L}}g(L-d)]A = \rho_{\text{B}}LAg$

A and g divide out and $\rho_{\text{w}}d + \rho_{\text{L}}(L-d) = \rho_{\text{B}}L$

$d(\rho_{\text{w}} - \rho_{\text{L}}) = (\rho_{\text{B}} - \rho_{\text{L}})L$

$$d = \left(\frac{\rho_{\text{L}} - \rho_{\text{B}}}{\rho_{\text{L}} - \rho_{\text{w}}}\right)L$$

(c) $d = \left(\dfrac{13.6\times10^3 \text{ kg/m}^3 - 7.8\times10^3 \text{ kg/m}^3}{13.6\times10^3 \text{ kg/m}^3 - 1000 \text{ kg/m}^3}\right)(0.100 \text{ m}) = 0.0460 \text{ m} = 4.60 \text{ cm}$

EVALUATE: In the expression derived in part (b), if $\rho_{\text{B}} = \rho_{\text{L}}$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_{\text{L}} \to \rho_{\text{w}}$ the block continues to float with a fraction $1-\rho_{\text{B}}/\rho_{\text{w}}$ above the water as water is added, and the water never reaches the top of the block $(d \to \infty)$.

12.84. IDENTIFY: For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y = \dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so $\Delta y = \dfrac{\Delta V}{A} = \dfrac{w/(\rho_{\text{water}}g)}{A} = \dfrac{w}{\rho_{\text{water}}gA} = \dfrac{(2.50\times10^6 \text{ N})}{(1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)[(60.0 \text{ m})(20.0 \text{ m})]} = 0.213 \text{ m}.$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 9.00\rho_{\text{water}}$, which gives $\Delta y' = \dfrac{\Delta y}{9}$, and $\Delta y - \Delta y' = \dfrac{8}{9}\Delta y = 0.189 \text{ m}$; the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

12.85. IDENTIFY: Consider the fluid in the horizontal part of the tube. This fluid, with mass ρAl, is subject to a net force due to the pressure difference between the ends of the tube.

SET UP: The difference between the gauge pressures at the bottoms of the ends of the tubes is $\rho g(y_{\text{L}} - y_{\text{R}})$.

EXECUTE: The net force on the horizontal part of the fluid is $\rho g(y_{\text{L}} - y_{\text{R}})A = \rho Ala$, or, $(y_{\text{L}} - y_{\text{R}}) = \dfrac{a}{g}l.$

(b) Again consider the fluid in the horizontal part of the tube. As in part (a), the fluid is accelerating; the center of mass has a radial acceleration of magnitude $a_{\text{rad}} = \omega^2 l/2$, and so the difference in heights between the columns is $(\omega^2 l/2)(l/g) = \omega^2 l^2/2g$. An equivalent way to do part (b) is to break the fluid in the horizontal part of the tube into elements of thickness *dr*; the pressure difference between the sides of this piece is $dp = \rho(\omega^2 r)dr$ and integrating from $r = 0$ to $r = l$ gives $\Delta p = \rho\omega^2 l^2/2$, the same result.

EVALUATE: **(c)** The pressure at the bottom of each arm is proportional to ρ and the mass of fluid in the horizontal portion of the tube is proportional to ρ, so ρ divides out and the results are independent of the density of the fluid. The pressure at the bottom of a vertical arm is independent of the cross-sectional area of the arm. Newton's second law could be applied to a cross-sectional of fluid smaller than that of the tubes. Therefore, the results are independent and of the size and shape of all parts of the tube.

12.86. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to a small fluid element located a distance *r* from the axis.

SET UP: For rotational motion, $a = \omega^2 r$.

EXECUTE: **(a)** The change in pressure with respect to the vertical distance supplies the force necessary to keep a fluid element in vertical equilibrium (opposing the weight). For the rotating fluid, the change in pressure with respect to radius supplies the force necessary to keep a fluid element accelerating toward the axis; specifically, $dp = \frac{\partial p}{\partial r}dr = \rho a\, dr$, and using $a = \omega^2 r$ gives $\frac{\partial p}{\partial r} = \rho\omega^2 r$.

(b) Let the pressure at $y = 0, r = 0$ be p_a (atmospheric pressure); integrating the expression for $\frac{\partial p}{\partial r}$ from part (a) gives $p(r, y = 0) = p_a + \frac{\rho\omega^2}{2}r^2$.

(c) In Eq. (12.5), $p_2 = p_a, p = p_1 = p(r, y = 0)$ as found in part (b), $y_1 = 0$ and $y_2 = h(r)$, the height of the liquid above the $y = 0$ plane. Using the result of part (b) gives $h(r) = \omega^2 r^2/2g$.

EVALUATE: The curvature of the surface increases as the speed of rotation increases.

12.87. **IDENTIFY:** Follow the procedure specified in part (a) and integrate this result for part (b).

SET UP: A rotating particle a distance r' from the rotation axis has inward acceleration $\omega^2 r'$.

EXECUTE: **(a)** The net inward force is $(p + dp)A - pA = Adp$, and the mass of the fluid element is $\rho Adr'$. Using Newton's second law, with the inward radial acceleration of $\omega^2 r'$, gives $dp = \rho\omega^2 r' dr'$.

(b) Integrating the above expression, $\int_{p_0}^{p} dp = \int_{r_0}^{r} \rho\omega^2 r' dr'$ and $p - p_0 = \left(\frac{\rho\omega^2}{2}\right)(r^2 - r_0^2)$, which is the desired result.

(c) The net force on the object must be the same as that on a fluid element of the same shape. Such a fluid element is accelerating inward with an acceleration of magnitude $\omega^2 R_{cm}$, and so the force on the object is $\rho V\omega^2 R_{cm}$.

(d) If $\rho R_{cm} > \rho_{ob} R_{cm\,ob}$, the inward force is greater than that needed to keep the object moving in a circle with radius $R_{cm\,ob}$ at angular frequency ω, and the object moves inward. If $\rho R_{cm} < \rho_{ob} R_{cm\,ob}$, the net force is insufficient to keep the object in the circular motion at that radius, and the object moves outward.

(e) Objects with lower densities will tend to move toward the center, and objects with higher densities will tend to move away from the center.

EVALUATE: The pressure in the fluid increases as the distance *r* from the rotation axis increases.

12.88. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: Let increasing *x* correspond to moving toward the back of the car.

EXECUTE: **(a)** The mass of air in the volume element is $\rho dV = \rho Adx$, and the net force on the element in the forward direction is $(p + dp)A - pA = Adp$. From Newton's second law, $Adp = (\rho Adx)a$, from which $dp = \rho adx$.

(b) With ρ given to be constant, and with $p = p_0$ at $x = 0$, $p = p_0 + \rho ax$.

(c) Using $\rho = 1.2\text{ kg/m}^3$ in the result of part (b) gives

$(1.2\text{ kg/m}^3)(5.0\text{ m/s}^2)(2.5\text{ m}) = 15.0\text{ Pa} = 15\times 10^{-5}\,p_{atm}$, so the fractional pressure difference is negligible.

(d) Following the argument in Section 12.3, the force on the balloon must be the same as the force on the same volume of air; this force is the product of the mass ρV and the acceleration, or $\rho V a$.

(e) The acceleration of the balloon is the force found in part (d) divided by the mass $\rho_{\text{bal}} V$, or $(\rho/\rho_{\text{bal}})a$. The acceleration relative to the car is the difference between this acceleration and the car's acceleration, $a_{\text{rel}} = [(\rho/\rho_{\text{bal}}) - 1]a$.

(f) For a balloon filled with air, $(\rho/\rho_{\text{bal}}) < 1$ (air balloons tend to sink in still air), and so the quantity in square brackets in the result of part (e) is negative; the balloon moves to the back of the car. For a helium balloon, the quantity in square brackets is positive, and the balloon moves to the front of the car.

EVALUATE: The pressure in the air inside the car increases with distance from the windshield toward the rear of the car. This pressure increase is proportional to the acceleration of the car.

12.89. **IDENTIFY:** After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: From Example 12.8, the speed of efflux is $\sqrt{2gh}$.

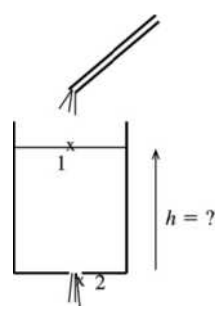
EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H - h, h'(H - h') = (H - h)h,$ and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2$, $h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free fall is greater.

12.90. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.

SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times10^{-4}\ \text{m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.



Let points 1 and 2 be chosen as in Figure 12.90.

Figure 12.90

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40\times10^{-4}\ \text{m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times10^{-4}\ \text{m}^3/\text{s}}{1.50\times10^{-4}\ \text{m}^2} = 1.60\ \text{m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_{\text{a}}$ (air pressure)

Then $p_{\text{a}} + \rho g h = p_{\text{a}} + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60\ \text{m/s})^2/2(9.80\ \text{m/s}^2) = 0.131\ \text{m} = 13.1\ \text{cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

12.91. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 12.8 says the speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\text{ m/s}^2)(8.00\text{ m})}(0.0160\text{ m}^2) = 0.200\text{ m}^3\text{/s}.$

(b) Since p_3 is atmospheric pressure, the gauge pressure at point 2 is

$p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3)$, using the expression for v_3 found above.

Substitution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

12.92. **IDENTIFY:** Apply Bernoulli's equation to the air in the hurricane.

SET UP: For a particle a distance r from the axis, the angular momentum is $L = mvr$.

EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\text{ km/h})\left(\frac{30}{350}\right) = 17\text{ km/h}.$

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\text{ kg/m}^3)((200\text{ km/h})^2 - (17\text{ km/h})^2)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 1.8\times10^3\text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160$ m.

(d) The pressure difference at higher altitudes is even greater.

EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

12.93. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 12.8 shows that the speed of efflux at point D is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at C and D gives that the fluid speed is $\sqrt{8gh_1}$ at C. Applying Bernoulli's equation to points A and C gives that the gauge pressure at C is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at E. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at C is less than the gauge pressure ρgh_1 at the bottom of tank A because of the speed of the fluid at C.

12.94. **IDENTIFY:** Apply Bernoulli's equation to points 1 and 2. Apply $p = p_0 + \rho gh$ to both arms of the U-shaped tube in order to calculate h.

SET UP: The discharge rate is $v_1A_1 = v_2A_2$. The density of mercury is $\rho_m = 13.6\times10^3\text{ kg/m}^3$ and the density of water is $\rho_w = 1.00\times10^3\text{ kg/m}^3$. Let point 1 be where $A_1 = 40.0\times10^{-4}\text{ m}^2$ and point 2 is where $A_2 = 10.0\times10^{-4}\text{ m}^2$. $y_1 = y_2$.

EXECUTE: **(a)** $v_1 = \frac{6.00\times10^{-3}\text{ m}^3\text{/s}}{40.0\times10^{-4}\text{ m}^2} = 1.50\text{ m/s}.$ $v_2 = \frac{6.00\times10^{-3}\text{ m}^3\text{/s}}{10.0\times10^{-4}\text{ m}^2} = 6.00\text{ m/s}$

(b) $p_1 + \rho gy_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \tfrac{1}{2}\rho v_2^2.$

$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) = \tfrac{1}{2}(1000\text{ kg/m}^3)([6.00\text{ m/s}]^2 - [1.50\text{ m/s}]^2) = 1.69\times10^4\text{ Pa}$

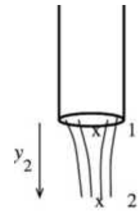
(c) $p_1 + \rho_w gh = p_2 + \rho_m gh$ and

$$h = \frac{p_1 - p_2}{(\rho_m - \rho_w)g} = \frac{1.69\times10^4\text{ Pa}}{(13.6\times10^3\text{ kg/m}^3 - 1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.137\text{ m} = 13.7\text{ cm}.$$

EVALUATE: The pressure in the fluid decreases when the speed of the fluid increases.

12.95. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply Eq. (12.10) to relate the speed to the radius of the stream.

SET UP:



Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 12.95.

Figure 12.95

Consider the free fall of the liquid. Take $+y$ to be downward.

Free fall implies $a_y = g$. v_y is positive, so replace it by the speed v.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1 A_1 = v_2 A_2$

And since $A = \pi r^2$ this becomes $v_1 \pi r_1^2 = v_2 \pi r_2^2$ and $v_2 = v_1 (r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1(r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1 \sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be r and y_2 be y. The equation we have derived then becomes

$r = r_0 \sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$

(b) $v_0 = 1.20$ m/s

We want the value of y that gives $r = \frac{1}{2} r_0$, or $r_0 = 2r$.

The result obtained in part (a) says $r^4(v_0^2 + 2gy) = r_0^4 v_0^2$.

Solving for y gives $y = \dfrac{[(r_0/r)^4 - 1]v_0^2}{2g} = \dfrac{(16-1)(1.20 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 1.10$ m.

EVALUATE: The equation derived in part (a) says that r decreases with distance below the end of the pipe.

12.96. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the rock.

SET UP: In the accelerated frame, all of the quantities that depend on g (weights, buoyant forces, gauge pressures and hence tensions) may be replaced by $g' = g + a$, with the positive direction taken upward.

EXECUTE: **(a)** The volume V of the rock is

$$V = \frac{B}{\rho_{\text{water}} g} = \frac{w - T}{\rho_{\text{water}} g} = \frac{((3.00 \text{ kg})(9.80 \text{ m/s}^2) - 21.0 \text{ N})}{(1.00\times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 8.57\times 10^{-4} \text{ m}^3.$$

(b) The tension is $T = mg' - B' = (m - \rho V)g' = T_0 \dfrac{g'}{g}$, where $T_0 = 21.0$N. $g' = g + a$. For $a = 2.50$ m/s^2,

$T = (21.0 \text{ N})\dfrac{9.80 + 2.50}{9.80} = 26.4$ N.

(c) For $a = -2.50$ m/s^2, $T = (21.0 \text{ N})\dfrac{9.80 - 2.50}{9.80} = 15.6$ N.

(d) If $a = -g$, $g' = 0$ and $T = 0$.

EVALUATE: The acceleration of the water alters the buoyant force it exerts.

12.97. **IDENTIFY:** The sum of the vertical forces on the object must be zero.

SET UP: The depth of the bottom of the styrofoam is not given; let this depth be h_0. Denote the length of the piece of foam by L and the length of the two sides by l. The volume of the object is $\frac{1}{2}l^2L$.

EXECUTE: **(a)** The tension in the cord plus the weight must be equal to the buoyant force, so $T = Vg(\rho_{\text{water}} - \rho_{\text{foam}}) = \frac{1}{2}(0.20\text{ m})^2(0.50\text{ m})(9.80\text{ m/s}^2)(1000\text{ kg/m}^3 - 180\text{ kg/m}^3) = 80.4\text{ N}$.

(b) The pressure force on the bottom of the foam is $(p_0 + \rho g h_0)L\left(\sqrt{2}l\right)$ and is directed up. The pressure on each side is not constant; the force can be found by integrating, or using the results of Problem 12.53 or Problem 12.55. Although these problems found forces on vertical surfaces, the result that the force is the product of the average pressure and the area is valid. The average pressure is $p_0 + \rho g(h_0 - (l/(2\sqrt{2})))$, and the force on one side has magnitude $(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll$ and is directed perpendicular to the side, at an angle of $45.0°$ from the vertical. The force on the other side has the same magnitude, but has a horizontal component that is opposite that of the other side. The horizontal component of the net buoyant force is zero, and the vertical component is

$B = (p_0 + \rho g h_0)Ll\sqrt{2} - 2(\cos 45.0°)(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll = \rho g \dfrac{Ll^2}{2}$, the weight of the water displaced.

EVALUATE: The density of the object is less than the density of water, so if the cord were cut the object would float. When the object is fully submerged, the upward buoyant force is greater than its weight and the cord must pull downward on the object to hold it beneath the surface.

12.98. **IDENTIFY:** Apply Bernoulli's equation to the fluid in the siphon.

SET UP: Example 12.8 shows that the efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upward before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H + h)$, where the assumption that the cross-sectional area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h$.

EVALUATE: The analysis shows that $H + h < \dfrac{p_a}{\rho g}$, so there is also a limitation on $H + h$. For water and normal atmospheric pressure, $\dfrac{p_a}{\rho g} = 10.3\text{ m}$.